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3. *Calibration of an Electromagnetic Seismograph by Means of the Frequency Analysis*

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Abstract

The tapping test method for obtaining the response curve of an electromagnetic seismograph by the application of the theory of frequency analysis is described in this paper.

A slight tapping is applied to the pendulum, and the motion of the pendulum and the galvanometer activated by it are recorded at the same time. The spectra of both the recorded motions are calculated, and then the magnification, the frequency characteristics and the phase angle of the electromagnetic seismograph are reckoned.

This method enables us to get the response curve for both the amplitude and the phase angle for all frequency ranges and for all kinds of electromagnetic seismographs having arbitrary damping and arbitrary period both for the galvanometer and the pendulum without any shaking table or another special apparatus. This simple method with a high accuracy good enough for ordinary purposes would be very useful for the calibration of the electromagnetic seismograph in field work as well as in routine work.

1. Ever since Galitzin[1] developed the first electromagnetic seismograph, valuable studies on this subject have been carried out by many authorities.[1)–8)]

1) B. GALITZIN, *Vorlesungen über Seismometrie* (*Leipzig: Teubner*).
2) J. COULOMB and G. GRENET, *Annales de Physique*, **11** (1935), 321–369.
3) G. SCHMERWITZ, *Annalen der Physik*, **5** (1939), 209–223.
4) J. ROYBNER, *Gerlands Beitr. z. Geophy.*, **55** (1939), 303–313.
5) F. T. WORRELL, *Bull. Seism. Soc. Amer.*, **32** (1942), 31–48.
6) S. K. CHAKRABARTY, *Bull. Seism. Soc. Amer.*, **39** (1949), 205–218.
7) K. TAZIME, *Zisin*, **7** (1954), 96–115.
8) J. P. EATON, *Bull. Seism. Soc. Amer.*, **47** (1957), 37–75.

In the use of electromagnetic seismographs, it is very important to know the characteristics of the seismograph exactly, but the method of obtaining these characteristics is not so simple. According to the ordinary theoretical method, we have to measure at least five constants to know the characteristics of the electromagnetic seismogragh as Coulomb and Grenet[2)] have pointed out. This method may include a considerably large error of measurement because the characteristics of the electromagnetic seismograph are expressed by the product or some complicated form of these constants, and especially this error on the characteristics may become larger when some of these constants are expressed by the damping constants of the pendulum and the galvanometer, which is liable to include a comparatively large error of measurement.

The direct metheod applying the shaking table test gives us a considerably good accuracy of measurement, but the usual seismograph such as the Galitzin type in which the mass of the pendulum is 7 kg. in weight is too large to apply the shaking table test. Further, if the characteristics of the seismograph extend over a considerably long period, it is also difficult to apply this direct method. Therefore it is desirable to find some simple method which enables us to make the calibration for all kinds of seismographs and for all ranges of period.

In this paper, a simple method applying the theory of frequency analysis to the tapping test of the electromagnetic seismograph is offered. This method is applicable for all kinds of seismographs and for all ranges of period.

2. The block diagram of the electromagnetic seismograph is illustrated in Fig. 1, in which $F(t)$ means the ground movement while $P(t)$ and $G(t)$ represent respectively the motion of the pendulum and that of the galvanometer caused by $F(t)$.

Expressing the spectra of $F(t), P(t)$ and $G(t)$ by $f(\omega), p(\omega)$ and $g(\omega)$ respectively, we get

$$\left.\begin{matrix} F(t) \\ P(t) \\ G(t) \end{matrix}\right\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left\{\begin{matrix} f(\omega) \\ p(\omega) \\ g(\omega) \end{matrix}\right\} e^{i\omega t}\, dw \tag{1}$$

and

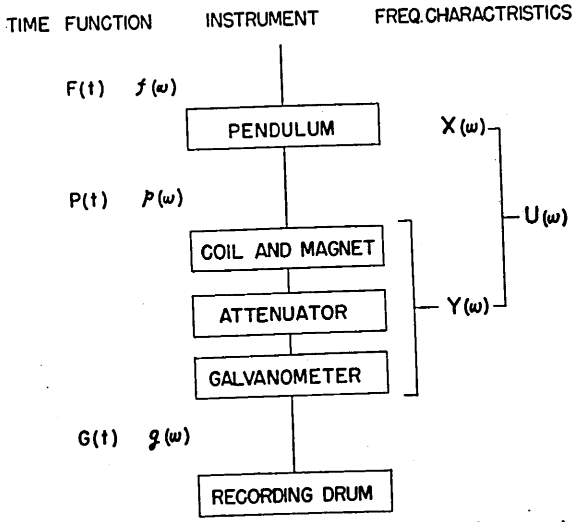

Fig. 1. The time functions, the frequency characteristics and the block diagram of the electromagnetic seismograph.

$$\left.\begin{matrix} f(\omega) \\ p(\omega) \\ g(\omega) \end{matrix}\right\} = \int_{-\infty}^{\infty} \left\{\begin{matrix} F(t) \\ P(t) \\ G(t) \end{matrix}\right\} e^{-i\omega t} dt \tag{2}$$

We denote the frequency characteristics of the instrument as follows

$X(\omega)$: frequency characteristics of the pendulum against the ground movement.

$Y(\omega)$: frequency characteristics of the galvanometer against the motion of the pendulum.

$U(\omega)$: frequency characteristics of the galvanometer against the ground movement.

Then we get the following relations

$$P(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) f(\omega) e^{i\omega t} d\omega \tag{3}$$

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega) p(\omega) e^{i\omega t} d\omega \tag{4}$$

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(\omega) f(\omega) e^{i\omega t} d\omega \tag{5}$$

The function $U(\omega)$ is the amplitude-frequency characteristics of the electromagnetic seismograph that we want to obtain. From equation

(1) and equation (5), we can get the following expression

$$U(\omega)=\frac{g(\omega)}{f(\omega)}=\frac{\int_{-\infty}^{\infty} G(t)e^{-i\omega t}dt}{\int_{-\infty}^{\infty} F(t)e^{-i\omega t}dt} \tag{6}$$

In other words, $U(\omega)$ is expressed by the ratio of $g(\omega)$ and $f(\omega)$ which are the spectra of $G(t)$ and $F(t)$ respectively.

If we were to apply the shaking table test, $F(t)$ and $G(t)$ must be a simple harmonic motion with a constant frequency, while in our present test $F(t)$ and $G(t)$ can be any arbitrary function of time.

In the actual test, it may be difficult to operate $F(t)$ by the form of displacement, therefore it would be wise to consider the case in which $F(t)$ is given by the form of acceleration.

If $F(t)$ is given by the unit impulse, $G(t)$ represents the response of the galvanometer against the unit impulsive acceleration of the ground, and $g(\omega)$ represents the acceleration-frequency characteristics of this seismograph.

If we get the acceleration-frequency characteristics, it will be easy to calculate the amplitude-frequency characteristics.

On the basis of the above theory, it would be preferable to adopt the following process to determine the magnification, the amplitude-frequency characteristics and the phase angle of the electromagnetic seismograph.

(1) When we want to record the corresponding $G(t)$ and $P(t)$ on a recording drum, we have to put an attenuator circuit by which we can control $G(t)$ to a suitable amplitude, keeping both the dampings of the pendulum and of the galvanometer in the original state.

(2) From the records of $P(t)$ and $G(t)$, we get the corresponding spectra $p(\omega)$ and $g(\omega)$, and therefore we get

$$Y(\omega)=\frac{g(\omega)}{p(\omega)}=\frac{\int_{-\infty}^{\infty} G(t)e^{-i\omega t}dt}{\int_{-\infty}^{\infty} P(t)e^{-i\omega t}dt} \tag{7}$$

(3) Although it is difficult to realise a perfect impulse, if the duration time of $F(t)$ is short enough compared to T_1 (the natural period of the pendulum) and T_2 (the natural period of the galvanometer), we can treat $F(t)$ as a perfect impulse in practice. If we calculate the damping constant of the pendulum h_1[9] from the recorded $P(t)$ which is activated

9) *loc. cit.* 8)

by an impulsive force $F(t)$, the frequency characteristics of the pendulum $X(\omega)$ is given by the well known formula following

$$X(\omega)=\frac{1}{\sqrt{(u^2-1)^2+4h_1^2u^2}}e^{-i\delta} \tag{8}$$

where

$$\delta=\frac{2h_1u}{u^2-1}, \qquad u=\frac{T}{T_1}.$$

T is the period of the ground movement, and T_1 is the natural period of the pendulum.

(4) From the foregoing procedure, we can get $X(\omega)$ and $Y(\omega)$, then the frequency characteristics of the electromagnetic seismograph for the displacement of the ground are expressed ultimately in the following form.

$$U(\omega)=X(\omega)Y(\omega) \tag{9}$$

(5) If the force $F(t)$ is a perfect impulse, the spectrum $f(\omega)$ has zero phase shift through the whole frequency ranges[10].

In this case, the phase angle for the displacement of the ground $\theta(\omega)$ is expressed in the following form

$$\theta(\omega)=\arg g(\omega)-\pi \tag{10}$$

Table I.

	Transducer	Galvanometer
Name of the instrument	HES-H	HES-R-1
Natural period	1.0 sec.	1.2 sec.
Resistance of the coil	1,523 ohm	500 ohm
Shunt resistance of critical damping	1.400 ohm	1.800 ohm
Total flux of the magnet	1.0×10^5 maxwell	2.5×10^4maxwell
Sensitivity	*2.47×10^{-5} A/mm	0.70×10^{-8} A/mm
Weight of the mass	1×10^4 gram	0.251 gram
Moment of inertia	3.78×10^5 gram cm^2	

* The length of the optical lever is 100 cm.

3. As an example, a practical test of an electromagnetic seismograph will be described below.

10) S. GOLDMAN, *Information Theory*, New York (1953,) 220.

The electromagnetic seismograph used in this measurement was the same kind of instrument which has been working in the seismological station of our institute at Mt. Tsukuba for the routine observation since May, 1956. The constants of this instrument are shown in Table 1. The HES–H transducer is a moving-coil type instrument employing a single magnet for transducing and damping.

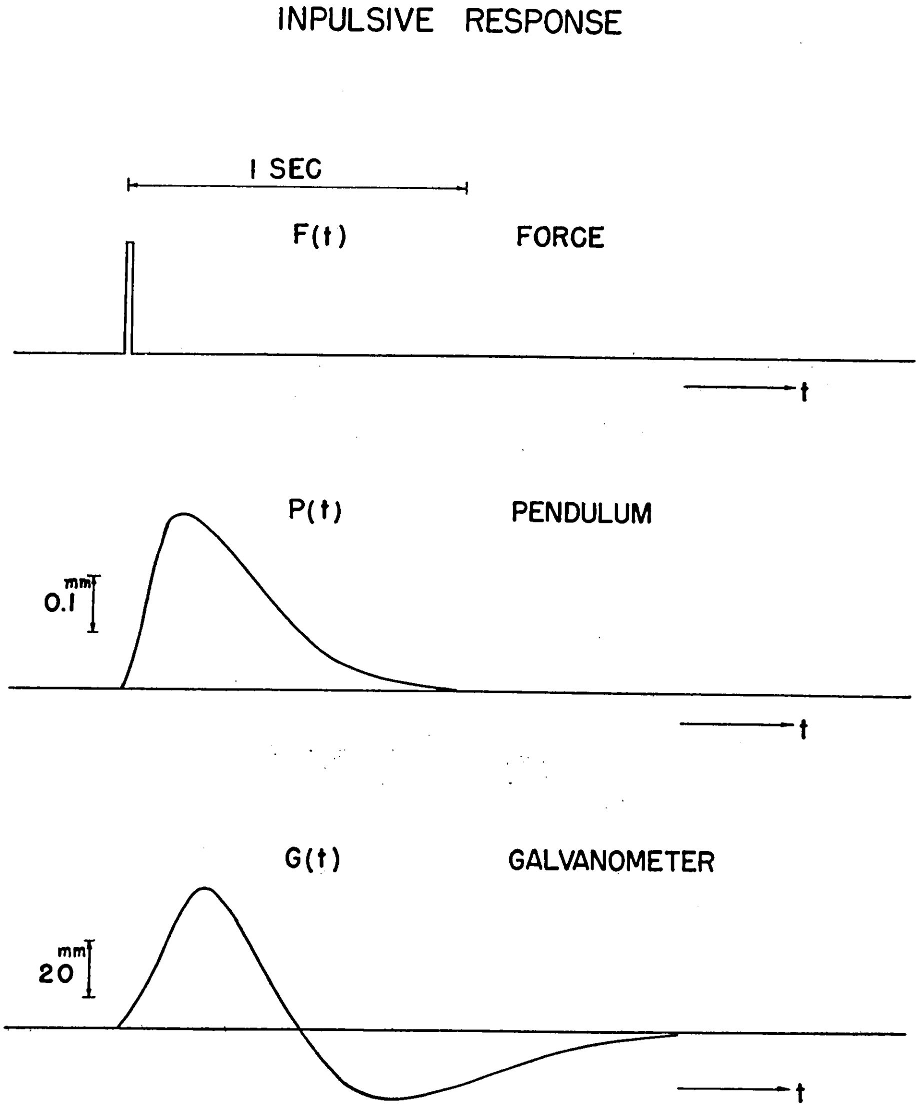

Fig. 2 The motion of the pendulum and the galvanometer activated by an impulsive force. The scale of versissa of $P(t)$ is refered to the centre of the percussion of the pendulum.

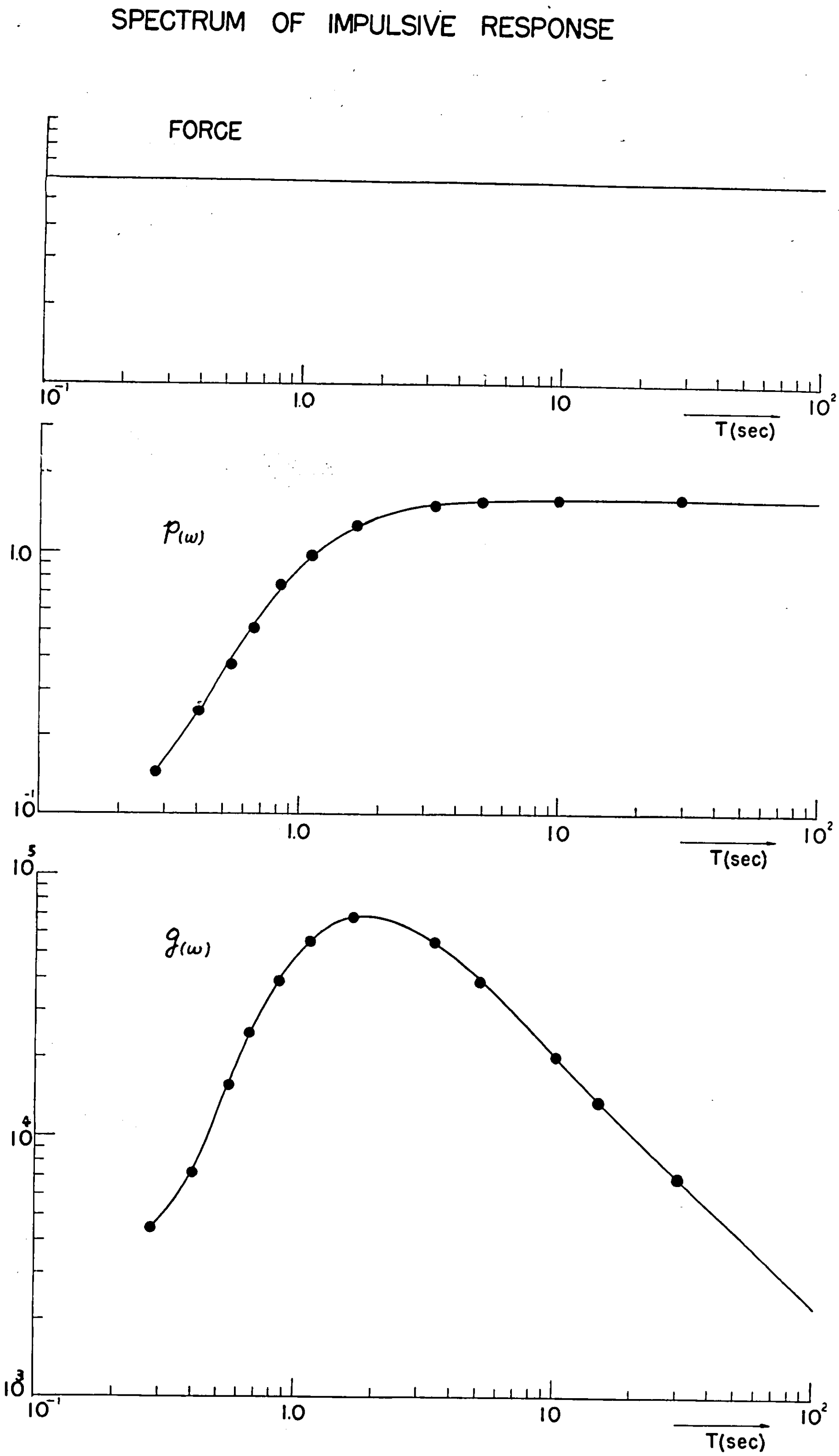

Fig. 3. The spectra $f(T)$, $p(T)$ and $g(T)$ corresponding impulsive response.

In routine work, the damping constants of transducer and galvanometer are adjusted equal to unity, and the ratio of reduction by the attenuator is 1/10. In this calibration, the attenuator was selected so as to make the damping constants remain at unity both for pendulum and galvanometer, the ratio of reduction being 1/1540 to control the amplitude of $G(t)$.

The tapping force was given by use of a small relay operated by the discharge current of a condenser, and the duration time of $F(t)$ was about 50 milliseconds.

The record of the motion of the pendulum and the galvanometer caused by this tapping are shown in Fig. 2.

The motion of the pendulum was recorded by an optical lever mothod. For this purpose, a small mirror was attached near the axis of rotation of the pendulum. In this figure, the scale of versissa is referred to the centre of the percussion of the pendulum. From these records, the

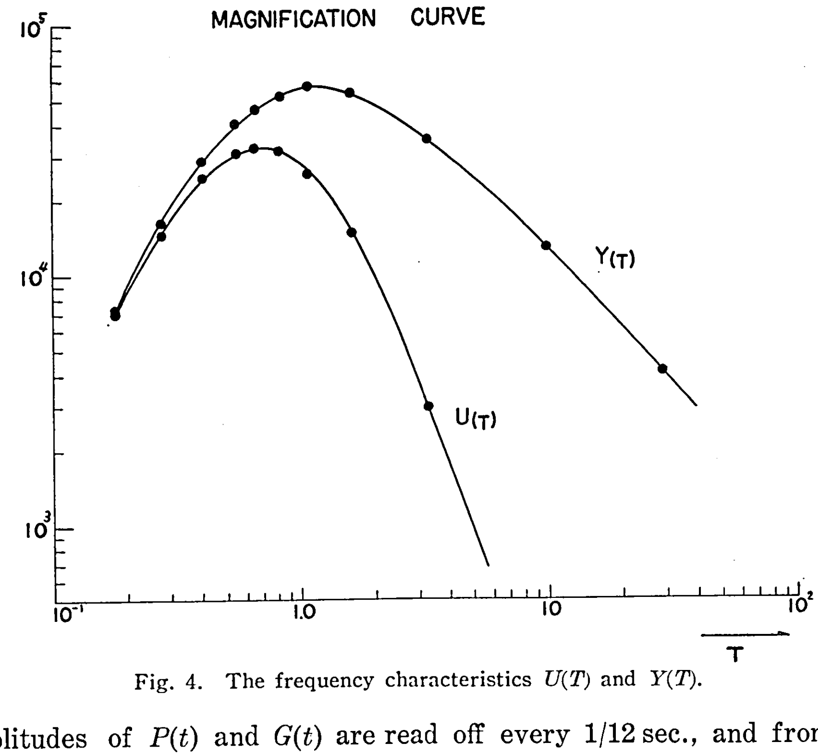

Fig. 4. The frequency characteristics $U(T)$ and $Y(T)$.

amplitudes of $P(t)$ and $G(t)$ are read off every 1/12 sec., and from this time series the spectra $p(\omega)$ and $g(\omega)$ were calculated. These spectra are shown in Fig. 3. In this figure the versissa of $g(\omega)$ is refered to the state when the ratio of reduction by the attenuator is 1/10.

The frequency characteristics $Y(\omega)$ and $U(\omega)$ calculated by equations (8) and (9) are shown in Fig. 4.

Also the phase angle of this electromagnetic seismograph $\theta(\omega)$ driven by equation (10) are shown in Fig. 5.

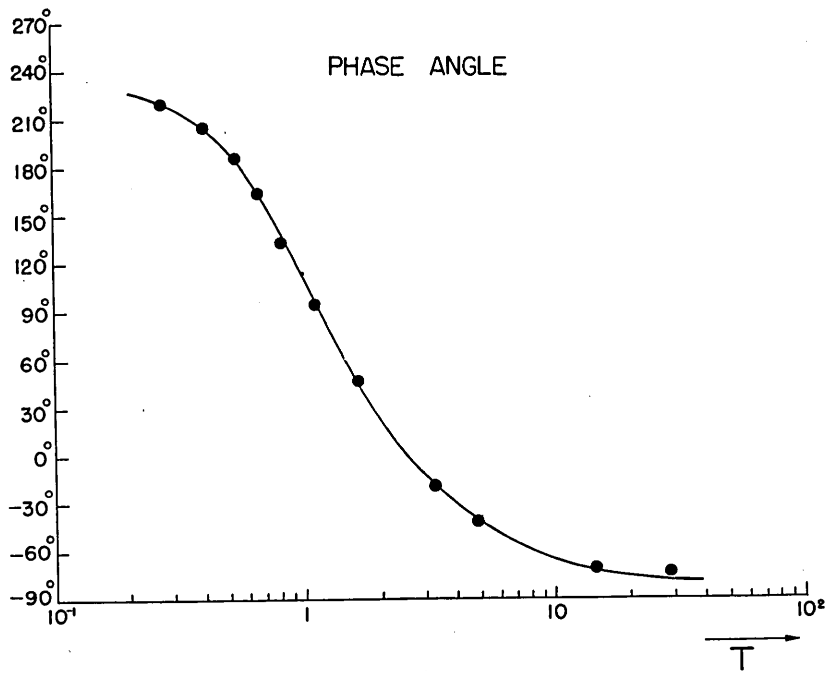

Fig. 5. Phase Angle of HES-R-1 seismograph

4. In this method, it is essential to record $P(t)$ and $G(t)$ caused by the same tapping, therefore we are obliged to choose a ratio of reduction by the attenuator which is, in general, highter than the ratio used in routine observation. For this reason, it must be remarked that the coupling factor differs from the usual state and we cannot measure the effect of coupling by this method. However, since the seismograph are usually designed to make the coupling factor very small, consideration of coupling will be unnecessary in usual case.

The accuracy of this method was found to be satisfactory, and the results obtained by this method agree well with the results obtained by the dynamical test which was carried out by giving the pendulum a sinusoidal force instead of impulsive force in the range from 4 sec.

to 0.3 sec.. Also these results agree well with the response calculated by the ordinary method based on the measurement of the period and the damping of the pendulum and the galvanometer[11]. In conclusion, the writer wishes to express his hearty thanks to Prof. T. Hagiwara who gave him valuable suggestions and advice about this study.

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3. 電磁地震計の特性を周波数分析によつて求める方法

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換振器に軽いパルスを与え，対応する換振器と検流計の運動を同時に記録し，その記録の周波数分析を行い，その周波数成分を比較することによつて，電磁地震計の周波数特性，倍率，および位相角を求めた．

従来行われている方法では，これらの特性は少くとも5個以上の測定値の組合せから導かれるが，測定が繁雑であると共に多くの誤差が生じやすい．

ここに述べられた方法は，広い帯域にわたつて，あらゆる型の地震計に適用できるものであり．また測定の精度も充分に良好である．

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11) A description of this method is now being printed by T. Hagiwara.