33. *The Influence of the Magnetic Field on Spectra of Seismic Core Waves.*

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Summary

The influence of the magnetic field on seismic waves being propagated in the electrically conductive core has been studied for various values of magnetic field intensity and electrical conductivity. It has resulted in knowledge that seismic waves passing through the core are not affected by the magnetic field unless the field intensity exceeds 10^6 gauss or the electrical conductivity is smaller than 10^{-8} emu. The spectral analyses of an actual seismogram indicated that the spectrum of seismic waves should be considered as not having been changed owing to the propagation through the core. Hence possible limits of the magnetic field intensity, (H), and the electrical conductivity, (σ), in the core would be inferred as $H<10^7$ gauss and $\sigma>10^{-8}$ emu.

1. Introduction

Possible interactions between magnetic field and seismic waves have been recently noticed by some geophysicists in the hope of investigating the magnetic field and the electrical state in the earth's core. If the magnetic field in the electrically conductive core were strong enough, both transmitted and reflected waves would be strongly affected by the electromagnetic force associated with magneto-hydrodynamic or magneto-elastic couplings between motions and magnetic fields. From the fact that *ScS* waves do not seem to be affected by magneto-hydrodynamic waves which will be generated in the electrical conductor permeated by magnetic flux, Rikitake[1] has determined the possible limit or range of the magnetic field intensity (H) and the electrical conductivity (σ) in the core. His results are $H<10^4$ gauss and $10^{-8}<\sigma<10^{-5}$ emu. Al-

* Communicated by T. Rikitake.

1) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **30** (1952), 191-205.

though the details have not yet been published, Hide and Runcorn[2] have also obtained $H<10^6$ gauss from the study of electro-magnetic damping of P wave within the earth.

Some attempts based on solid or liquid state physics have been also made in order to estimate the electrical conductivity in the core as accurately as possible. Elsasser has obtained $\sigma=1.3\times10^{-5}$ emu from a calculation under the assumption that the core is composed of metallic iron under high pressure and high temperature. Meanwhile, Bullard has reached $\sigma=1\times10^{-6}$ emu by taking into account the pressure and temperature variations empirically determined in the earth.

In this paper the writer studies the characteristics of core waves by applying spectral analyses to various phases of seismic waves observed at Tokyo on the occasion of a particular earthquake that occurred in the Atlantic near Argentina. On the basis of the study, the writer also considers the possibility of inferring the possible range of the intensity of the magnetic field and the electrical conductivity in the earth's core. Although no exact results are obtained, the possible range suggested by the present study is almost the same as those given by Rikitake, Hide and Runcorn.

2. Theory

As has been studied in an electrically conductive fluid, motions in an elastic conductor permeated by magnetic flux are affected by so-called Lorentz force. When elastic waves are propagated through a magnetic field, the velocity and the amplitude of waves are to be modified by couplings between motions and fields. If it is supposed that there were an extraordinarily strong field in the core which is highly conductive, frequency characteristics of seismic core waves might become different from those of the waves which do not pass through the core. By comparing the core waves, for example P' waves, with the other phase, PP, we might be able to obtain some information about the intensity of the magnetic field in the core.

The equation of motion when elastic waves are progagated through an elastic conductor placed in a magnetic field has been studied by Knopoff[4]. He has applied his theory to the longitudinal waves passing through the core which is assumed to occupy a region only in which

2) R. HIDE, *Physics and Chemistry of the Earth*, *vol.* **1** (Pergamon Press, 1956), p. 116.

4) L. KNOPOFF, *Jour. Geophys. Res.*, **60** (1955), 441–456.

the magnetic fluxes are confined and outside which no magnetic flux exists. He concluded that the attenuation of seismic waves caused by the magnetic field is not significant and the interaction with the magnetic field is not an important mechanism in the propagation of seismic waves through the earth's core even for considerably high values of the intensity of the magnetic field and the conductivity. But Knopoff's assumption that the fluxes terminate at the boundary does not seem to fit the condition in the core. In this paper the writer will adopt some different model as the earth's core, that is, only the electrical conductivity is discontinuous at the boundary and the intensity of the magnetic field is constant everywhere.

Combining the following, the fundamental equation is obtained; one is the principal equation of elasticity and the other that of electromagnetic force,

$$\frac{\partial^2 \boldsymbol{U}}{\partial t^2}=\alpha^2 \text{ grad div } \boldsymbol{U}-\beta^2 \text{ rot rot } \boldsymbol{U}+\frac{\boldsymbol{F}}{\rho}\,, \tag{1}$$

$$\boldsymbol{F}=[\sigma(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})]\times\boldsymbol{B}\,, \tag{2}$$

where α and β are the velocity of the longitudinal and the transverse waves. ρ and σ denote density and electrical conductivity respectively, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric field and the magnetic induction, $\boldsymbol{U}(u_x, u_y, u_z)$ the displacement vector, $\boldsymbol{v}$ is the velocity, i.e. $\partial \boldsymbol{U}/\partial t$, and $\boldsymbol{F}$ represents the body force, the electromagnetic force in this case, exerting on the elastic body. With the aid of Maxwell's equations, $\boldsymbol{F}$ may be expressed in terms of $\boldsymbol{v}$ and $\boldsymbol{B}$, as follows;

$$\left(\nabla^2-\sigma\mu\frac{\partial}{\partial t}\right)\boldsymbol{F}=-\sigma\{\text{rot rot}(\boldsymbol{v}\times\boldsymbol{B})\}\times\boldsymbol{B}\,, \tag{3}$$

where μ is magnetic permeability. When the magnetic field is located in y-direction, substituting (1) into (3), the following equations will be obtained,

$$\left(\frac{\partial^2}{\partial z^2}-\sigma\mu\frac{\partial}{\partial t}\right)\left(\beta^2\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}\right)u_x=0\,, \tag{4}$$

$$\left(\frac{\partial^2}{\partial z^2}-\sigma\mu\frac{\partial}{\partial t}\right)\left(\beta^2\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}\right)u_y=0\,, \tag{5}$$

$$\left(\frac{\partial^2}{\partial z^2}-\sigma\mu\frac{\partial}{\partial t}\right)\left(\alpha^2\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}\right)u_z=\frac{\sigma B_y^2}{\rho}\frac{\partial^2 u_z}{\partial z^2\partial t}\,. \tag{6}$$

For the compressional plane waves

$$u_z = Ae^{ip(t-z/c)}, \tag{7}$$

which are propagated in z-direction, equation (6) becomes

$$\left(\frac{p^2}{c^2}+ip\sigma\mu\right)\left(p^2-\frac{\alpha^2 p^2}{c^2}\right)=ip\frac{\sigma B^2}{\rho}\frac{p^2}{c^2}. \tag{8}$$

According to Knopoff's expression, (8) and its solution are written as follows;

$$(c^2-\alpha^2)(c^2-iV_e^2)=c^2V_h^2, \tag{9}$$

$$2c^2=V_h^2+\alpha^2+iV_e^2\pm\sqrt{(V_h^2+\alpha^2+iV_e^2)^2-4iV_e^2\alpha^2}, \tag{10}$$

where V_h is the phase velocity of magneto-hydrodynamic waves, that is,

$$V_h=\frac{B}{\sqrt{\rho\mu}}, \tag{11}$$

and

$$V_e=\sqrt{\frac{p}{\mu\sigma}}. \tag{12}$$

In the case of $V_h^2 \ll \alpha^2$, the roots of equation (10) can be expressed in a simpler form,

$$C_1 \approx \alpha\left\{1+\frac{V_h^2}{2(\alpha^2-iV_e^2)}\right\}, \tag{13}$$

$$C_2 \approx i^{1/2}V_e\left\{1-\frac{V_h^2}{2(\alpha^2-iV_e^2)}\right\}. \tag{14}$$

On the other hand, if $V_e^2 \ll \alpha^2$, they will be approximated in somewhat different form,

$$C_1' \approx \sqrt{\alpha^2+V_h^2}\left\{1+\frac{iV_e^2V_h^2}{2(\alpha^2+V_h^2)^2}\right\}, \tag{15}$$

$$C_2' \approx i^{1/2}\frac{V_e}{\sqrt{1+\frac{V_h^2}{\alpha^2}}}. \tag{16}$$

From (13), (14), (15) and (16) it is obvious that the waves in the electrical conductor are attenuated owing to the existence of the magnetic field. For the waves whose velocities are C_1, C_2, C_1' and C_2', the attenuation factors are expressed respectively as follows;

$$C_1: \quad \exp\left[-pd\frac{V_h^2 V_e^2}{2\alpha^5\left(1+\dfrac{V_h^2}{\alpha^2}+\dfrac{V_e^4}{\alpha^4}\right)}\right] \tag{17}$$

$$C_2: \quad \exp\left[-\frac{pd}{\sqrt{2}\,V_e}\left\{1+\frac{(\alpha^2-V_e^2)V_h^2}{2\alpha^4\left(1-\dfrac{V_h^2}{\alpha^2}+\dfrac{V_e^4}{\alpha^4}\right)}\right\}\right] \tag{18}$$

$$C_1': \quad \exp\left[-pd\frac{V_h^2 V_e^2}{2(\alpha^2+V_h^2)^{5/2}}\right]$$

$$=\exp\left[-p^2d\frac{B^2}{2\mu^2\rho\sigma\left(\alpha^2+\dfrac{B^2}{\rho\mu}\right)^{5/2}}\right] \tag{19}$$

$$C_2': \quad \exp\left[-\frac{pd}{V_e}\sqrt{\frac{1}{2}\left(1+\frac{V_h^2}{\alpha^2}\right)}\right]$$

$$=\exp\left[-d\sqrt{\frac{\mu\sigma p}{2}\left(1+\frac{B^2}{\alpha^2\mu\rho}\right)}\right] \tag{20}$$

The second type wave specified by C_2 or C_2' is the rapidly attenuative wave. The depth of penetration δ where the amplitude of the wave diminishes to as small as $1/e$ of that of the incident one becomes,

$$\delta \doteqdot \left\{\frac{2}{\mu\sigma p}\times\frac{1}{1+\dfrac{B^2}{\alpha^2\mu\rho}}\right\}^{1/2} \tag{21}$$

for the wave C_2'. As for the first type wave C_1 or C_1', the attenuation is not so remarkable as for the second type wave, but the stronger the magnetic field intensity grows or the lower the electrical conductivity becomes, the more notable the attenuation will become.

Now let us consider an infinitely extended slab of an elastic conductor bounded by the planes $z=0$ and $z=d$. Outside this region the conductivity is zero everywhere. The compressional plane wave being propagated in z-direction yields at the boundary $z=0$ two modes of oscillation. One has the velocity of the ordinary elastic wave type C_1, and the other is the wave attenuating rapidly in the conductive region. These excited waves likewise produce two modes of waves at the other boundary $z=d$, so that four types of waves are in the slab. These four waves, together with the incident, refracted and reflected waves in the non-conducting region, should satisfy the following conditions at the boundary:

1) the displacements due to these waves are continuous,
2) the tangential component of the magnetic field is continuous,
3) the total stress composed of the mechanical stress and the electromagnetic stress is continuous.

Now that the intensity of the toroidal field in the core which has no radial component is inferred to be much stronger than that of the poloidal field[5], the seismic waves being propagated in the radial direction are nearly perpendicular to the field. In the calculation below we shall take as actual a model as possible.

When the magnetic field $(1/\mu)B(=H)$ is parallel to the boundary and perpendicular to the direction of propagation of the wave, we can easily derive the following relation in the electric conductor from Maxwell's equations.

$$\left(\frac{\partial}{\partial t}-\frac{1}{\sigma\mu}\Gamma^2\right)\boldsymbol{b}=\mathrm{rot}\left(\frac{\partial \boldsymbol{U}}{\partial t}\times \boldsymbol{B}\right), \tag{22}$$

where $\boldsymbol{b}$ is the induced magnetic flux density. Assuming

$$\boldsymbol{b}=\begin{pmatrix}0\\ b_y\\ 0\end{pmatrix}=\begin{pmatrix}0\\ Se^{ip(t-z/c_l)}\\ 0\end{pmatrix}, \tag{23}$$

the solution of (22) is obtained as follows,

$$b_y=\frac{-\dfrac{p}{c_l}BA_l}{i-\dfrac{1}{\sigma\mu}\dfrac{p}{c_l}}e^{ip(t-z/c_l)}, \tag{24}$$

5) E. C. BULLARD and H. GELLMAN, *Phil. Trans. Roy. Soc. London, A*, **247** (1954), 213-278.

where A_i is the amplitude of the compressional wave. To satisfy the boundary condition the total sum of these additive magnetic induction must be zero across the boundaries, because the magnetic induction $\boldsymbol{B}$ is constant everywhere and does not change with the time.

As mentioned above, the total stress consists of two kinds, the elastic stress and the electromagnetic one, which are in general represented by a tensor (T_{ij}). In this case it is sufficient only to satisfy the condition of continuity of the component T_{zz}, the conditions for the other components being automatically satisfied provided the condition for the component T_{zz} is fulfilled. T_{zz} is written as

$$T_{zz}=\rho\alpha^2\frac{\partial u_z}{\partial z}-\frac{1}{2\mu}(B+b_y)^2 . \tag{25}$$

The first term of this equation represents the elastic stress and the second Maxwell's stress due to the magnetic field. The value of the second term being continuous at the boundaries, because the tangential component of the magnetic field does not change there, so that we see that we have only to bring the first term into question. On taking into account the continuity conditions of the quantities calculated in (24) and (25) together with the continuity of the displacements at the boundaries $z=0$ and $z=d$, we have six equations to be solved.

From these equations the ratio K of the amplitude of the transmitted wave to that of the incident one is obtained as follows;

$$K=\frac{4q_2(q_1r_2-q_2r_1)e^{ip(d/\alpha)}}{\{q_2(r_1-1)-q_1(r_2+1)\}^2e^{-ip(d/c_1)}-\{q_2(r_1+1)-q_1(r_2+1)\}^2e^{ip(d/c_1)}}, \tag{26}$$

where

$$q_k=\frac{\dfrac{p}{c_k}}{i-\dfrac{V_e^2}{c_k^2}}, \tag{27}$$

$$r=\frac{\alpha}{c_k} . \tag{28}$$

In the course of the derivation of the transmission coefficient K, the writer assumed that the waves (specified by the velocity C_2), which are generated at one side of the boundaries, are attenuated so strongly that they have practically no effects upon the other boundary. The assumption will be justified by the fact that the depth of penetration of these waves is very small and will never reach the opposite boundary if the width of the conductive region is wide. In reality the earth's core is so large that the depth of penetration δ given in (21) is negligibly small compared with the dimension of the core, so that the above-stated assumption could safely be adopted.

Although we do not know any accurate values of the intensity of the magnetic field and the electrical conductivity in the core, it has been presumed that the magnetic field intensity is less than 10^3 gauss[6], and the electrical conductivity is $10^{-8}<\sigma<10^{-5}$ emu[7] from various points of view. In the calculation, it will be assumed that the velocity of seismic waves is 10 km/sec, the density $\rho=10\,\mathrm{g/cm^3}$, the permeability $\mu=1$ cgs emu, and the diameter of the core is 7000 km. With these values we see that the velocity of the magneto-hydrodynamic wave V_h is less than 1 m/sec, hence it will be allowable to make use of the approximations (13) and (14). Then the transmission coefficient will be approximately written as

$$K \doteqdot \frac{1}{1+\dfrac{2}{r_2}} \exp\left[ip\frac{d}{\alpha}\left(1+\frac{\alpha}{c_1}\right)\right]. \tag{29}$$

From this the following equation will be derived for $\sigma=10^{-6}$ emu.

$$|K|^2 \doteqdot \frac{1}{1+2\sqrt{\dfrac{1}{T}\times 10^{-3}}+\dfrac{2}{T}\times 10^{-6}}, \tag{30}$$

where T is the period of the seismic wave and $p=2\pi/T$. As a consequence it may be concluded that seismic waves having periods longer than 1 sec would be little affected by the magnetic field the intensity of which is less than 10^3 gauss.

By taking the product of the complex conjugate in (26), we can obtain the following real function for the transmission coefficient,

$$|K|^2=\frac{C}{A-B\cos\left(\dfrac{2\pi}{T}\dfrac{d}{c_1}+\theta\right)}, \tag{31}$$

6) E. C. BULLARD and H. GELLMAN, *loc. cit.*, 5).
7) T. RIKITAKE, *loc. cit.*, 1).

where A, B and C are all real-positive. Although A, B and C are the functions of period, they vary little provided the variation of period is not very large, while the cosine-term is subject to oscillations with small changes in the period. Accordingly we may regard A, B and C as constants comparing with the cosine-term in a small range of period and the transmission coefficient K takes the value between K_{min} and K_{max}.

$$\frac{C}{A+B} \leqq |K|^2 \leqq \frac{C}{A-B} \tag{32}$$

Table I. Mean values of square of transmission coefficient; K_0^2.

T (sec.)	σ (emu) / B (gauss)	10^{-9}	10^{-8}	10^{-5}	10^{-2}	10
10	10^3	0.980	0.994	1.000	1.000	1.000
	10^4	0.980	0.994	1.000	1.000	1.000
	10^5	0.980	0.994	1.000	1.000	1.000
	10^6	0.980	0.994	1.000	1.000	1.000
	10^7	0.948	0.949	0.957	0.957	0.957
4	10^3	0.969	0.990	1.000	1.000	1.000
	10^5	0.969	0.990	1.000	1.000	1.000
	10^6	0.970	0.990	1.000	1.000	1.000
	10^7	0.978	0.955	0.957	0.957	0.957

Table II. Minimum values of square of transmission coefficient; K_{min}^2.

T (sec.)	σ (emu) / B (gauss)	10^{-9}	10^{-8}	10^{-5}	10^{-2}	10
10	10^3	0.980	0.994	1.000	1.000	1.000
	10^4	0.980	0.994	1.000	1.000	1.000
	10^5	0.980	0.994	1.000	1.000	1.000
	10^6	0.980	0.994	1.000	1.000	1.000
	10^7	0.941	0.916	0.918	0.918	0.918
4	10^3	0.969	0.990	1.000	1.000	1.000
	10^5	0.969	0.990	1.000	1.000	1.000
	10^6	0.970	0.990	1.000	1.000	1.000
	10^7	0.934	0.916	0.918	0.918	0.918

Table III. Maximum values of square of transmission coefficient; K^2_{max}.

T (sec.)	B (gauss) \ σ (emu)	10^{-9}	10^{-8}	10^{-5}	10^{-2}	10
10	10^3	0.980	0.994	1.000	1.000	1.000
	10^4	0.980	0.994	1.000	1.000	1.000
	10^5	0.980	0.994	1.000	1.000	1.000
	10^6	0.981	0.994	1.000	1.000	1.000
	10^7	0.995	0.996	0.999	0.999	0.999
4	10^3	0.970	0.990	1.000	1.000	1.000
	10^5	0.970	0.990	1.000	1.000	1.000
	10^6	0.970	0.990	1.000	1.000	1.000
	10^7	1.026	0.998	1.000	1.000	1.000

Table I, II and III show these K_0, K_{min} and K_{max} respectively, where K_0 represents the transmission coefficient in case of the sinusoidal term being zero, that is to say, $K_0=C/A$. In Fig. 1 the relation between the transmission coefficient K_0 and the electrical conductivity is shown taking the intensity of the magnetic field as the parameter, and in Fig. 2 the relation between K_0 and the magnetic field is also shown by taking the conductivity as the parameter.

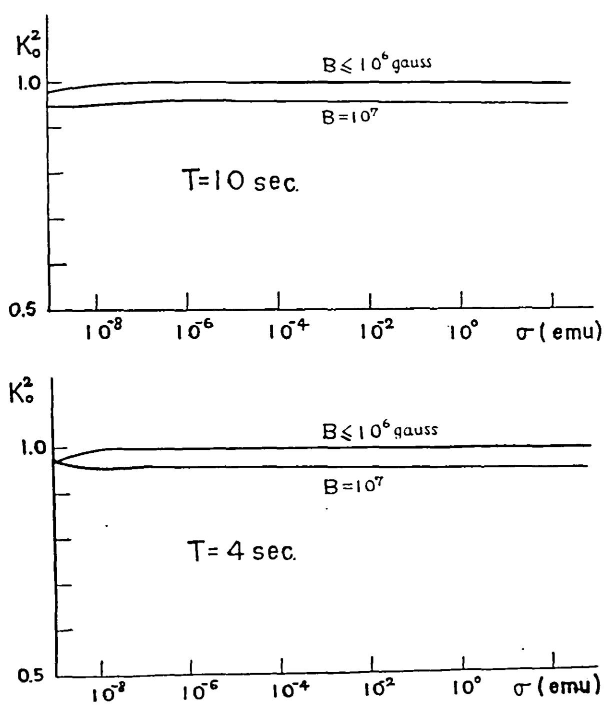

Fig. 1. Relation between the transmission coefficient (K_0) and the electrical conductivity (σ) for T=10 sec. and 4 sec.

From these it might well be said that the intensity of the magnetic field should exceed 10^6 gauss in order to have a significant influence upon the seismic waves passing through the core and that the electrical conductivity would not show any effect as long as it takes a value larger than 10^{-8} emu.

3. Spectral analyses of core waves.

It has been so far believed that predominating periods of seismic core waves cover a range roughly from $T=1$ to 5 sec., though, as far as the writer knows, no detailed analysis has been carried out. In this context, the writer would here like to analyse a seismogram of a distant earthquake as one example of the records of core waves. Since only one seismogram is available, the reliability of the results will not be very great. However, this sort of study would serve to make clear the differences, if any, between the properties of core waves from those of the other waves such as propagated in the mantle. Since no detailed study has been published yet, it would be also of interest to investigate the characteristics of core waves by means of spectral analysis.

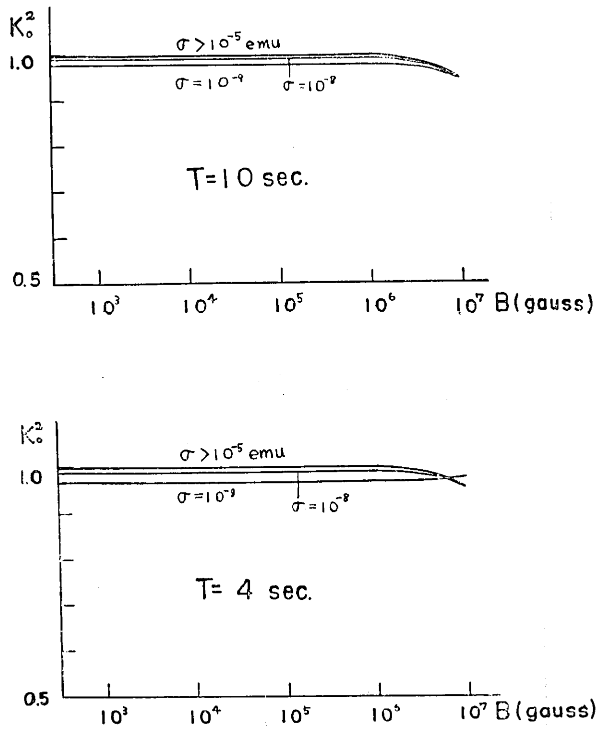

Fig. 2. Relation between the transmission coefficient (K_0) and the magnetic field (B) for $T=10$ sec. and 4 sec.

The earthquake taken up here occurred near South Georgia Island (the epicenter was at 29.5°W, 54°S) at $12^h47^m05^s$ (GMT) on June 27th, 1929 and its magnitude was $M=7.8$. Seismograms used for the analyses are those obtained at Hongô station with Ewing's vertical seismograph ($T=10.8$ sec., $h^2=0.05$) and Oomori's horizontal one ($T=31.1$ sec., $h^2=0.11$) (Fig. 3, Fig. 4). The amplitude spectra of this earthquake was computed in accordance with Kasahara's method[8]. The results are shown

8) K. KASAHARA, *Bull. Earthq. Res. Inst.*, **35** (1957), 473–532.

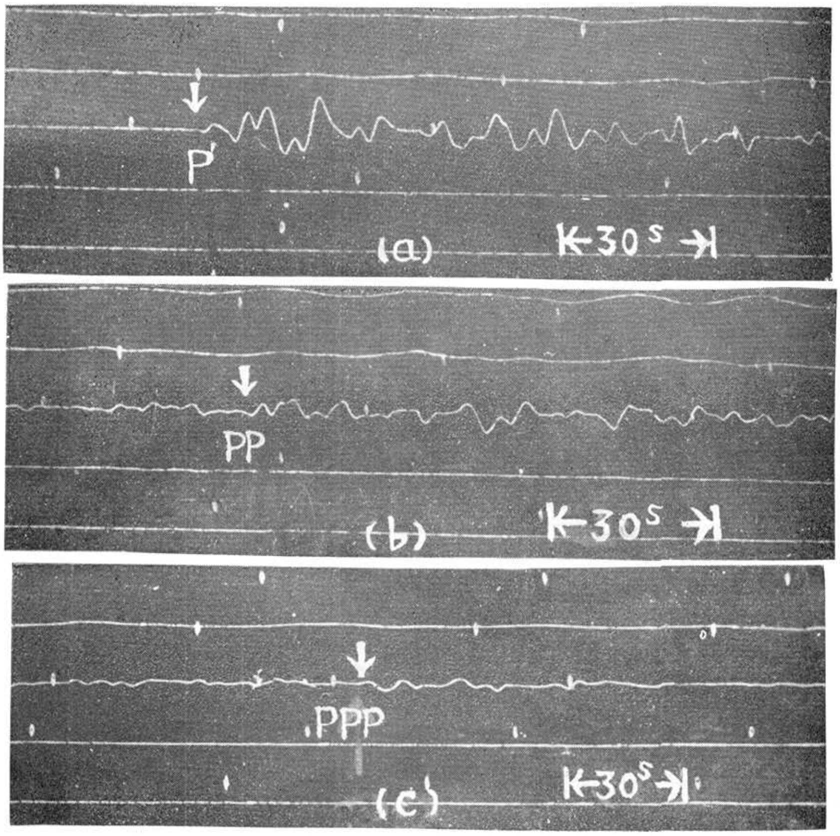

Fig. 3. Seismograms used for analyses of the vertical components: (a) P' wave, (b) PP wave, (c) PPP wave.

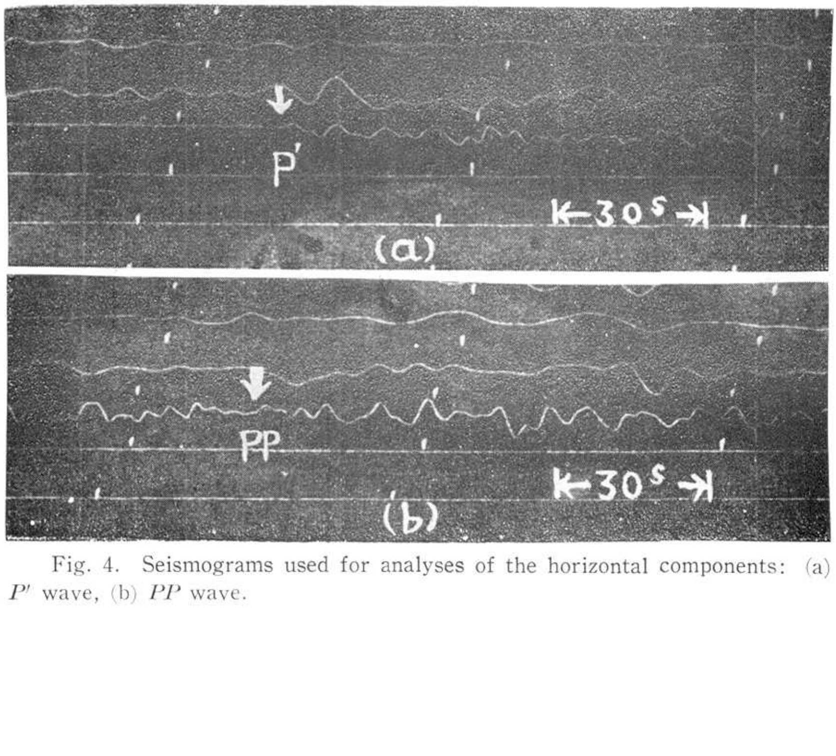

Fig. 4. Seismograms used for analyses of the horizontal components: (a) P' wave, (b) PP wave.

in Fig. 5 and Fig. 6 in which, as a matter of course, the corrections for alteration of spectra due to the characteristics of seismographs are made. In this case P' wave seems to have the maxima of the amplitude at about $T=2.2$, 4 and 10 sec., while it does not seem that the short period waves are predominating in particular. It may rather be said that the waves around $T=10$ sec. are most predominating for P' as we can also see in the other phases such as PP.

In order to study the characteristics of P' wave, it is necessary to depict the idealized spectrum of the longitudinal wave that can be regarded as the one which would travel along the same path as that of P' without being affected by the core and to compare it with the spectrum of P' wave itself. For the purpose, PP or PPP which are propagated through the mantle of the earth and recorded on the same seismogram would serve to take the place of such a virtual wave. The spectra of PP and PPP waves are also shown in Fig. 5 and Fig. 6 together with that of P'. As for the up and down component of the seismogram which seems to give more precise information about the longitudinal waves than the horizontal component, the amplitude of PP is smaller than that of P' wave on the whole, and moreover the amplitude of PPP is smaller than that of PP. In addition to these differences that can be seen in the amplitude ratio, the periods of maximum amplitude of PPP seem to have shifted from those of PP, a similar tendency being also observed between P' and PP. From these facts it is apparent that on reflection of seismic waves at the earth's surface, their spectra are likely to be transformed by some unknown mechanism; the amplitude is diminished and the period is some what lengthened. Therefore the spectra of PP and PPP are not to be used by themselves as the standard of comparison and the spectrum corrected for the effect of reflection is now required.

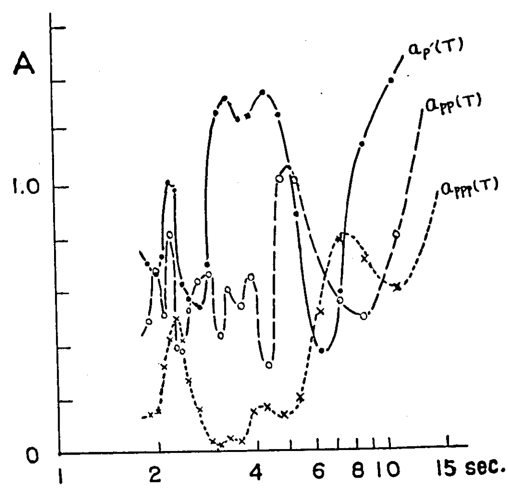

Fig. 5. The amplitude spectra of the vertical component: $a_{p'}(T)$ is the spectrum of P' wave, $a_{pp}(T)$ that of PP wave, $a_{ppp}(T)$ that of PPP wave: ordinate in arbitrary unit.

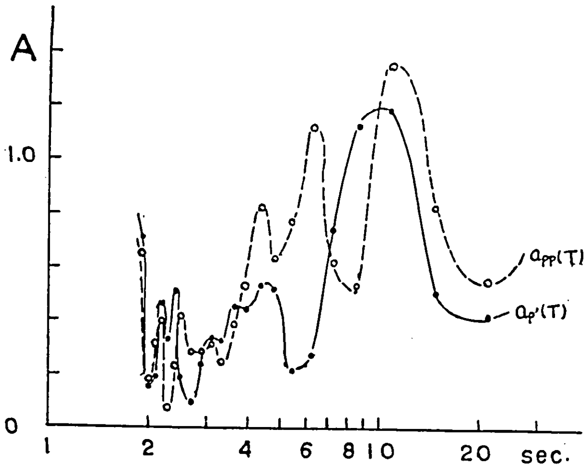

Fig. 6. The amplitude spectra of the horizontal component: $a_{P'}(T)$ is the spectrum of P' wave, $a_{PP}(T)$ is that of PP wave: ordinate in arbitrary unit.

It is assumed here that the every reflection has the same influence upon the amplitude spectra of seismic waves. The effect of reflection would be surmised from the appearence of the spectra of PP and PPP, though by what mechanism it is exerted on the seismic waves are unknown. Before proceeding further, we must pay attention to the attenuation of amplitude with the length of the path. According to Gutenberg's study[9] the square of amplitudes of P that has travelled in the mantle is proportional to $e^{-0.00012D}$, that is to say,

$$u^2 \propto e^{-0.00012D} ,$$

where D is the whole path of the waves. Taking this absorption into account and taking the path length of P' as a unit of length, the spectra of PP and PPP are redrawn in Fig. 7, in which the small fluctuations of the spectra are neglected and the curves are smoothed.

In Fig. 7 good correspondence can be seen between the individual traits of curves of PP and PPP, a_1 might be transferred to a_2, b_1 to b_2 and c_1 to c_2. To express this mathematically, the following relation will be assumed for the first approximation between the spectrum of PP, $A_{PP}(T)$ and that of PPP, $A_{PPP}(T)$,

$$A_{PPP}(T+kT)=f(T)\cdot A_{PP}(T) , \tag{33}$$

where $f(T)$ is the operator of real function which represents the characteristics of reflection. As the value of k

$$k=0.2$$

9) B. GUTENBERG, *Bull. Seis. Soc. Amer.*, **35** (1945), 57-69.

is obtained from Fig. 7. Fig. 8 shows the function $f(T)$ as calculated numerically. If the same relation as in the expression (33) is also taken for granted between the spectra of PP and the virtual wave suffering no effect of reflection, namely,

$$A_{PP}(T+kT)=f(T)\cdot A_{OP}(T) \tag{34}$$

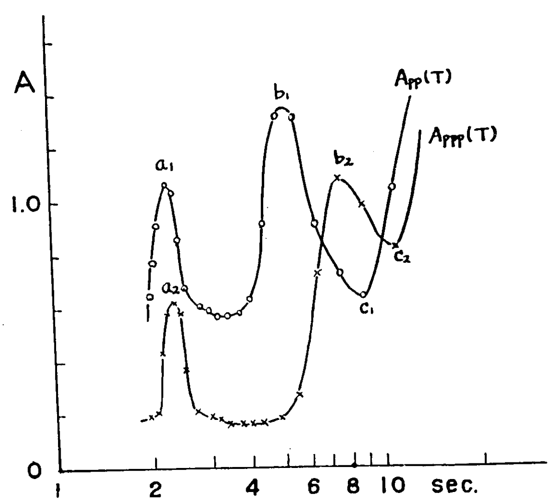

Fig. 7. The corrected curves of spectra, $A_{pp}(T)$ and $A_{ppp}(T)$, for the effect of absorption due to the difference of the path length: ordinate in arbitrary unit.

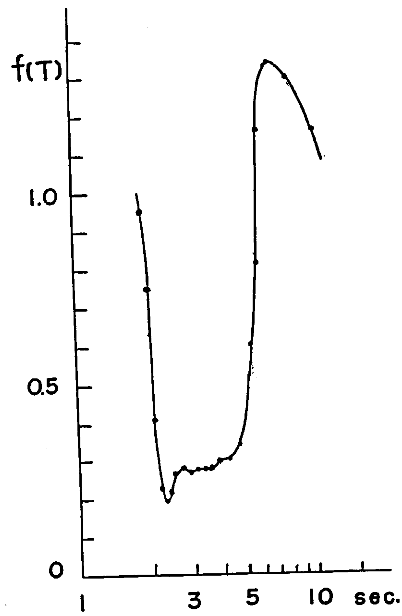

Fig. 8. $f(T)$

where $A_{OP}(T)$ is the spectrum of the virtual wave, $A_{OP}(T)$ may be easily calculated. $A_{OP}(T)$ thus obtained is the spectrum of the wave arriving at the surface with the same angle of incidence with that of PP. Since the amplitude of the seismic wave differs also on account of the difference of the angle of incidence and the epicentral distance, the spectrum of the virtual wave $A_0(T)$ is not the same as $A_{OT}(T)$. For this reason amplitude ratio of P' to PP is calculated to become 0.375 at $\Delta=155°$ by Dana[10]. Taking it into account, $A_0(T)$, the standard spectrum of the longitudinal wave assumed to have passed through the core without any influence of it is obtained in Fig. 9.

10) S. W. DANA, *Bull. Seis. Soc. Amer.*, **35** (1945), 27–35.

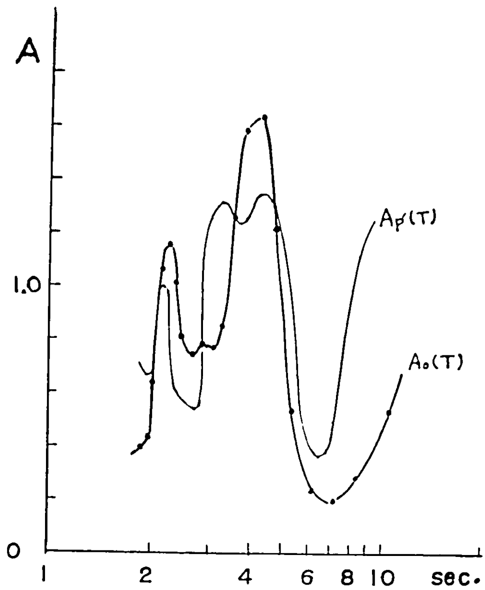

Fig. 9. Comparison of the spectrum of the virtual wave $A_0(T)$ assumed to have travelled through the core without any influence of it with that of P' wave $A_{p'}(T)$ calculated from the seismogram: ordinate in arbitrary unit.

In Fig. 9 it will be noticed that the periods of maximum or minimum amplitude of $A_0(T)$ curve agree with those of $A_{P'}(T)$ curve fairly well, which is the curve of P' wave, though some discrepancy can be seen in the value of the amplitude. Taking into account the errors which unavoidably accompany the seismogram and are included in the semi-empirical formula used for the calculation of $A_0(T)$, the above discrepancy might not be regarded as very significant. Accordingly it might be concluded that the amplitude of the compressional wave would not be much affected by travelling through the core and it would be unlikely that the magnetic field or some other mechanism would be playing an important rôle in the propagation of the seismic waves in the earth's core.

The above statement would be also supported by Gutenberg's study[11]. He has investigated the absorption of the seismic waves in the core using $P'P'$ and P, as well as in the mantle, and obtained almost the same absorbing factor as in the mantle. This means that the core would be exerting no significant influence upon the propagation of seismic waves.

4. Concluding remarks

As stated before, if the core waves were subject to some remarkable modification owing to the propagation through the core, the physical state in the core would be brought to light partly. The results of analysis of core waves, however, have clarified that there seems to be no effect of travelling through the core, if any, there must be little. According to the calculation of interactions between seismic waves and

11) B. GUTENBERG, *loc. cit.*, 9).

magnetic field, as shown in Fig. 1 and Fig. 2 the amplitude ratio of the transmitted wave to the incident one is not affected unless the intensity of the magnetic field exceeds 10^6 gauss, or the electrical conductivity is smaller than 10^{-8} emu. Therefore as a limit of the magnetic field intensity and the electrical conductivity in the core $H<10^7$ gauss and $\sigma>10^{-8}$ emu could be safely adopted. As for the conductivity the above value has agreed with the lower limit obtained by Rikitake.

Also in case of using the model that the earth's core is highly conductive comparing with the mantle while the magnetic field is continuous across the core-mantle boundary, the influence of the magnetic field is not to be considered very important for the propagation of seismic waves through the core.

The spectral analyses of P', PP and PPP waves brought forth an interesting tendency that the spectra of these waves change by the reflection at the earth's surface. As far as we are concerned with the theory of elastic waves, no reasonable explanation can be offered for such a phenomenon. Although it is beyond the scope of the present study to examine the cause or mechanism of the reflection effect, it will be highly desirable to conduct ample investigations of the effect in connection with the structure of the crust or upper part of the mantle.

5. Acknowledgments

The writer would like to express his hearty thanks to Dr. T. Rikitake who has given him numerous helpful suggestions and valuable advice, and to Mr. K. Kasahara for his kind advice and instructions about the spectral analysis of seismic waves. He also wishes to thank Messrs. I. Yokoyama and S. Uyeda for their incessant encouragement and counsels in the course of this study.

33. 地球核を通る地震波に対する磁場の影響

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最近，地震波と地球磁場との相互作用を利用して地球核内の磁場や電気伝導度を求める試みが二三なされてきた．核内においてトロイダル磁場はポロイダル磁場より遙かに強い磁場を作つていると推定されるので，核と同程度の厚さを持つ電気伝導体中を磁場に垂直に伝播する地震波に対して，磁場の強さと電気伝導度をパラメーターとして透過係数を求めた．その結果，地震波に対する磁場の影響は殆んどないが，磁場の強さが 10^6 gauss を越えるか，電気伝導度が 10^{-8} emu より小さくなれば僅かに振幅の減少が期待される．

従来 P′ 波の振幅は短周期において卓越していると考えられていたが，アルゼンチン沖に起つた一つの地震をスペクトル分析によつて調べた結果，核を通過したために生じたと推定される地震波の特性は認められないことがわかつた．このことより，核内磁場の上限および電気伝導度の下限として，10^7 gauss, 10^{-8} emu，という値が得られる．

なお P′ 波の特性を求める際に併せて得られた PP, PPP 波のスペクトルをみると，何に起因するかは不明であるが，地表面の反射によつて地震波は振幅が著しく減衰し，卓越周期がのばされているという興味ある事実が認められた．