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10. *Rocking and Elastic Vibrations of Actual Building. I. (Experiments by Vibration Generator).*

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1. Introduction

The property of building vibration varies according to the ground property even if the same vibrational force acts on buildings of the same structure, and the property of earthquake motion depends on the nature of the ground. Therefore it can be said that the buildings in an earthquake are doubly influenced by the ground property.

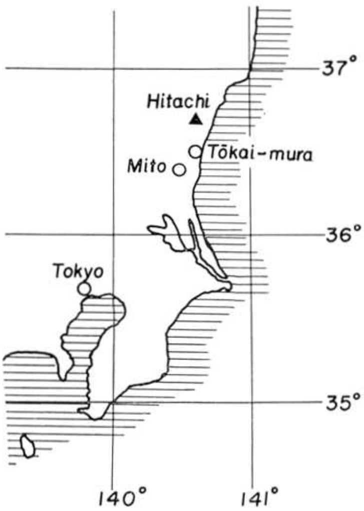

Fig. 1.

In the field of experimental studies, the vibration of actual buildings caused by artificial vibrational forces as well as by actual earthquakes has been measured many times.[1] These investigations showed us that the vibration problem of buildings in a destructive earthquake is very complex and many subjects concerning this problem remain to be treated.

Nevertheless it can be said that, since these investigations threw light on the problem of earthquake-proof construction to a considerable extent, its remarkable developement will be realized in the near future.

1) K. KANAI, T. SUZUKI, T. HISADA and K. NAKAGAWA, "Vibration Experiments in the Buildings on Various Kinds of Ground. I", *Report Archit. Inst. Japan*, **33** (1955), 185.

K. KANAI, T. TANAKA and T. SUZUKI, "*ditto*. II", *ditto*, 187.

K. KANAI and T. SUZUKI, "Relation between the Property of Building Vibration and the Nature of Ground", *Bull. Earthq. Res. Inst.*, **31** (1953), 305: Part II, *ditto*, **33** (1955), 109: Part III, *ditto*, **34** (1956), 61.

With a view of gaining further insight into the problem just mentioned, we tried the investigation of the vibration problem of actual buildings standing on rock and on crushed stone. We carried out the experiments on the buildings by means of a vibration generator.

The buildings submitted to these experiments are three-storied reinforced concrete apartment-houses, located at Hitachi Mine site, Ibaraki prefecture. One building called No. 14 house stands on crushed stone with six meter concrete piles. Another building called No. 6 house stands directly on fresh rock.

2. The case of No. 14 house

(i) Method of experiments.

Figs. 2~5 are the photographs and the rough sketches of the plane view, side view and parts of foundation of No. 14 house. The vibration generator used in this case consists of three rotating wheels with two horizontal axes. The total weight of the eccentric masses used is $5.4\text{kg} \times 2 + 10.8\text{kg} \times 1 = 21.6\text{kg}$ and the length of each eccentric mass from the axis is 21.5cm.


Fig. 2. Photograph of No. 14 house.

The vibration generator was installed on the roof-floor and vibrational force was given in the beam direction, that is, short breadth direction. The position of the vibration generator is indicated by the mark × in Figs. 3 and 4. The vibration generator was driven manually faster than the natural period of a test building and then set free in a state of free rotation. As it keeps on running for quite a long time diminishing its speed gradually, it can be considered as the first approximation that the vibrational force acted on a test building for each period is sinusoidal.

Four horizontal component vibrographs and two vertical component vibrographs were used. The former have a mechanical magnification system with smoked paper for recording, and the latter have an electromagnetic magnification system with smoked paper for recording. The constants of these are shown in Tables 1 and 2 respectively. The positions of the horizontal and vertical vibrographs are indicated by marks ○ and ● in Figs. 3 and 4, respectively. The number of vibrographs used at each position are shown in Table 3 and 4.

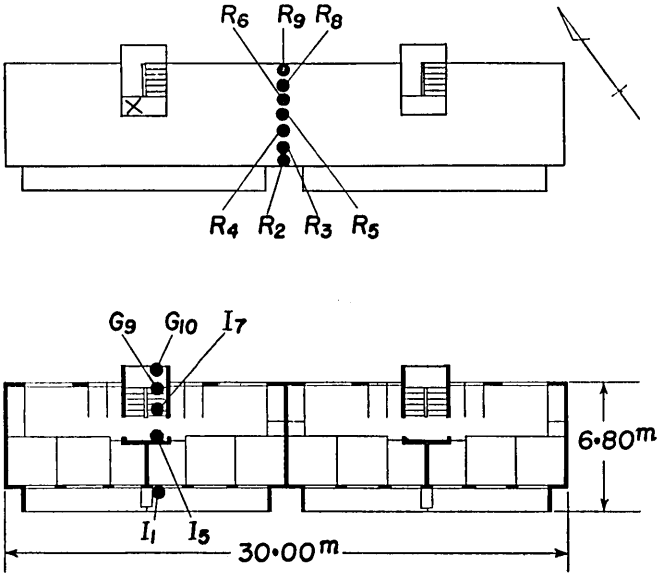

Fig. 3. Sketch of the plane view of No. 14 house.

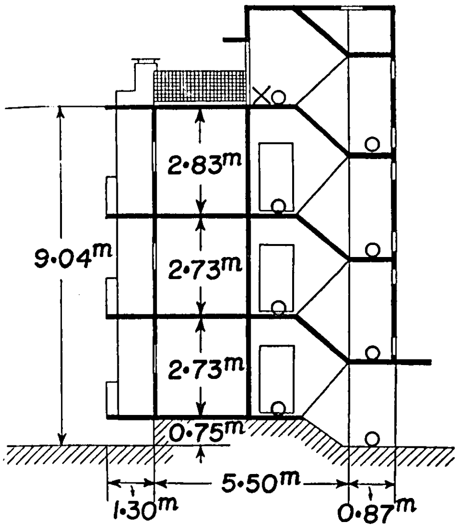

Fig. 4. Sketch of the side view of No. 14 house.

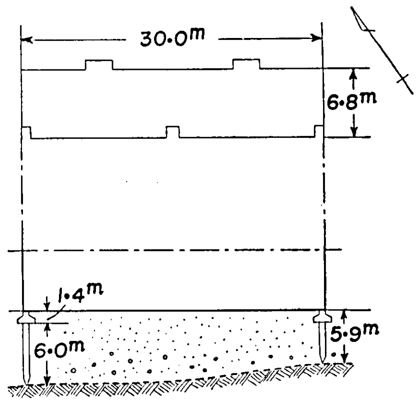

Fig. 5. Sketch of the foundation parts of No. 14 house.

Table 1. Constants of the mechanical type vibrographs.

No.	Natural period (sec)	Damping ratio	Magnification
1	1.0	13	139
2	〃	〃	118
3	〃	〃	123
4	〃	〃	137

Table 3. Locations of the mechanical type vibrographs.

No. of exper.	Horizontal vibrographs			
	No. 1	No. 2	No. 3	No. 4
1	III$\frac{1}{2}$	III$\frac{1}{2}$	III$\frac{1}{2}$	III$\frac{1}{2}$
2	II$\frac{1}{2}$	〃	R	—
3	II	〃	III	I$\frac{1}{2}$
4	I	—	—	G

Table 2. Constants of the electro-magnetic type vibrographs.

Unit		Constants
Transducer	Natural period	1.0 sec
	Damping ratio (electro-magnetic)	10
Amplifier	Conductance between input and output	28 mho
Recorder (smoke writing type)	Natural period	0.045 sec
	Sensitivity	2 mm/mA
Overall magnification		max. $=10^5$ (attenuater, 2db step, −80db)

(ii) Results of experiment.

The representative records are illustrated in Fig. 6. The relation between the amplitudes of horizontal component and the vibration periods at each position is shown in Fig. 7. The vibrational force produced by the vibration generator is represented by $f=m(4\pi^2/T^2)r$, where m=eccentric mass, r=radius, T=period. Therefore, the observed values represent the amplitudes corresponding to the applied force which changes according to frequency. This makes theoretical consideration rather complicated. So the values are converted into the amplitudes, supposing that the constant force of 5.5×10^6 dyne, that is, force corresponding to the period of 0.18 sec, acts regardless of period.

From Fig. 7, it is found that the resonance period of the beam direction of No. 14 house was 0.182 sec, and we obtain the value of the fraction of critical damping of No. 14 house in h, that is, $h=0.032$.[2] The frequency-period relation of micro-tremors at the roof-floor of this

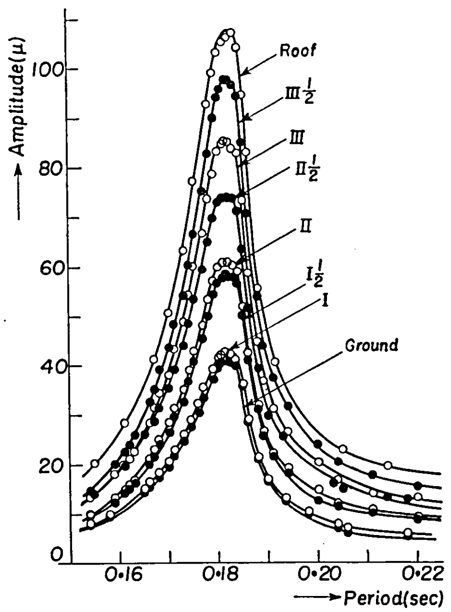

Fig. 7. Amplitude-period relations of horizontal component of No. 14 house.

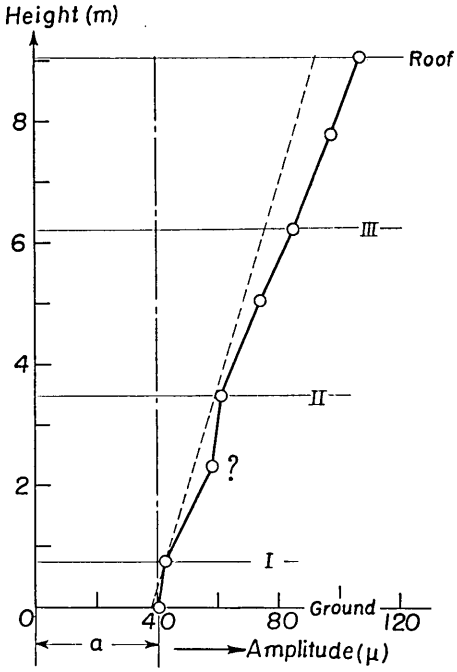

Fig. 8. Amplitude distribution of horizontal component of No. 14 house.

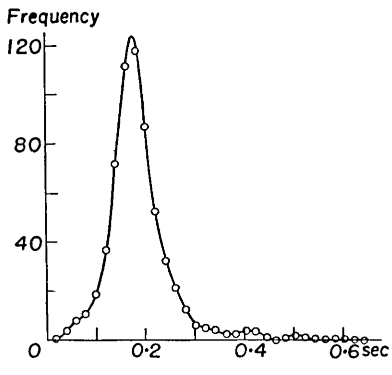

Fig. 9. Frequency-period relation of micro-tremors.

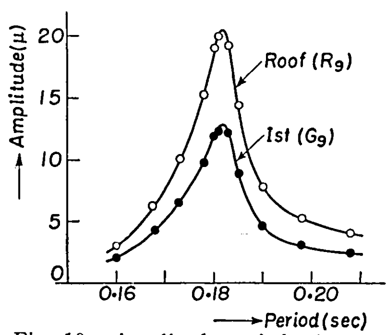

Fig. 10. Amplitude-period relations of vertical component of No. 14 house.

2) $h=\{(1/T_2)^2-(1/T_1)^2\}T_0^2/4$, in which T_1, T_2 and T_0 are the periods of resonance curve at $1/\sqrt{2}$ times resonance amplitude and resonance period

house is illustrated in Fig. 9. Figs. 7 and 9 tell us that the predominant period of micro-tremors at the top of a building coincides fairly with the resonance period of the same building. From Fig. 7, we obtain

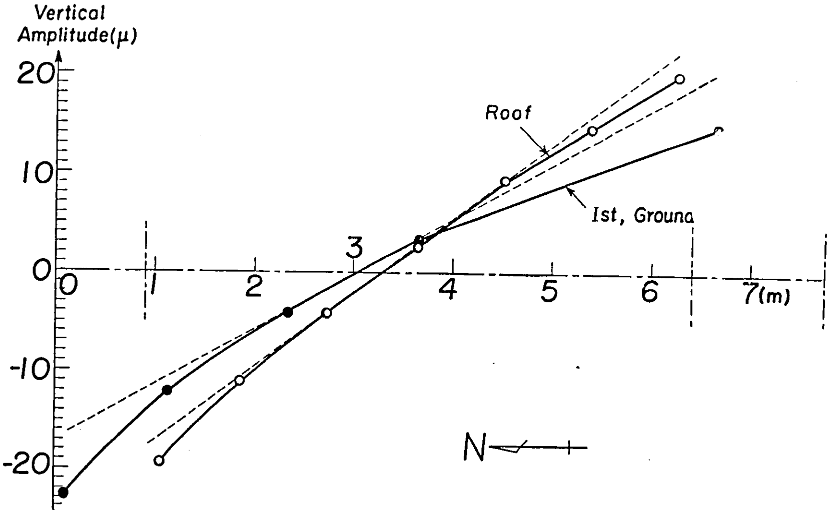

Fig. 11. Distributions of resonance amplitudes of vertical component of No. 14 house.

the distribution of resonance amplitude of horizontal component at each position as shown in Fig. 8. Next, Fig. 10 shows the examples of the relation between the amplitudes of vertical component and vibration periods at the roof-floor and at the first floor. From such resonance curves as in Fig. 10, we obtain the distribution of resonance amplitudes of vertical component at each position as shown in Fig. 11. Fig. 11 tells us that the building underwent rocking vibration on its axis which is situated a little north of the center line. From the broken line in Fig. 11, the values of the inclination near the axis were found to be 6×10^{-6} radian at the first-floor and 7.5×10^{-6} radian at the roof-floor.

It is also a noteworthy fact that the elastic deformation of horizontal members including the basement of the building took place in the course of vibration caused by the vibration generator as shown in Fig. 11. The angles formed by the broken lines and the chain lines in Figs. 8 and 11 are equal. Therefore, Figs. 8 and 11 tell us that there, in the building, took place the relative displacement at the foundation to the ground, a, the rocking vibration of the angle θ together with the elastic deformation corresponding to the displacement between the broken line and the full line, in the course of the vibration caused by the vibration generator.

In case of vibrational experiments of actual buildings by means of the vibration generator, by substituting the data of the elastic deformation as well as the vibration period in the following equation (1), it is possible to find out the stiffness resisting to horizontal vibrations at each storey of a n-storied building.[3] That is,

$$E_sI_s=\frac{Kl_s^3\left(\frac{2\pi}{T}\right)^2\sum_{r=s}^{n}m_ry_r}{y_s-y_{s-1}}, \quad [s=1,\ 2,\cdots n] \tag{1}$$

in which, n=number of storeys, s=number of floors for which the stiffness is sought, E=Young's modulus, I=total sum of the moments of inertia of sections, l=height between the s-1th and sth floors, m=concentrated mass at floor, y=horizontal displacement at floor, T= vibration period, K=constant determined by the fixing conditions of vertical members. That is, in case the vertical members are clamped at the horizontal members as well as at the base; K=1/12. In this equation, for simplicity, the effect of damping has been neglected.

The constants of No. 14 and No. 6 houses are as follows:

$$\begin{aligned}&l_1=2.73\times10^2,\ l_2=2.73\times10^2,\ l_3=2.83\times10^2,\\&m_1=m_2=m_3=3.3\times10^8. \qquad \text{(C. G. S. unit)}\end{aligned} \tag{2}$$

From the present investigation, we obtained the following data:

$$\begin{aligned}&y_1=4\times10^{-4},\ y_2=9\times10^{-4},\ y_3=15\times10^{-4}\\&T=0.18 \text{ and } K=1/12. \qquad \text{(C. G. S. unit)}\end{aligned} \tag{3}$$

Substituting (2) and (3) in (1), it is possible to get easily the total effective stiffness of the vertical members of each floor as follows:

$$E_1I_1=5\times10^{18},\ E_2I_2=3\times10^{18},\ E_3I_3=2\times10^{18}. \tag{4}$$

3. The case of No. 6 house

(i). Method of experiments.

3) K. KANAI, "A Method of Determining the Stiffness of Each Storey of a n-storied Building", *Bull. Earthq. Res. Inst.*, **28** (1950), 161.

Figs. 12～15 are the photograph and the rough sketches of the plane view, side view and parts of foundation of No. 6 house.

Fig. 12. Photograph of No. 6 house.

The vibration generator used in this case consists of two rotating wheels with two horizontal axes. The total weight of the eccentric masses used is 1.4kg × 2 = 2.8kg and the length of each eccentric mass from the axis is 21.5cm. The vibration generator was installed on the roof-floor and vibrational force was given in the beam direction, that is, short breadth direction. The position of the vibration generator is indicated by the mark × in Figs. 13 and 14. The vibration generator was driven by A.C. motor of 1/2HP faster than the natural period of a test-building and then set free in a state of

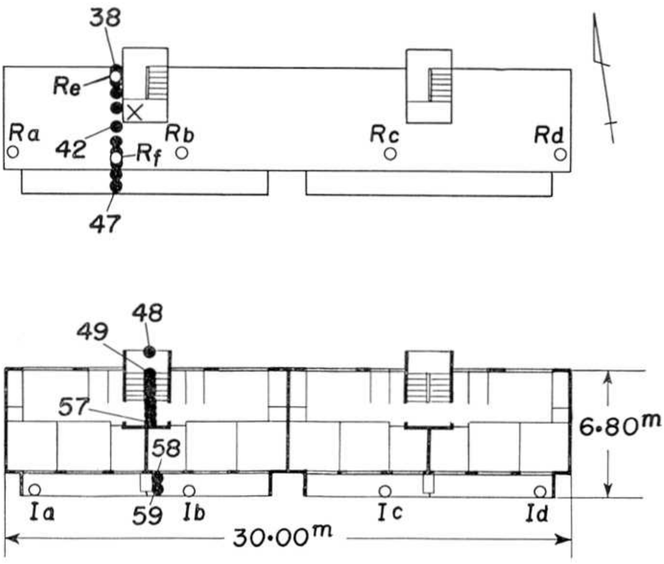

Fig. 13. Sketch of the plane view of No. 6 house.

free rotation. For the first approximation, the vibrational force acted on a test-building for each period may be considered sinusoidal.

Six horizontal and two vertical component electro-magnetic type vibrographs and one mechanical magnification type vibrograph were used. The electro-magnetic type vibrographs were used in combination

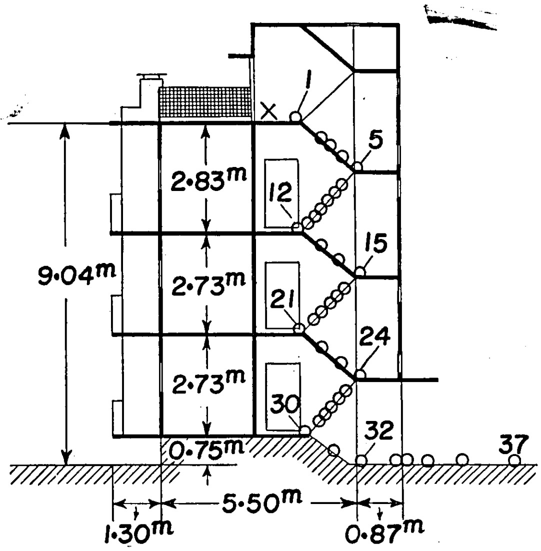

Fig. 14. Sketch of the side view of No. 6 house.

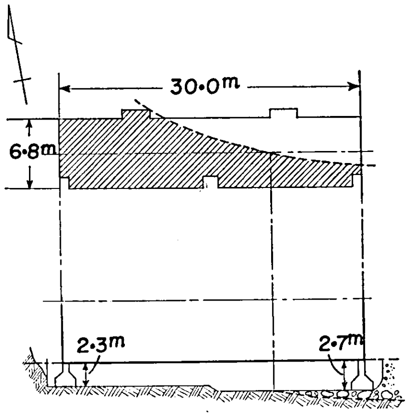

Fig. 15. Sketch of the foundation parts of No. 6 house.

with a six-element oscillograh, and a mechanical type vibrograph used as the standard was combined with smoked paper recording apparatus. The constants of vibrographs are shown in Table 5.

The positions of the horizontal and vertical vibrographs are indicated by marks ○ and ● in Figs. 13 and 14, respectively. The locations of the vibrographs are shown in Table 6.

Observation on the stairs was made in order to measure the vibration of horizontal component at the parts between each floor. Only a few measurements of vertical component were made on the south side of the building, because the rooms on that side had Japanese mats on the floor.

Table 4. Locations of the electro-magnetic type vibrographs.

No. of exper.	Vertical vibrographs	
	No. 1	No. 2
1	G_9	G_9
2	G_{10}	〃
3	I_7	〃
4	I_5	〃
5	I_1	〃
6	R_9	〃
7	R_3	R_2
8	R_4	〃
9	R_5	〃
10	R_6	〃
11	R_8	〃
12	R_9	〃
13	R_2	〃

(ii) Results of experiment.

The representative records are illustrated in Fig. 16. The relations between the amplitudes of horizontal component and the vibration periods at the roof-floor and the ground-floor are shown in Fig. 17. Fig. 17 shows plainly that there are two peaks in the amplitude-period curve, in other words, the vibration amplitudes predominated at two peculiar periods. We obtained the relation between the amplitudes and the periods of the vertical component which was similar to that of the horizontal component. It will be seen from Fig. 17, that the amplitude-period

Table 5. Constants of the electro-magnetic type vibrograph.

Unit		Constants
Transducer	Natural period	1.0 sec
	Damping ratio (electro-magnetic)	10
Amplifier	Conductance between input and output	10 mho (at 0.2 sec)
Oscillograph	Natural period	0.01 sec
	Sensitivity	14 mm/mA
Overall magnification		1.5×10^5 (attenuater, 2db step, −80db)
Overall period characteristics		Displacement type (in a period range between 0.05 sec and 1.0 sec.)

Table 6. Locations of the vibrographs.

No. of exper.	Vertical vibrographs		Horizontal vibrographs			
	V_1	V_2	H_3	H_4	H_5	H_6
1	5	5	5	5	5	5
2	38	47	〃	1	12	15
3	39	46	〃	21	24	30
4	40	45	〃	32	33	34
5	41	44	〃	35	36	37
6	42	43	R_d	R_c	R_b	R_a
7	49	59	I_d	I_c	I_b	I_a
8	〃	〃	〃	〃	〃	〃
9	51	58	28	26	4	2
10	49	38	23	22	10	7
11	53	55	20	17	14	13
12	50	52	25	31	3	11
13	54	56	27	29	6	8
14	48	57	18	19	9	16
15	R_e	R_f	R_e(B)	R_e(G)	R_f(B)	R_f(G)
16	R_e	R_e	R_e	R_e	R_e	R_e

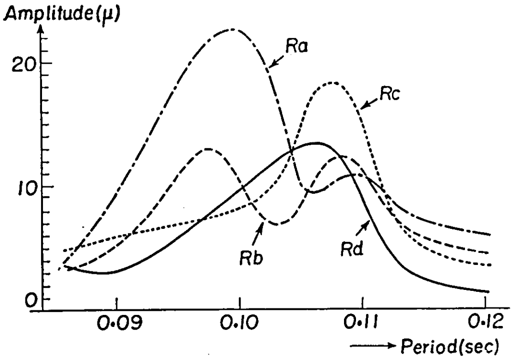

Fig. 17a. Amplitude-perio drelations at roof-floor of No. 6 house.

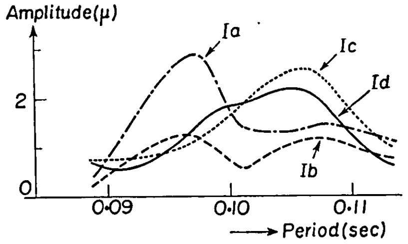

Fig. 17b. Amplitude-period relations at first-floor of No. 6 house.

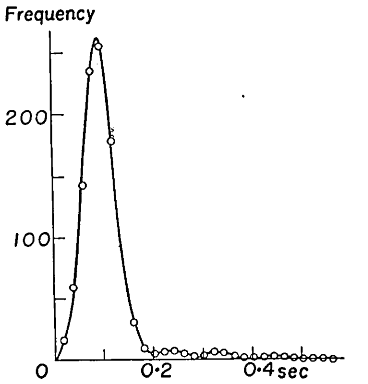

Fig. 18. Frequency-period relation of micro-tremors.

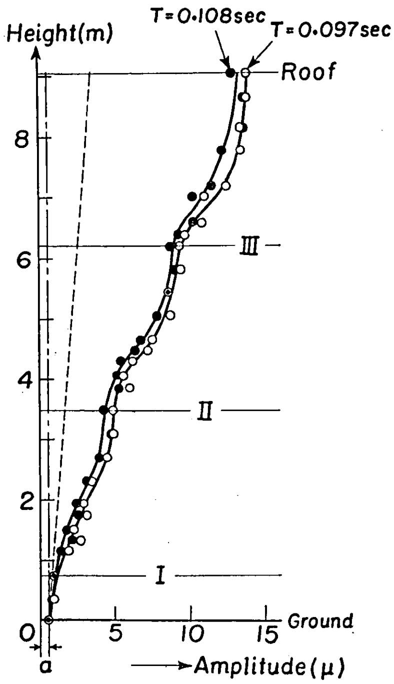

Fig. 19. Amplitude distributions of horizontal component of No. 6 house.

relation of No. 6 house is very complex, and then it is too difficult to obtain the value of h of No. 6 house. Therefore, in next time, we will estimate it by adopting various means. The frequency-period relation of micro-tremors at the roof-floor of this house is illustrated in Fig. 18. It is found from Figs. 17 and 18 that the frequency-period relation of micro-tremors at the top of No. 6 house differs from the amplitude-period relation of sinusoidal-forced oscillation of the house and the peak in the former curve nearly coincides with one of the peaks in the latter curve. The amplitude distribution of two peaks of both horizontal and vertical components of each position is shown in Figs. 19 and 20. In Figs. 17, 19 and 20, the amplitudes were converted into the values supposing that

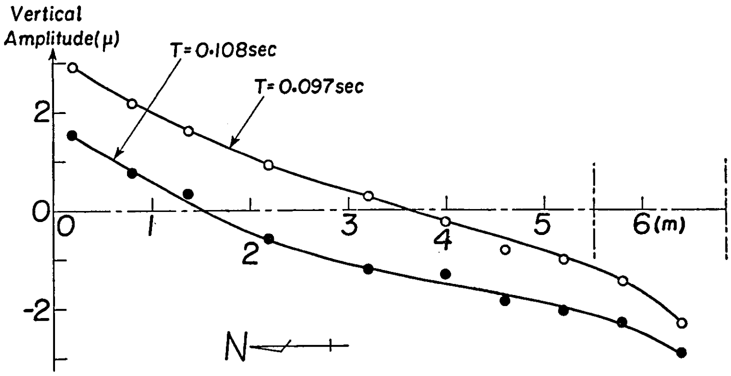

Fig. 20a. Amplitude distributions of vertical component at roof-floor of No. 6 house.

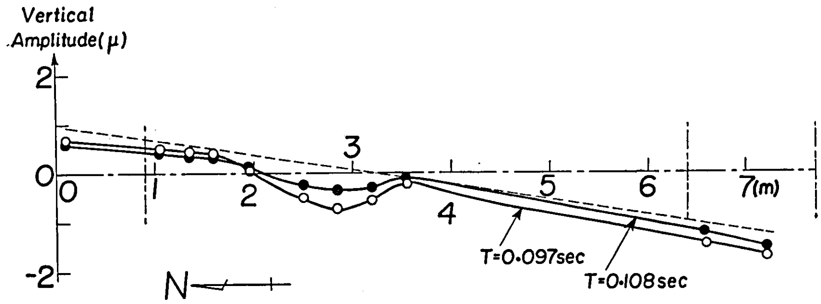

Fig. 20b. Amplitude distributions of vertical component at first-floor of No. 6 house.

the constant force of 2×10^{8} dyne, that is, force corresponding to the period of 0.108 sec., acted regardless of period. It will be seen from Fig. 20b that the foundation beam of the No. 6 house was given a considerably complicated deformation by the horizontal vibrational force. It is especially a noteworthy result that the type of bending oscillation of vertical members of the house has appeared so clearly in such experiments as shown in Fig. 19.

At any rate, Figs. 19 and 20 show that in No. 6 house, that is, a house standing on the rock, there also took place the relative displacement at the foundation to the ground, rocking vibration and elastic deformation in the course of the sinusoidal forced oscillation. Roughly speaking, the angles formed by the broken lines and the chain lines in Figs. 19

and 20b mean the inclination of the foundation, and the values between the broken lines and the curves of full line correspond to elastic deformation.

Next, we try to estimate the total stiffness of the vertical members of No. 6 house by using equation (1). In this case, the observed values of y and T are as follows:

$$\left.\begin{array}{l} y_1=2\times10^{-4},\ y_2=5\times10^{-4},\ y_3=8\times10^{-4} \\ T=0.108,\ K=1/12\quad \text{(C. G. S. unit)} \end{array}\right\} \qquad (5)$$

Substituting (5) and (2) in (1), we obtain the following values:

$$E_1I_1=18\times10^{18},\ E_2I_2=10\times10^{18},\ E_3I_3=7\times10^{18}\quad \text{(C. G. S. unit)} \qquad (6)$$

The each value in (6) are about 1/3 of the one in (4). This is considered to be natural because two houses have the same construction except the foundation parts. (The theoretical investigations will be presented in the forthcoming Bulletin.)

Next, we investigate the horizontal vibration modes at four positions on the roof-floor as well as the first-floor, that is, *Ra, Rb, Rc* and *Rd* and *Ia, Ib, Ic* and *Id*. The amplitude-period relations obtained there as shown in Figs. 17a and 17b show us plainly that at the period of 0.097 sec the amplitudes at *Ra, Rb, Ia* and *Id* which are on the west

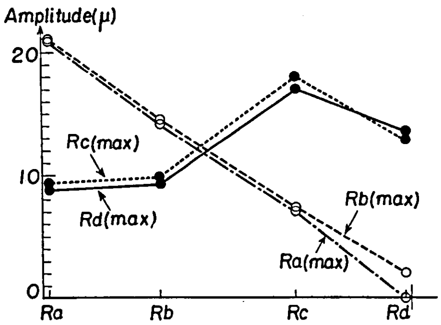

Fig. 21 a. Amplitude distributions at the moment when the horizontal amplitude of each position at the roof-floor of No. 6 house became maximum.

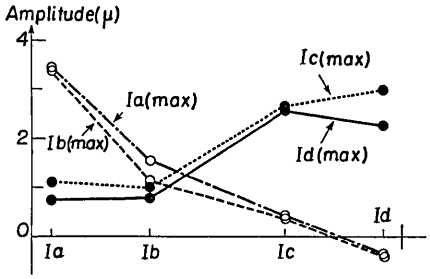

Fig. 21 b. Amplitude distributions at the moment when the horizontal amplitude of each position at the first-floor of No. 6 house became maximum.

side grew large, on the other hand, at the period of 0.108 sec the amplitudes at *Rc, Rd, Ic* and *Id* on east side predominated. The amplitude distributions at the moment when the horizontal amplitude of each position at the roof-floor and the first-floor became maximum are shown

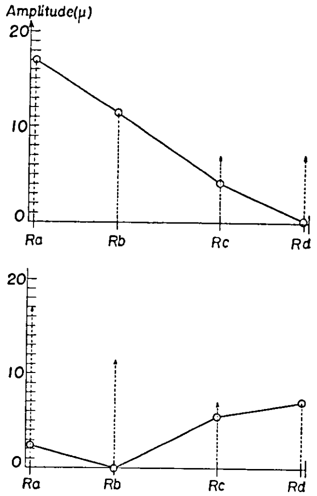

Fig. 22a. Horizontal amplitude distributions at the moments within 1/4 cycle of vibration, including the moment when the amplitude of position *Rb* (roof-floor) of No. 6 house became maximum.

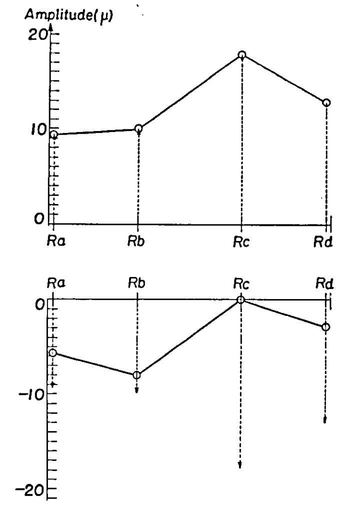

Fig. 22b. Horizontal amplitude distributions at the moments within 1/4 cycle of vibration, including the moment when the amplitude of position *Rc*(roof-floor) of No. 6 house became maximum.

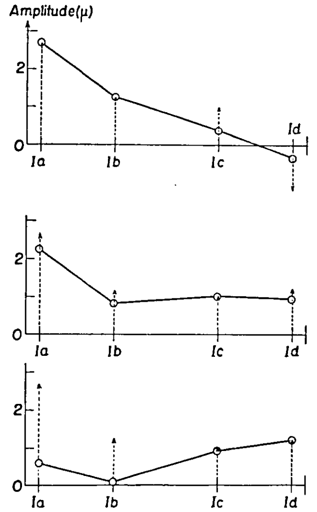

Fig. 22c. Horizontal amplitude distributions at the moments within 1/4 cycle of vibration, including the moment when the amplitude of position Ib (first-floor) of No. 6 house became maximum.

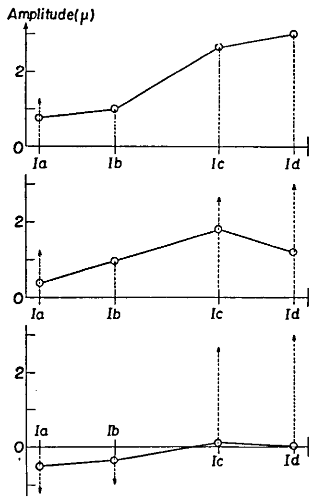

Fig. 22 d. Horizontal amplitude distributions at the moments within 1/4 cycle of vibration, including the moment when the amplitude of position Ic (first-floor) of No. 6 house became maximum.

in Figs. 21a and 21b, respectively. From Figs. 21a and 21b, it will be seen that the mode of the horizontal vibration of which the amplitudes at c and d became maximum is approximately parallel to the long breadth direction of the house, on the other hand, that of the vibration of which the amplitudes at a and b became maximum is the horizontal vibration on a axis lying near the east end of the house. Figs. 22a, 22b, 22c and 22d show the horizontal amplitude distributions along the long breadth direction of the house at peculiar moments within 1/4 cycle of vibration, including the moment when the amplitudes of *Rb*, *Rc*, *Ib* and *Ic* became maxima, respectively. The amplitude-period relations of both the beam and girder directions observed simultaneously and the figures of horizontal orbital motions at *Re* are shown in Figs. 23 and 24, respectively. It is found from Figs. 23 and 24 that No. 6 house vibrated in the beam direction at the long period of vibration,

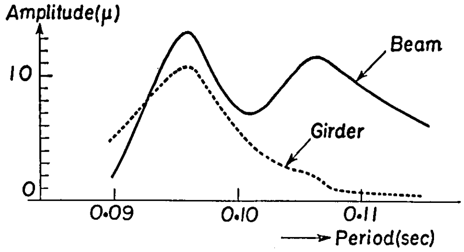

Fig. 23. Amplitude-period relation of both the beam and girder directions of No. 6 house observed simultaneously.

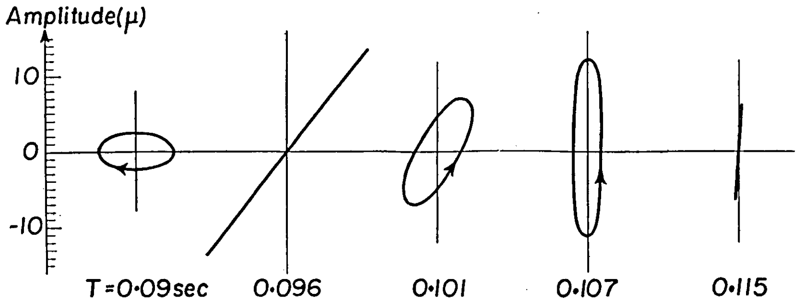

Fig. 24. Horizontal orbital motions at Re (roof-floor) of No. 6 house.

and in the phase difference of 90° and 0° between the amplitudes of two directions at the time of predominant vibrations of 0.108 sec and 0.098 sec periods, respectively. It seems to be natural to consider that the values of the proper periods of rocking and elastic vibrations of No. 6 house are near to each other, and the vibrations of the periods of 0.108 sec and 0.098 sec correspond to the vibrations of the beam and the diagonal directions, respectively.

4. Conclusion

The results of experiments using a vibration generator made in two buildings of the same structure standing on ground of different properties, that is, rock and crushed stone, proved the following more evidently than ever.

(1) In the buildings there take place the relative displacement at the foundation to the ground and rocking vibration together with elastic deformation, in the course of the vibration caused by a vibration generator. The theoretical investigations of this subject will be presented in the forthcoming Bulletin.

(2) The vertical members of buildings perform almost perfect bending vibration.

(3) The horizontal members and the vertical members of the building are clamped almost perfectly.

(4) The smaller the rigidity of the ground, the greater the vibration damping of the building.

(5) In general, the longest proper period of the building corresponds to the resonance period of rocking vibration.

(6) In the case of the ununiform property of ground, the building undergoes complicated vibration.

In conclusion, we wish to express our thanks to the Science Section of the Educational Ministry, for the financial aid (Research Funds) granted us. Also hearty thanks are due to the members of Motoyama Office, Hitachi Mine for their contributions and to Mr. K. Osada and Miss S. Yoshizawa for their cooperation in the course of these investigations.

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10. 建物の動揺，弾性振動 第1報 (起振機による振動実験)

地震研究所 金井清 田中貞二 鈴木富三郎

茨城県日立鉱山の岩盤上並びに砕石上に建つ2棟の3階建鉄筋コンクリート造アパートについて，起振機による振動実験を行つて，次の事柄を明かにした．

なお，本実験では同時に6ケの振動計を使い，記録紙の速度も早くしたので，振動型についての，相当にたちいつた議論ができた．

(i) 建物は，振動の際に，基礎と地面との間で相対水平変位と傾斜運動をしながら，各部材に弾性変形を生ずる．

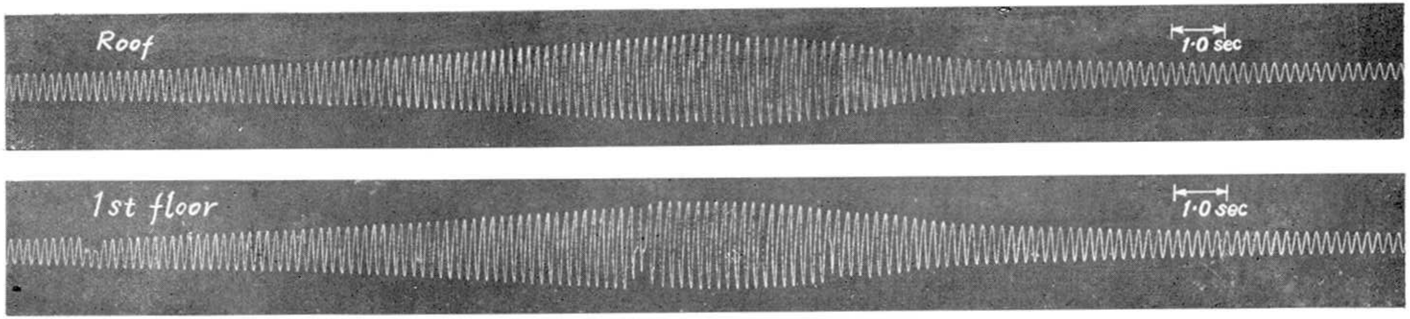

Fig. 6. Representative records of No. 14 house.

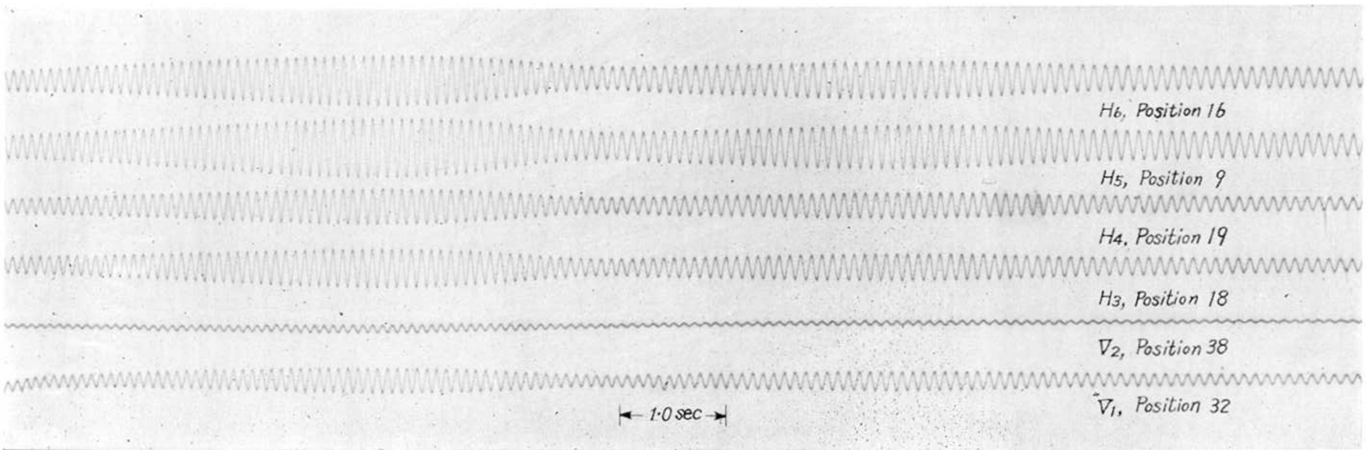

Fig. 16. Representative records of No. 6 house.

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(ii) 縦架材と横架材は完全固定の状態に近い.

(iii) 縦架材は両端固定の屈曲振動をする.

(iv) 一般に，建物の固有周期としては，動揺振動の周期があらわれやすい.

(v) 数多くの建物について，このような実験を行つた後でなければ決定的なことは言えないが，少くとも，本実験の結果だけからみると，基礎梁にも弾性変形が起り，特に，地盤が均質でない場合には基礎梁の変形の様子が非常に複雑である.