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8. *A Note on the Theory of the Electromagnetic Seismograph.*

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Abstract

An attempt was made to derive the expressions for the theory of the electromagnetic seismograph in a more convenient form for practical use.

The magnification of the electromagnetic seismograph of the moving coil type was found to be expressed in the following form.

$$\frac{y_m}{x_m}=Q\cdot f$$

$$Q=MH\frac{2\pi}{T_1}\left(\frac{2\pi}{T_2}\right)^2 S_1S_2\mu_1\frac{1}{Z_{11}}$$

where y_m is the amplitude on the recording drum, x_m the maximum amplitude of the simple harmonic motion of the ground, M the mass of the pendulum of the transducer, H the distance between the center of gravity and the axis of rotation of the pendulum, T_1 the transducer period, T_2 the galvanometer period, S_1 the sensitivity of the transducer when regarded as a sort of galvanometer (angular deflection of the pendulum when unit direct current is given to the transducer coil), and S_2 the sensitivity of the galvanometer (displacement of the light spot on the recording drum when unit direct current is given to the galvanometer coil). μ_1, the attenuation factor newly defined here, expresses the ratio of the current through the galvanometer coil and the current through the transducer coil when the galvanometer is cramped, and Z_{11} is the resistance of the transducer coil plus its external damping resistance. f, the period response of the seismograph, is given as a function of h_1, h_2, T_2/T_1, σ, and T_ω/T_1, where h_1 is the damping constant of the transducer, h_2 the damping constant of the galvanometer, σ the coupling factor, and T_ω the period of the ground motion.

The coupling factor σ was found to be expressed in the following form.

$$\sigma^2=\frac{(h_1-h_{01})(h_2-h_{02})}{h_1h_2}\frac{Z_{22}}{Z_{11}}\mu_1^2$$

where h_1 is the damping constant of the transducer while h_{01} is the same constant when the circuit is open and hence no current passes through the coil. h_2 and h_{02} are the analogous quantities for the galvanometer. Z_{11} is the resistance of the transducer coil plus its external damping resistance. Z_{22} is the analogous quantity for the galvanometer.

It should be noticed that the above expression for σ does not contain the mass or the moment of inertia of the pendulum as well as of the galvanometer. This means that we can never avoid the reaction of the galvanometer against the transducer simply by using a heavy mass as the pendulum. When no additional damping device is attached to the pendulum, the only way to make σ small is to reduce the attenuation factor μ_1. If the transducer has no additional damping device and the damping is caused simply by the current through the coil, then $h_1 \gg h_{01}$, and for the galvanometer also $h_2 \gg h_{02}$. The seismograph is designed, in many cases, so that $Z_{11} \approx Z_{22}$. In such a case, σ is approximately equal to μ_1.

Numerical calculation for the period response of the seismograph, of which the result is indicated in Fig. 3~20, shows that if we make $\sigma < 1/3$ the reaction of the galvanometer will be neglected even in the case when T_1 coincides with T_2 and the reaction of the galvanometer is expected to be most effective.

1. Introduction

Since the first investigation of Galitzin, the theory of the electromagnetic seismograph of the moving-coil type has been developed by Wenner, Coulomb and Grenet, Schmerwitz, Eaton and other investigators. The historical outline of the development is stated in Eaton's paper[1].

Tazime also investigated theoretically the coupling factor, which is a standard of the reaction of the galvanometer on the pendulum, replacing mechanical quantities by electrical ones[2]. Although the theory of the electromagnetic seismograph has been almost completed by these investigators, the author faced the necessity of writing the mathematical expression in a more understandable form for the convenience of designing and calibrating the seismograph. When the electromagnetic seismograph is treated mathematically, we used to consider simply the shunt resistance between the transducer and the galvanometer. However, as Neumann[3] pointed out, it is necessary in routine seismological

1) EATON, J. P., *Bul. Seism. Soc. America*, **47** (1957), 37–75.
2) TAZIME, K., *Jour. Fac. Sci. Hokkaido Univ.*, **[vii]**, **1** (1957), 55–67.
3) NEUMANN, F., *Trans. American Geophys. Union*, **37** (1956), 483–490.

observation to control the magnification of the seismograph from time to time, because the microseisms predominate in Winter while they become quiet in Summer except when a typhoon or cyclone appears, and therefore the seismograph must have an attenuation circuit reducing the magnification to a desired value keeping the damping constants of both the transducer and the galvanometer unchanged. Then it would be more practical to consider an attenuation circuit rather than the simple shunt resistance at the start of the mathematical treatment of the electromagnetic seismograph. In this paper, the attenuation factor was defined as the ratio of the current through the galvanometer coil and the current through the transducer coil when the electromotive force is given simply to the transducer coil. Introducing such a new quantity, and guided by the previous investigators, an attempt was made to find out the mathematical formulae in convenient form for the design and calibration of the electromagnetic seismograph.

2. Formulae for computing the magnification of the elertromagnetic seismograph

We consider the attenuation circuit of T-type inserted between the transducer and the galvanometer as shown in Fig. 1. The circuit diagram shows that

R_1 is the resistance of the transducer coil;
I_1 is the current through the transducer coil;
R_2 is the resistance of the galvanometer;
I_2 is the current through the galvanometer coil;
X_1, X_2, and X_3 are the resistances of the T-type attenuator;

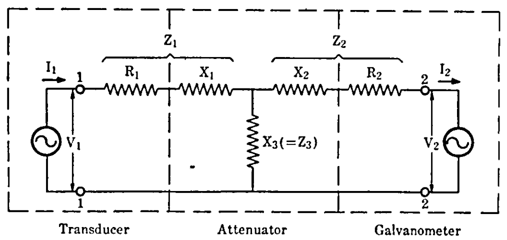


Fig. 1.

V_1 is the electromotive force induced in the transducer coil; and
V_2 is the electromotive force induced in the galvanometer coil.
V_1 is given by

$$V_1 = G\frac{d\theta}{dt} \tag{1}$$

where

θ is the angular deflection of the pendulum of the transducer;
t is the time; and
G is the electrodynamical constant of the transducer.

For the arrangement as shown in Fig. 2, G is the circumference times the number of turns of the coil, times the field strength of the magnet, times the distance from the coil and the axis of rotation of the pendulum. If the galvanometer is cramped and no electromotive force is induced in the galvanometer coil, then

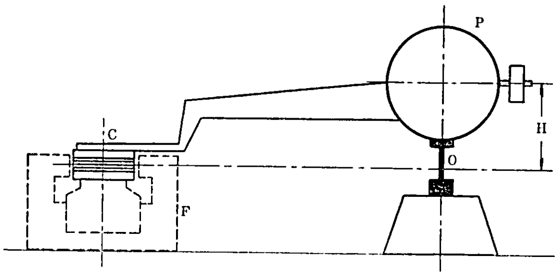


Fig. 2.

$$(I_1)_{V_2=0}=\frac{V_1}{Z_{11}}=\frac{G}{Z_{11}}\frac{d\theta}{dt} \tag{2}$$

where

Z_{11} is the resistance measured from the terminals 11 when the terminals 22 are shorted and corresponds to the resistance of the transducer coil plus its external damping resistance.

Then the current passing through the galvanometer under the same conditions is given by

$$(I_2)_{V_2=0}=\frac{\mu_1 G}{Z_{11}}\frac{d\theta}{dt} \tag{3}$$

where μ_1 is the attenuation factor defined as in the following.

$$\mu_1=\left(\frac{I_2}{I_1}\right)_{V_2=0} \tag{4}$$

The suffix $V_2=0$ in the above expressions means that the terminals 22 in the diagram are shorted.

When the galvanometer is not cramped the motion of the coil will generates the electromotive force which is given by

$$V_2 = -g\frac{d\phi}{dt} \tag{5}$$

where

ϕ is the angular deflection of the galvanometer; and

g is the electrodynamical constant of the galvanometer and is given by the product of the area, the number of turns of the coil, and the field strength of the magnet.

If the transducer coil is cramped and no electromotive force is induced in it, then the current through the galvanometer coil is given by

$$(I_2)_{V_1=0} = \frac{V_2}{Z_{22}} = -\frac{g}{Z_{22}}\frac{d\phi}{dt} \tag{6}$$

where

Z_{22} is the resistance measured from the terminals 22 when the terminals 11 are shorted and corresponds to the resistance of the galvanometer coil plus its external damping resistance.

The suffix $V_1=0$ in the above expression means that the terminals 11 are shorted. Then the current passing through the transducer coil under the same condition is given by

$$(I_1)_{V_1=0} = -\frac{\mu_2 g}{Z_{22}}\frac{d\phi}{dt} \tag{7}$$

where μ_2 is defined as

$$\mu_2 = \left(\frac{I_1}{I_2}\right)_{V_1=0} \tag{8}$$

μ_2 is a sort of attenuation factor seen from the side of the galvanometer.

When both the transducer and the galvanometer are not cramped, the current passing through both the coils is therefore given by

$$\left.\begin{aligned} I_1 &= (I_1)_{V_2=0} + (I_1)_{V_1=0} \\ &= \frac{G}{Z_{11}}\frac{d\theta}{dt} - \frac{\mu_2 g}{Z_{22}}\frac{d\phi}{dt} \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} I_2 &= (I_2)_{V_2=0} + (I_2)_{V_1=0} \\ &= \frac{\mu_1 G}{Z_{11}}\frac{d\theta}{dt} - \frac{g}{Z_{22}}\frac{d\phi}{dt} \end{aligned}\right\} \tag{10}$$

The equation of motion for the transducer and the galvanometer are written according to Wenner's notations as follows:

$$K\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+U\theta=-MH\frac{d^2x}{dt^2}-GI_1\,, \tag{11}$$

$$k\frac{d^2\phi}{dt^2}+d\,\frac{d\phi}{dt}+u\phi=gI_2 \tag{12}$$

where

K is the moment of inertia of the pendulum about the axis of the rotation;
D is the coefficient of the damping of the pendulum, including all forms of damping when the electric circuit is open;
U is the coefficient of the restitutive force;
M is the mass of the pendulum;
H is the distance between the center of gravity and the axis of rotation of the pendulum;
x is the displacement of the ground;
k is the moment of inertia of the moving system of the galvanometer;
d is the coefficient of the damping of the galvanometer with the circuit open; and
u is the coefficient of the restitutive force of the galvanometer.

Putting (9) into (11), and (10) into (12), we obtain

$$K\frac{d^2\theta}{dt^2}+\left[D+\frac{G^2}{Z_{11}}\right]\frac{d\theta}{dt}+U\theta=-MH\frac{d^2x}{dt^2}+\frac{\mu_2 Gg}{Z_{22}}\,\frac{d\phi}{dt} \tag{13}$$

$$k\frac{d^2\phi}{dt^2}+\left[d+\frac{g^2}{Z_{22}}\right]\frac{d\phi}{dt}+u\phi=\frac{\mu_1 Gg}{Z_{11}}\,\frac{d\theta}{dt} \tag{14}$$

or

$$\frac{d^2\theta}{dt^2}+2\varepsilon_1\frac{d\theta}{dt}+n_1^2\theta=-\frac{MH}{K}\,\frac{d^2x}{dt^2}+2\sigma_1\varepsilon_1\frac{d\phi}{dt} \tag{15}$$

$$\frac{d^2\phi}{dt^2}+2\varepsilon_2\frac{d\phi}{dt}+n_2^2\phi=2\sigma_2\varepsilon_2\frac{d\theta}{dt} \tag{16}$$

where

$$\left.\begin{aligned} 2\varepsilon_1 &= \frac{D}{K} + \frac{G^2}{KZ_{11}} \\ 2\varepsilon_2 &= \frac{d}{k} + \frac{g^2}{kZ_{22}} \\ 2\sigma_1\varepsilon_1 &= \frac{\mu_2 Gg}{KZ_{22}} \\ 2\sigma_2\varepsilon_2 &= \frac{\mu_1 Gg}{kZ_{11}} \end{aligned}\right\} \tag{17}$$

We obtain the following equation, eliminating θ from (15) and (16).

$$\frac{d^4\phi}{dt^4} + m\frac{d^3\phi}{dt^3} + n\frac{d^2\phi}{dt^2} + o\frac{d\phi}{dt} + p\phi = -q\frac{d^3x}{dt^3} \tag{18}$$

where

$$\left.\begin{aligned} m &= 2(\varepsilon_1 + \varepsilon_2) \\ n &= n_1^2 + n_2^2 + 4\varepsilon_1\varepsilon_2(1-\sigma^2) \\ o &= 2(\varepsilon_1 n_2^2 + \varepsilon_2 n_1^2) \\ p &= n_1^2 n_2^2 \\ q &= \frac{MHGg\mu_1}{KkZ_{11}} \end{aligned}\right\} \tag{19}$$

and

$$\sigma^2 = \sigma_1\sigma_2 \tag{20}$$

σ, which is called the coupling factor, expresses the largeness of the reaction of the galvanometer.

If the motion of the ground is assumed to be a simple harmonic, with amplitude x_m, angular velocity ω, and phase angle α,

$$x = x_m e^{j\alpha} e^{j\omega t} \tag{21}$$

where j is $\sqrt{-1}$. If we limit the case to the stationary state, we can put

$$\phi = \phi_m e^{j\beta} e^{j\omega t} \tag{22}$$

where ϕ_m is the amplitude in angle, and β the phase angle of the motion of the galvanometer. Putting (21) and (22) into (18), we obtain

$$(\omega^4-j\omega^3m-\omega^2n+j\omega o+p)\phi_m e^{j\beta}e^{j\omega t}=j\omega^3 q x_m e^{j\alpha}e^{j\omega t} \tag{23}$$

Hence

$$\frac{\phi_m}{x_m}e^{j(\beta-\alpha)}=\frac{j\omega^3 q}{\omega^4-j\omega^3m-\omega^2n+j\omega o+p} \tag{24}$$

The right hand side of equation (24) is of complex form and the absolute value gives the amplitude ratio ϕ_m/x_m, and the quotient, the imaginary part divided by the real part, gives tan $(\beta-\alpha)$, namely,

$$\frac{\phi_m}{x_m}=\frac{q\omega^3}{\sqrt{(\omega^4-\omega^2n+p)^2+(-\omega^3m+\omega_0)^2}}$$

$$=\frac{q\omega^3}{\sqrt{[\omega^4-\{n_1^2+n_2^2+4\varepsilon_1\varepsilon_2(1-\sigma^2)\}\omega^2+n_1^2n_2^2]^2+[-2(\varepsilon_1+\varepsilon_2)\omega^3+2(\varepsilon_1n_2^2+\varepsilon_2n_1^2)\omega]^2}} \tag{25}$$

$$\tan(\beta-\alpha)=\frac{\omega^4-\omega^2n+p}{-\omega^3m+\omega o}$$

$$=\frac{\omega^4-\{n_1^2+n_2^2+4\varepsilon_1\varepsilon_2(1-\sigma^2)\}\omega^2+n_1^2n_2^2}{-2(\varepsilon_1+\varepsilon_2)\omega^3+2(\varepsilon_1n_2^2+\varepsilon_2n_1^2)\omega} \tag{26}$$

The equations (25) and (26) can be written in the following form.

$$\frac{\phi_m}{x_m}=\frac{q}{n_1}f(h_1, h_2, \nu, \sigma, u_1) \tag{27}$$

$$f(h_1, h_2, \nu, \sigma, u_1)$$

$$=\frac{u_1}{\sqrt{\left[1-\left\{\left(1+\frac{1}{\nu^2}\right)+4h_1h_2\frac{1}{\nu}(1-\sigma^2)\right\}u_1^2+\frac{1}{\nu^2}u_1^4\right]^2+\left[-2\left(h_1+\frac{1}{\nu}h_2\right)u_1+2\left(\frac{1}{\nu}h_1+h_2\right)\frac{1}{\nu}u_1^3\right]^2}} \tag{28}$$

$$\tan(\beta-\alpha)=\frac{1-\left\{\left(1+\frac{1}{\nu^2}\right)+4h_1h_2\frac{1}{\nu}(1-\sigma^2)\right\}u_1^2+\frac{1}{\nu^2}u_1^4}{-2\left(h_1+\frac{1}{\nu}h_2\right)u_1+2\left(\frac{1}{\nu}h_1+h_2\right)\frac{1}{\nu}u_1^3} \tag{29}$$

where

$h_1=\frac{\varepsilon_1}{n_1}$ (damping constant of the transducer)

$h_2=\frac{\varepsilon_2}{n_2}$ (damping constant of the galvanometer)

$\nu=\frac{n_1}{n_2}=\frac{T_2}{T_1}$

$u_1=\frac{n_1}{\omega}=\frac{T_\omega}{T_1}$

T_ω=period of the ground motion
T_1=period of the transducer
T_2=period of the galvanometer

If we consider the deflection of the light spot on the recording drum as in the usual case,

$$y_m=L\phi_m \tag{30}$$

where

y_m is the amplitude on the recording drum;
L is the length of optical lever, that is, twice the distance between the mirror of galvanometer and the recording drum.

If we use y_m in place of ϕ_m, the equation (27) can be written as follows:

$$\left.\begin{aligned} \frac{y_m}{x_m}&=Q\cdot f(h_1, h_2, \nu, \sigma, u_1) \\ Q&=\frac{qL}{n_1} \end{aligned}\right\} \tag{31}$$

Here f is the amplitude characteristics and $\beta-\alpha$ is the phase characteristics of the seismograph and both of them are the function of h_1, h_2, ν, σ, and u_1. y_m/x_m is the actual magnification of the seismograph.

Next the method of obtaining the quantity qL/n_1 will be examined. We can regard the transducer as a sort of galvanometer because if the electric current is given to the coil of the transducer the pendulum is deflected. We can measure the deflection of the pendulum just like a galvanometer if we attach a mirror near the axis of rotation of the pendulum and put a lamp scale at a suitable distance. If the deflection angle of the pendulum is θ when the direct current I is given to the coil,

$$n_1^2\theta = \frac{G}{K} I \tag{32}$$

G/n_1^2K is therefore the deflection angle of the pendulum when unit direct current passes through the transducer coil.

If the deflection angle of the galvanometer is ϕ when the direct current i passes through the galvanometer coil,

$$n_2^2\phi = \frac{g}{k} i \tag{33}$$

If we denote the displacement of the light spot on the recording drum by y, since $y=\phi L$,

$$n_2^2 y = \frac{gL}{k} i \tag{34}$$

gL/n_2^2k is therefore the displacement of the light spot on the recording drum when unit direct current passes through the galvanometer coil. The ampere sensitivity of a galvanometer in ordinary use is just the inverse of gL/n_2^2k. If we use such measurable quantities, Q can be written in the following form:

$$\left.\begin{aligned} Q &= \frac{MHGg\mu_1 L}{KkZ_{11}n_1} \\ &= MHn_1n_2^2 \frac{G}{n_1^2K} \frac{gL}{n_2^2k} \mu_1 \frac{1}{Z_{11}} \end{aligned}\right\} \tag{35}$$

$$\left.\begin{aligned} &= MH \frac{2\pi}{T_1}\left(\frac{2\pi}{T_2}\right)^2 S_1S_2\mu_1 \frac{1}{Z_{11}} \\ S_1 &= \frac{G}{n_1^2K} \\ S_2 &= \frac{gL}{n_2^2k} \end{aligned}\right\} \tag{36}$$

Since μ_1 and Z_{11} can be computed if the values of resistance of the electric circuit are known, all terms of the right hand side of equation (36) are measurable quantities. Since Q is proportional to μ_1, we can easily reduce to magnification of the seismograph to a desired value by reducing the value of μ_1 without changing any other state.

3. Expression for σ

From equations (17) and (20),

$$\left.\begin{aligned}\sigma^2&=\sigma_1\sigma_2\\&=\frac{(Gg)^2\mu_1\mu_2}{2\varepsilon_1 2\varepsilon_2 KkZ_{11}Z_{22}}\end{aligned}\right\} \tag{37}$$

From equation (17),

$$\left.\begin{aligned}h_1&=\frac{\varepsilon_1}{n_1}\\&=h_{01}+\frac{G^2}{2n_1KZ_{11}}\end{aligned}\right\} \tag{38}$$

$$\left.\begin{aligned}h_2&=\frac{\varepsilon_2}{n_2}\\&=h_{02}+\frac{g^2}{2n_2kZ_{22}}\end{aligned}\right\} \tag{39}$$

where

$$h_{01}=\frac{D}{2n_1K}$$

$$h_{02}=\frac{d}{2n_2k}$$

h_{01} is the damping constant of the transducer when the circuit is open and no current passes through the coil. h_{02} is the analogous quantity to the galvanometer. From (38) and (39),

$$\frac{G^2}{2n_1KZ_{11}}=h_1-h_{01} \tag{40}$$

$$\frac{g^2}{2n_2kZ_{22}}=h_2-h_{02} \tag{41}$$

Therefore

$$\left.\begin{aligned}\sigma^2&=\frac{n_1}{\varepsilon_1}\frac{n_2}{\varepsilon_2}\frac{G^2}{2n_1KZ_{11}}\frac{g^2}{2n_2kZ_{22}}\mu_1\mu_2\\&=\frac{(h_1-h_{01})(h_2-h_{02})}{h_1h_2}\mu_1\mu_2\end{aligned}\right\} \tag{42}$$

The last equation shows that σ is a dimensionless quantity and $0 \leqq \sigma \leqq 1$ because $0 \leqq \mu_1 \leqq 1$ and $0 \leqq \mu_2 \leqq 1$.

If we regard the circuit as a four-terminal network with terminals 11 and 22 as shown in Fig. 1 and denote its four parameters by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

then we have the well-known formulae written as follows.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \tag{43}$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}, \tag{44}$$

and

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1 . \tag{45}$$

Therefore

$$\left.\begin{aligned} \mu_1 &= \left(\frac{I_2}{I_1}\right)_{V_2=0} = \frac{1}{D} \\ \mu_2 &= \left(\frac{I_1}{I_2}\right)_{V_1=0} = \frac{1}{A} \\ Z_{11} &= \left(\frac{V_1}{I_1}\right)_{V_2=0} = \frac{B}{D} \\ Z_{22} &= \left(\frac{V_2}{I_2}\right)_{V_1=0} = \frac{B}{A} \end{aligned}\right\} . \tag{46}$$

Hence

$$\frac{\mu_2}{\mu_1} = \frac{D}{A} = \frac{Z_{22}}{Z_{11}} . \tag{47}$$

Hence

$$\mu_2 = \frac{\mu_1 Z_{22}}{Z_{11}} . \tag{48}$$

Equation (42) can be therefore written as follows.

$$\sigma^2 = \frac{(h_1 - h_{01})(h_2 - h_{02})}{h_1 h_2} \frac{Z_{22}}{Z_{11}} \mu_1^2 \,. \tag{49}$$

The interesting fact is that the above expression for σ does not contain the inertia term of the pendulum as well as of the galvanometer. This means that σ is independent of the inertia of the pendulum and the galvanometer but depends on the damping constants and the constants

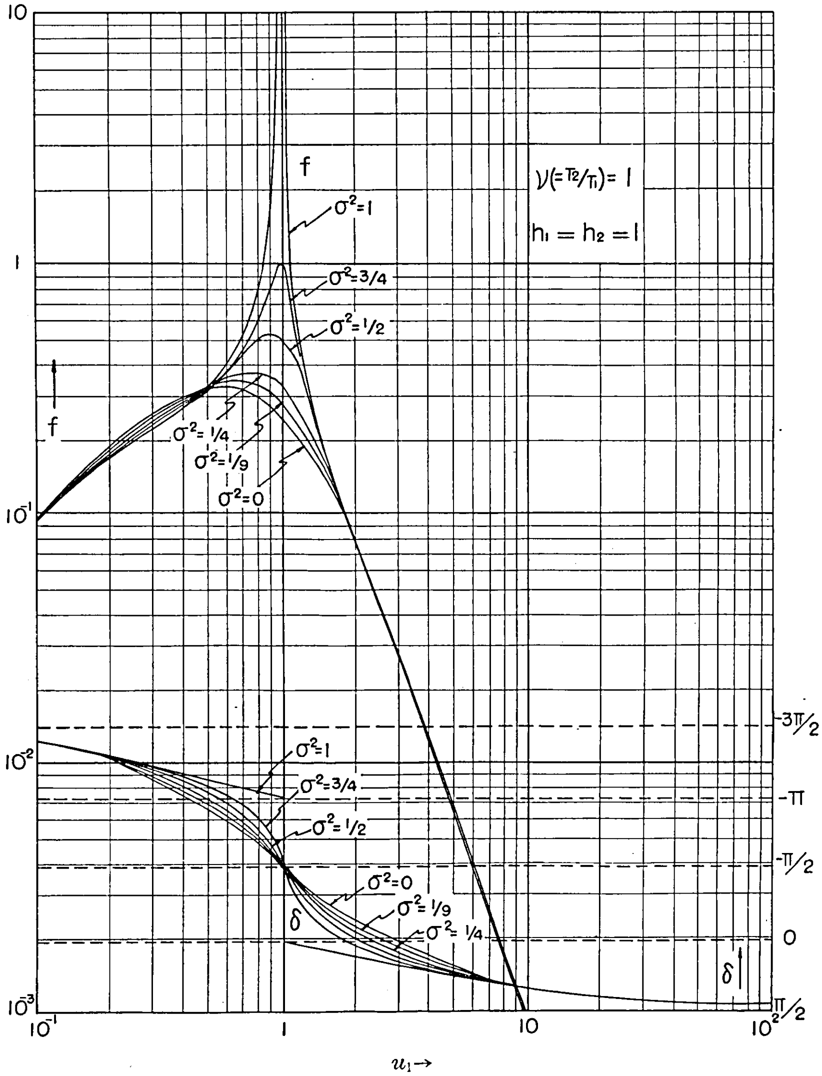
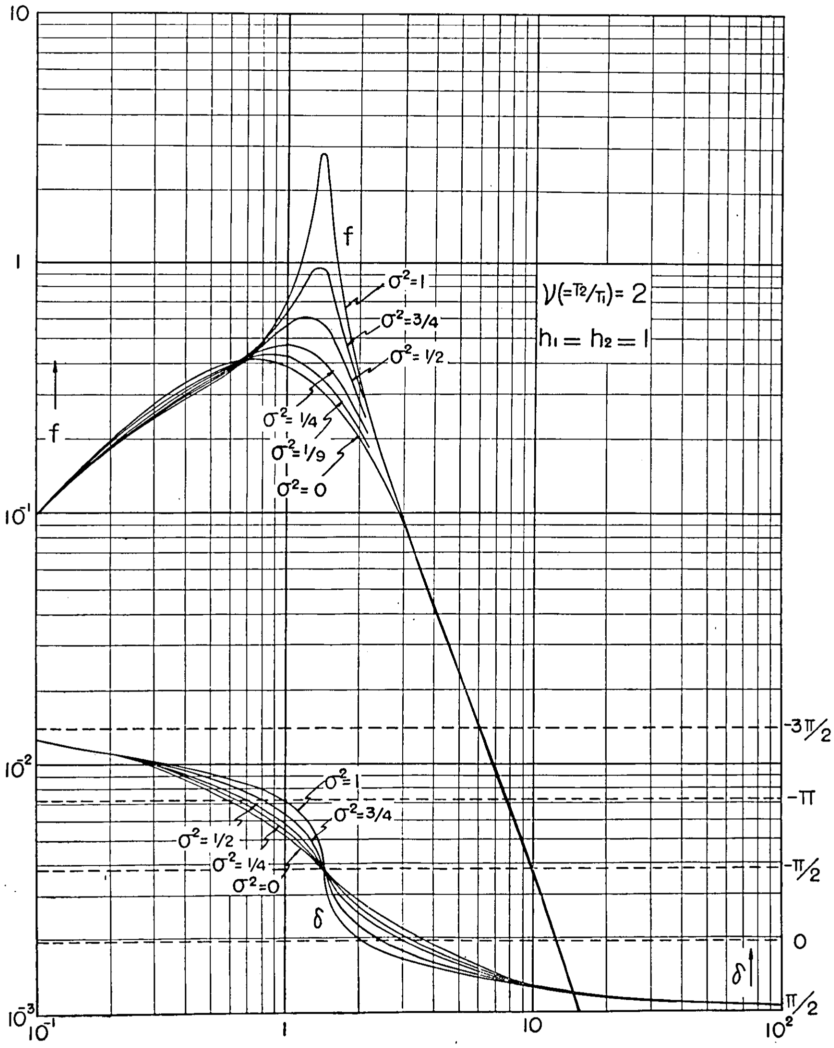


Fig. 3.

of the electrical circuit. We can never avoid the reaction of the galvanometer on the pendulum simply by adopting a heavy mass as the pendulum. If any additional damping device is not provided for the pendulum, the only way to avoid the reaction of the galvanometer is to decrease the attenuation factor of the circuit.

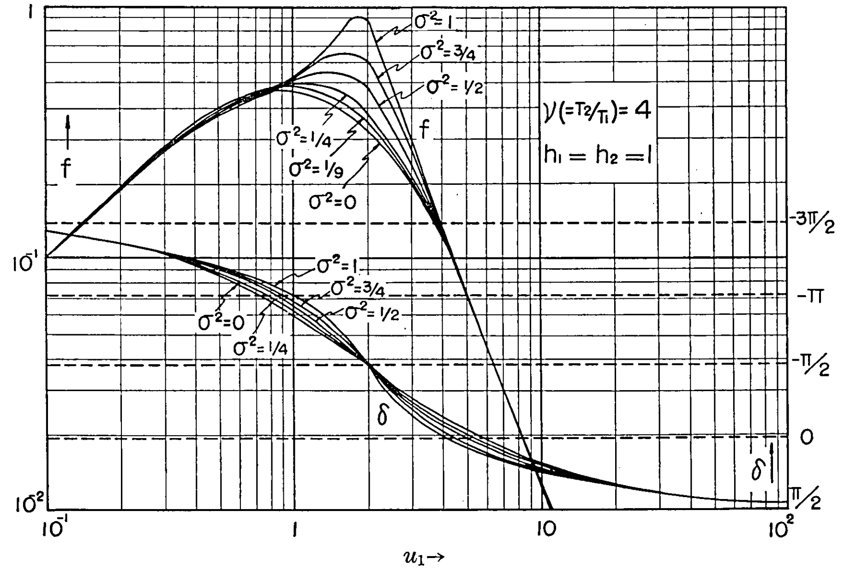
4. Effect of σ on the period responce

Some examples for the period response of the electromagnetic seismo-



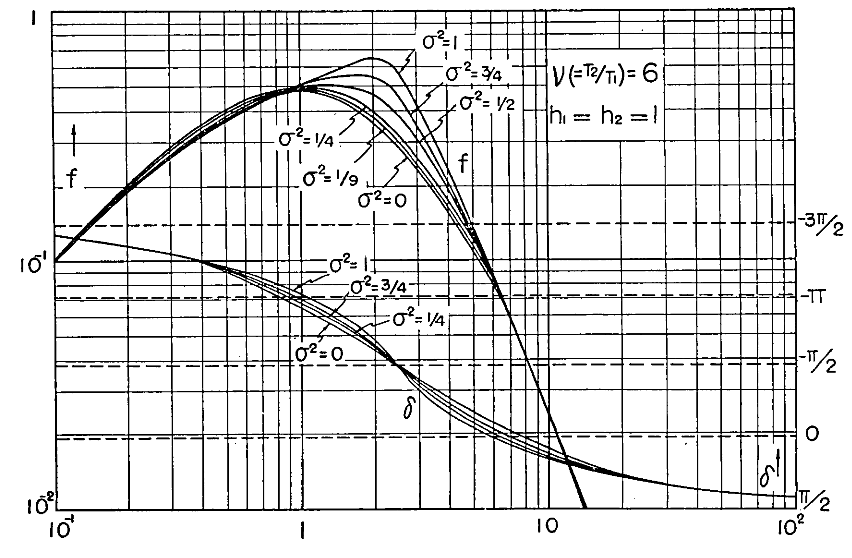
$u_1 \rightarrow$

Fig. 4.



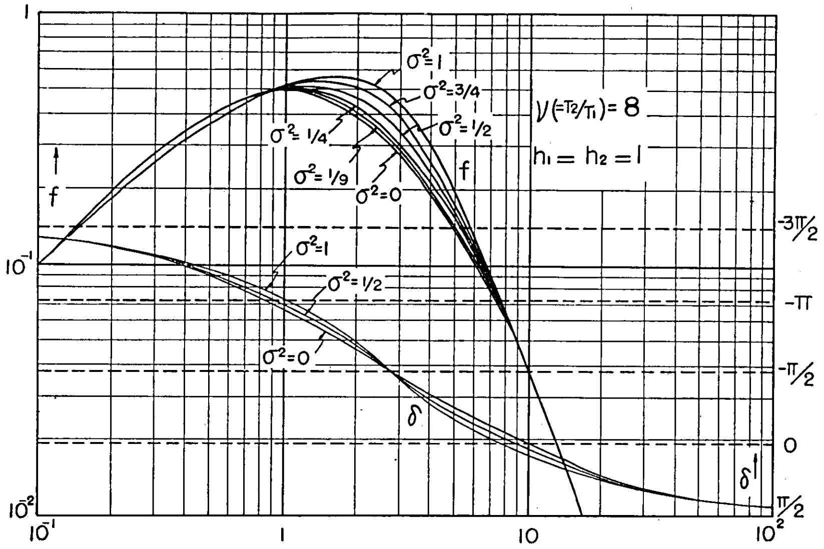
σ²=1
σ²=3/4
σ²=1/2
ν(=T2/T1)=4
h1 = h2 = 1
σ²=1/4
σ²=1/9
σ²=0
f
δ
-3π/2
-π
-π/2
0
π/2
1
10⁻¹
10⁻²
10
10²
u1→

Fig. 5.



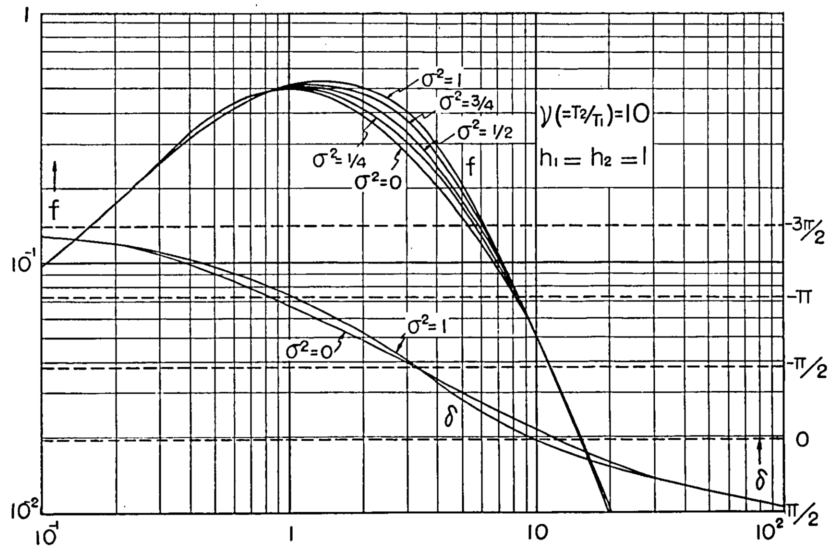
σ²=1
σ²=3/4
σ²=1/2
ν(=T2/T1)=6
h1 = h2 = 1
σ²=1/4
σ²=1/9
σ²=0
f
δ
-3π/2
-π
-π/2
0
π/2
1
10⁻¹
10⁻²
10
10²
u1→

Fig. 6.



$u_1 \rightarrow$

Fig. 7.



$u_1 \rightarrow$

Fig. 8.

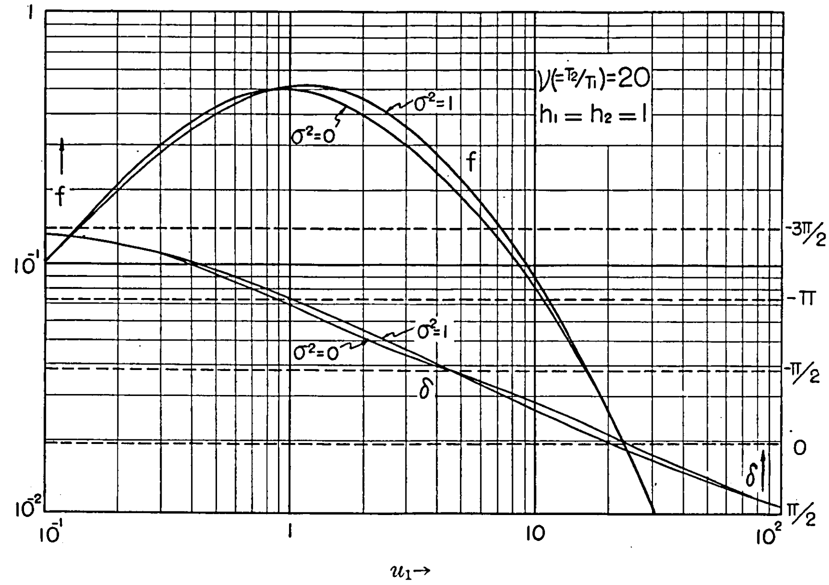

$\nu(=T_2/T_1)=20$
$h_1=h_2=1$
$\sigma^2=1$
$\sigma^2=0$
f
$\sigma^2=1$
$\sigma^2=0$
δ
1
10^{-1}
10^{-2}
10^{-1}
1
10
10^2
$-3\pi/2$
$-\pi$
$-\pi/2$
0
$\pi/2$
$u_1\rightarrow$

Fig. 9.

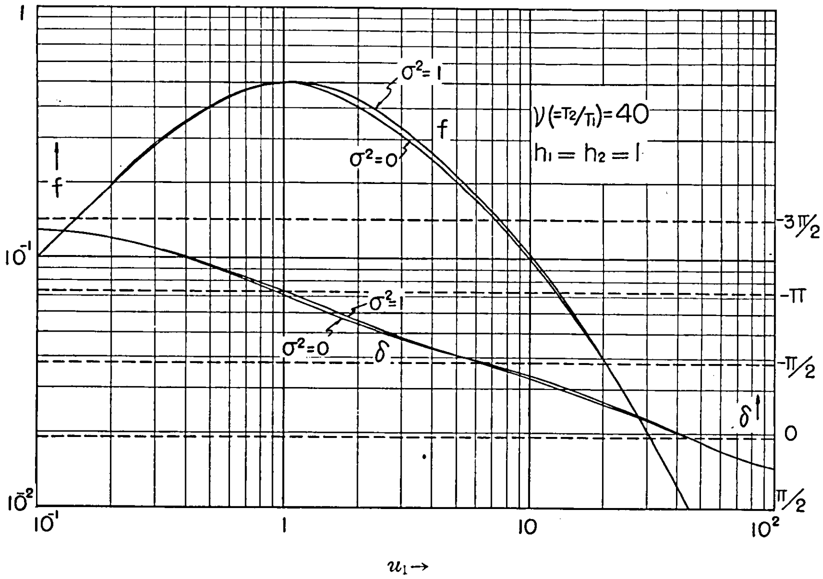
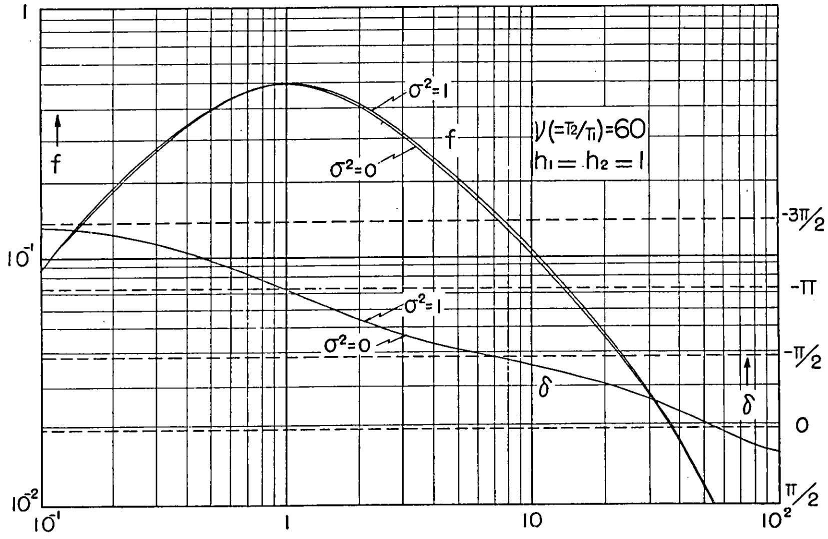
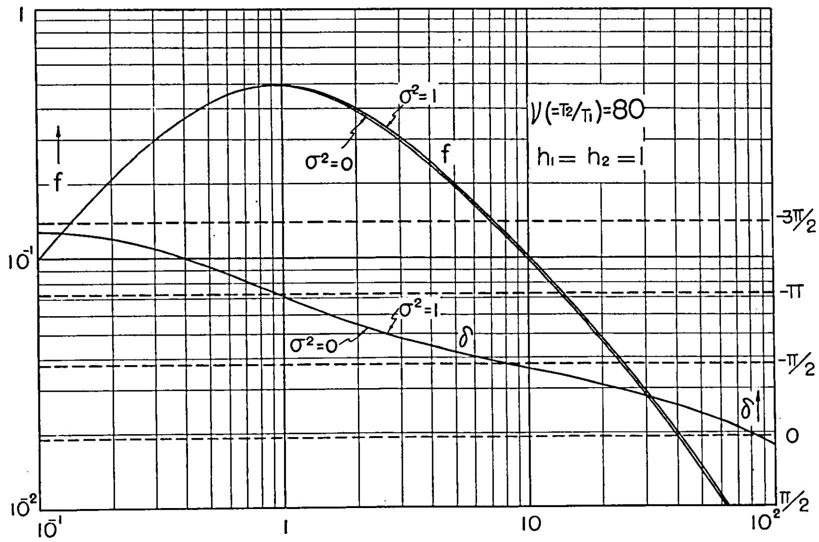

$\nu(=T_2/T_1)=40$
$h_1=h_2=1$
$\sigma^2=1$
$\sigma^2=0$
f
$\sigma^2=1$
$\sigma^2=0$
δ
1
10^{-1}
10^{-2}
10^{-1}
1
10
10^2
$-3\pi/2$
$-\pi$
$-\pi/2$
0
$\pi/2$
$u_1\rightarrow$

Fig. 10



$u_1 \rightarrow$

Fig. 11.



$u_1 \rightarrow$

Fig. 12.

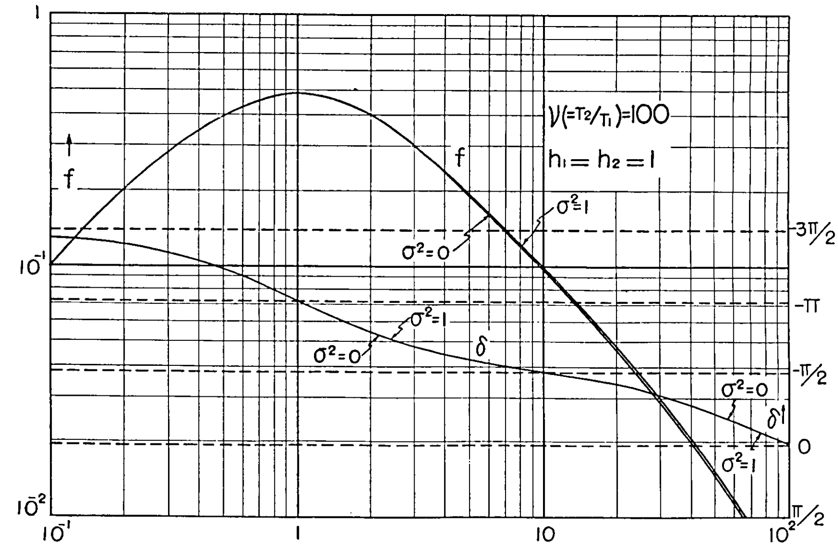

ν(=T2/T1)=100
h1 = h2 = 1
f
δ
σ²=0
σ²=1
-3π/2
-π
-π/2
0
π/2
1
10⁻¹
10⁻²
10
10²
u1→

Fig. 13.

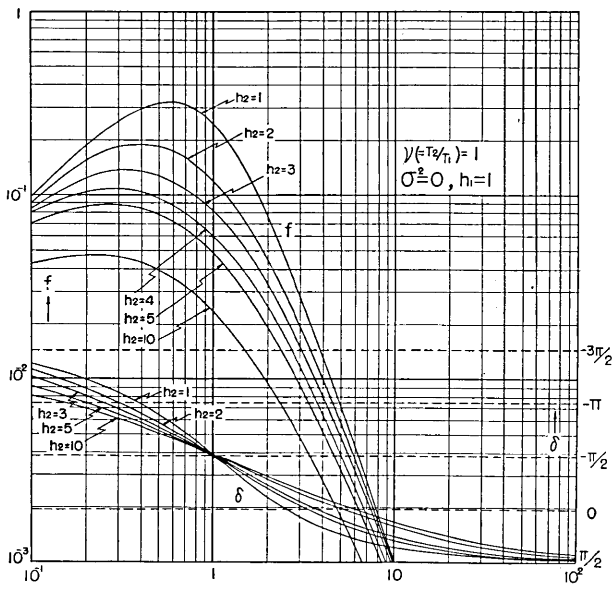

ν(=T2/T1)= 1
σ²=0 , h1=1
h2=1
h2=2
h2=3
h2=4
h2=5
h2=10
f
δ
-3π/2
-π
-π/2
0
π/2
1
10⁻¹
10⁻²
10⁻³
10
10²
u1→

Fig. 14.

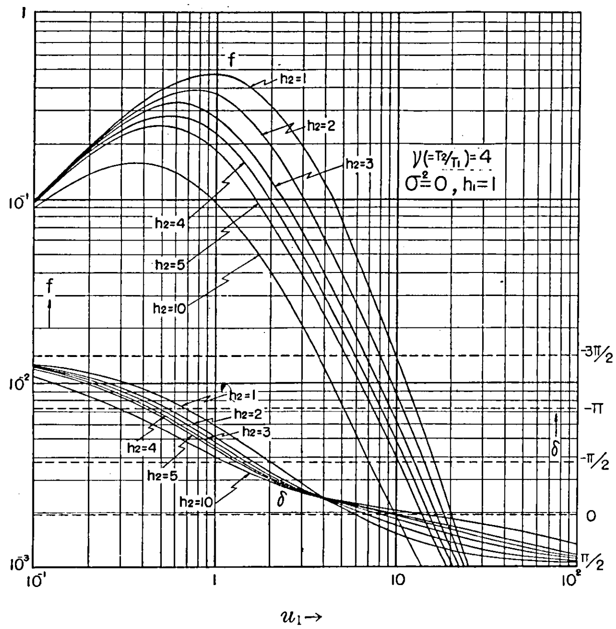

f
h2=1
h2=2
h2=3
h2=4
h2=5
h2=10
ν(=T2/T1)=4
σ²=0, h1=1
δ
-3π/2
-π
-π/2
0
π/2
u1→

Fig. 15.

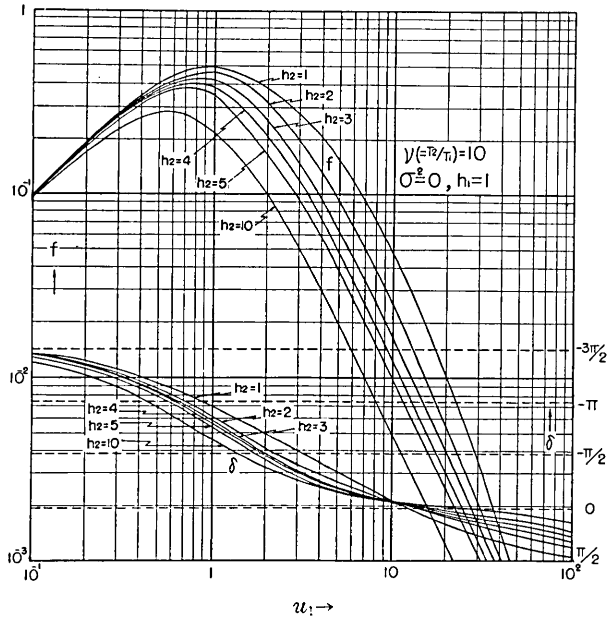
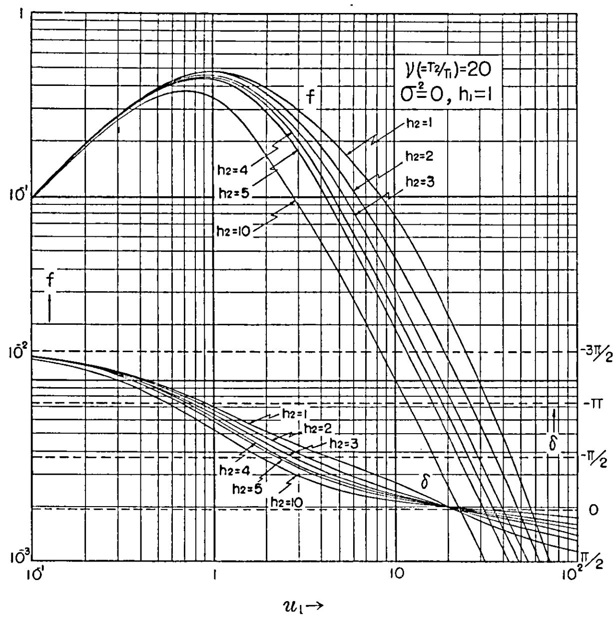
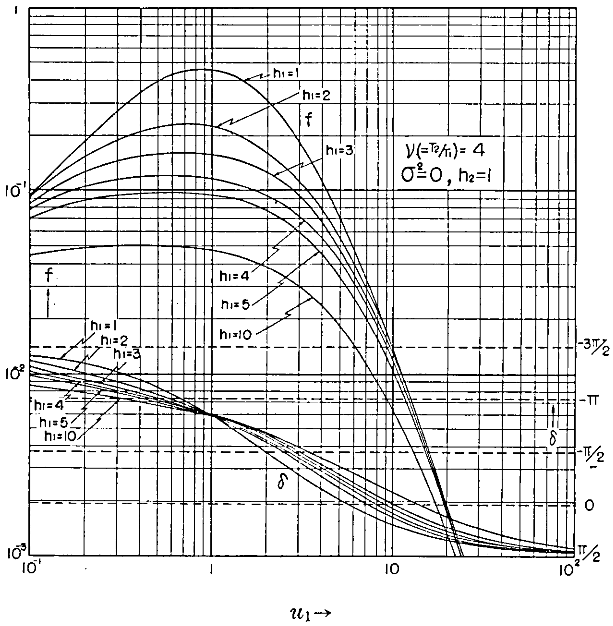

f
h2=1
h2=2
h2=3
h2=4
h2=5
h2=10
ν(=T2/T1)=10
σ²=0, h1=1
δ
-3π/2
-π
-π/2
0
π/2
u1→

Fig. 16.



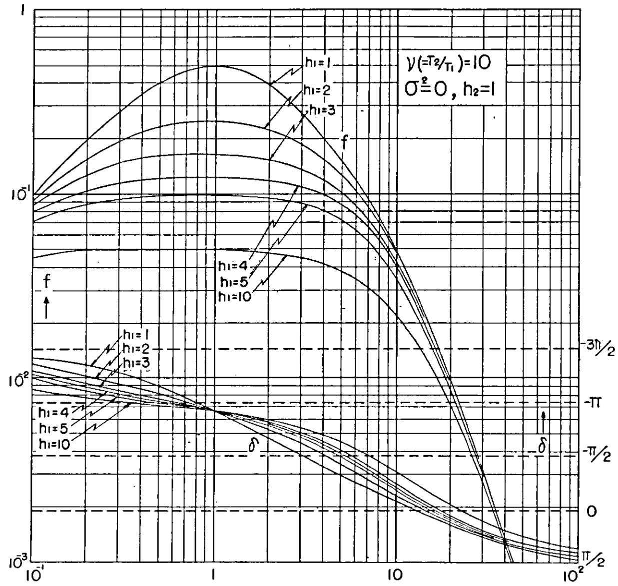
$\nu(=T_2/T_1)=20$
$\sigma^2=0,\ h_1=1$
f
$h_2=1$
$h_2=2$
$h_2=3$
$h_2=4$
$h_2=5$
$h_2=10$
δ
$-3\pi/2$
$-\pi$
$-\pi/2$
0
$\pi/2$
1
10^{-1}
10^{-2}
10^{-3}
10
10^2
$u_1 \rightarrow$

Fig. 17.



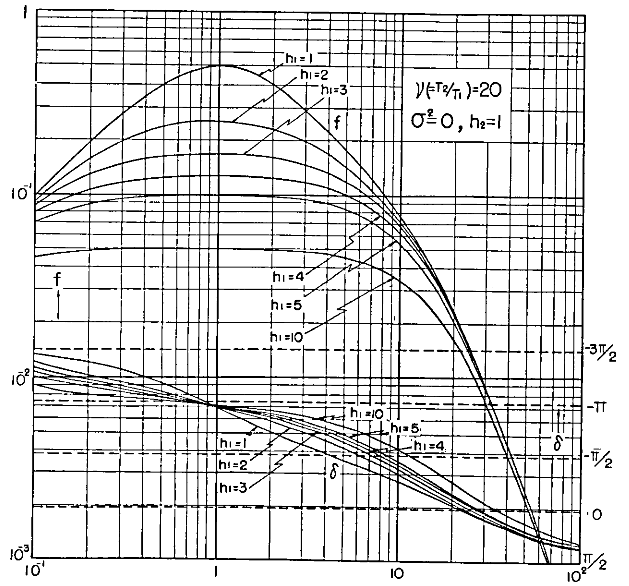
$\nu(=T_2/T_1)=4$
$\sigma^2=0,\ h_2=1$
f
$h_1=1$
$h_1=2$
$h_1=3$
$h_1=4$
$h_1=5$
$h_1=10$
δ
$-3\pi/2$
$-\pi$
$-\pi/2$
0
$\pi/2$
1
10^{-1}
10^{-2}
10^{-3}
10
10^2
$u_1 \rightarrow$

Fig. 18.



$u_1 \rightarrow$

Fig. 19.



$u_1 \rightarrow$

Fig. 20.

graph calculated from the equations (28) and (29) are shown in Figs. 3~20, in which f and $\delta(=\beta-\alpha)$ were taken in ordinate and u_1 in abscissa. The case for $h_1=h_2=1$, which is the most common in routine observation was indicated in rather more detail.

The figures shows us that the reaction of the galvanometer becomes practically negligible if $\sigma<1/3$ even in the case when T_1 equals T_2 and the reaction is expected to be most influential.

When the transducer is not provided with any additional damping device and the electromagnetical damping is caused simply by the current through the transducer coil, h_{01} is very small compared to unity, so that if the damping is critical or nearly critical it can be considered that $h_1 \gg h_{01}$. For the same reason, if the damping of the galvanometer is critical or nearly critical we can put $h_2 \gg h_{02}$. Hence the first term of the right hand side of equation (49) is approximately equal to unity. The seismograph is designed in many cases so that $Z_{11} \approx Z_{22}$ for the sake of matching resistances. In such a case, (49) can be written as $\sigma^2 \approx \mu_1^2$ or $\sigma \approx \mu_1$, so that we can say that the coupling factor is approximately equal to the attenuation factor. This means that when the additional damping device is not provided for the transducer, if we select the attenuation factor less than 1/3 inserting an attenuation circuit between the transducer and the galvanometer, the reaction of the galvanometer can be left out of consideration.

5. An example of the numerical computation

As an example, the computation of the magnification and the resistances of the attenuation circuit will be indicated in the following for the electromagnetic seismograph of Type HES-H which is used for routine observation at the Tsukuba seismological station of the Earthquake Research Institute. The seismograph is of horizontal component and the transducer is of the moving coil type. The seismogram is registered on 35 mm standard photographic film and the reading is made after that by means of a film-reader with enlarging ratio of 8.

The constants of the transducer and the galvanometer of the seismograph are as follows:

Transducer			Galvanometer		
M	1.09×10^4	gram			
H	5.0	cm			
T_1	1.00	sec	T_2	1.10	sec

$2\pi/T_1$	6.28		$(2\pi/T_2)^2$	3.26×10	
S_1	2.02×10^2	radian/unit current in c.g.s. e.m.u.	S_2	1.43×10^8	cm/unit current in c.g.s. e.m.u.
R_1	1520	ohm	R_2	500	ohm
Ω_1 (external damping resistance)	1400	ohm	Ω_2 (external damping resistanace)	1800	ohm
h_1	1.0		h_2	1.0	
h_{01}	0.0		h_{02}	0.0	

The following numericals are computed for the case $\mu_1=1/10$ using the above constants.

$$Z_{11}=R_1+\Omega_1=2920\ \text{ohm}=2.92\times 10^{12}\ \text{c.g.s. e.m.u.}$$

$$Z_{22}=R_2+\Omega_2=2300\ \text{ohm}$$

$$\left\{\begin{aligned} A&=\frac{1}{\mu_1}\frac{Z_{11}}{Z_{22}}=12.70\\ B&=\frac{1}{\mu_1}Z_{11}=29{,}200\\ C&=\frac{AD-1}{B}=4.315\times 10^{-3}\\ D&=\frac{1}{\mu_1}=10 \end{aligned}\right.$$

For the T-type network,

$$\left\{\begin{aligned} Z_1&=\frac{A-1}{C}=2710\ \text{ohm}\\ Z_2&=\frac{D-1}{C}=2090\ \text{ohm}\\ Z_3&=\frac{1}{C}=232\ \text{ohm} \end{aligned}\right.$$

Hence,

$$X_1=Z_1-R_1=1190\ \text{ohm},$$

$$X_2=Z_2-R_2=1590\ \text{ohm},$$

$$X_3=Z_3=232\ \text{ohm}.$$

And

$$\sigma=\sqrt{\frac{(h_1-h_{01})(h_2-h_{02})}{h_1h_2}\frac{Z_{22}}{Z_{11}}}\,\mu_1=0.089\,,$$

$$Q=1.09\cdot 10^4\times 5.0\times 6.28\times 3.26\cdot 10\times 2.02\cdot 10^2\times 1.43\cdot 10^8\times 10^{-1}\times\frac{1}{2.92\cdot 10^{12}}$$

$$=11{,}040$$

Maximum value of f is obtained as 0.330 from the period response curves when $h_1=h_2=1$ and $T_2/T_1=1.1$. Hence the maximum magnification of the seismograph becomes :

$V_{\max}$ (on the recording film) $=Q\cdot f_{\max}=11{,}040\times 0.330=3{,}640$
$V_{\max}$ (on the film-reader) $=3{,}640\times 8=29{,}100$

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8. 電磁地震計の倍率その他について

地震研究所　萩　原　尊　禮

直結式電磁地震計の理論は，Galitzin 以後 Wenner, Coulomb と Grenet, Schmerwitz, Eaton, 田治米などの研究者により進められ，ほぼ完成の域に達した．しかし，実際に地震計を設計したり，検定したりする場合になると，必要な公式がもう少し便利な形で表現されていることが望ましくなる．本文では，こういうことを考慮して，電磁地震計の理論式をできるだけ理解しやすく且つ簡単に測定できる量を使つて表わすことを試みた．

従来は，電磁地震計を理論的に取扱う場合に，換振器と電流計を結ぶ回路に一つのシャント抵抗を置くものとして出発したが，実際の観測では時により倍率を変化させる必要を生じるので，換振器と電流計の減衰常数を変えることなく倍率だけを変化できるアテニュエーター回路を初めから考えに入れて出発した方が便利である．本文では，電磁地震計の倍率を決める式を次のような形で表現した．

$$\frac{y_m}{x_m}=Q\cdot f$$

$$Q=MH\frac{2\pi}{T_1}\left(\frac{2\pi}{T_2}\right)^2 S_1S_2\mu_1\frac{1}{Z_{11}}$$

ここに，y_m は記録ドラム上の振動振幅，x_m は正弦波形をした地面の変位振幅，M は換振器の振子の質量，H は振子の重心と回転軸の間の距離，T_1 は換振器の自然周期，T_2 は電流計の自然周期，S_1 は振子のコイルに単位電流を与えた場合の振子のふれの角（これは，振子の回転軸付近に小さな鏡を取付け，ランプスケールを使つて，電流計の感度検定と全く同じ操作で測定すれば求められる)，S_2 は電流計のコイルに単位電流を与えた場合の記録ドラム上の光点の変位（ふつう電流計のアンペア感度と称されているものは，S_2 の逆数に相当する）である．μ_1 はここに新しく定義したもので，電流計をクランプしておいて換振器のコイルに起電力を与えた場合に電流計を流れる電流と換振器を流れる電流の比を表わす．即ち，換振器の電流の μ_1 倍が電流計に分流するわけである．μ_1 を attenuation factor と称することにする．Z_{11} は換振器のコイルの抵抗とその外部減衰抵抗（換振器に所定の減衰を起させる外部抵抗）との和である．f はこの地震計の周期特性であつて，h_1 (換振器の減衰常数), h_2 (電流計の減衰常数), T_2/T_1 (電流計と換振器の自然周期の比), σ (coupling factor), 及び T_ω/T_1 (地面の周期と換振器の周期の比）の函数で表わされる．

σ については次の式で表現した．

$$\sigma^2=\frac{(h_1-h_{01})(h_2-h_{02})}{h_1h_2}\frac{Z_{22}}{Z_{11}}\mu_1^2$$

ここに，h_{01} は換振器の外部抵抗を取去りコイルに電流が流れないようにした場合の換振器の減衰常数であつて，特別にダンパーを取付けてなければほとんど零に近い値を取る．h_{02} は電流計に対する同じ意味の値である．Z_{22} は電流計のコイルの抵抗とその外部減衰抵抗との和である．σ は電流計の運動が振子の運動に及ぼす影響の度合を示す量であつて，$0\leqq\sigma\leqq 1$ の値を取り，$\sigma=0$ のとき

は影響は無く，$\sigma=1$ のとき影響が最も大きい．換振器が特別にダンパーを備えていない場合は，$h_1 \geqq h_{01}$ であり，電流計についても $h_2 \geqq h_{02}$ である．また，ふつうの場合は，Z_{11} と Z_{22} とはとびはなれて違つた値を取ることはなく，$Z_{11} \approx Z_{22}$ のように設計されている．従つて，近似的には $\sigma \approx \mu_1$ と見てさしつかえない場合が多い．即ち，σ は近似的には attenuation factor そのものにほぼ等しいと見てもよい．このように考えると σ の物理的意味がかなりはつきりしてくる．

本文の終りに振幅特性 f と位相特性 δ の図を示してある．σ の影響の最も大きいのは $T_1=T_2$ の場合であるが，この場合でも $\sigma<1/3$ であれば，電流計の反作用の影響は実際上ほとんど無視してよいことがわかる．従つて，特別にダンパーを取付けず，減衰をコイルの電流の作用だけに頼る場合でも，attenuator により attenuation factor をある程度小さくすれば，電流計の反作用を無視できることになる．

注目すべきことは，上に示した σ の表現に，振子や電流計の慣性が含まれていないことである．これは，振子の質量を如何に大きくしても，ただそれだけでは，電流計の反作用の影響を小さくすることは決してできないということを意味している．特別にダンパーを取付けない場合は，attenuation factor を小さくするということが，電流計の反作用から免れる唯一の道である．もつとも，振子の質量を大きくすれば，それに比例して地震計の倍率は大きくすることができるから，あらかじめ決められた倍率を得るためには attenuation factor を充分小さくできることになるから，こういう意味では振子の質量を大きくすることが電流計の反作用を小さくすることにはなる．

最後に，倍率，σ，μ_1 などを望む値にするための attenuator の抵抗値の決め方などについて実例を示した．実際に地震計測に従事する方々の参考になれば幸である．