# 23. *The Requisite Conditions for the Predominant Vibration of Ground.*

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## 1. Introduction

The vibrational characteristics of ground became clear from the observational studies of earthquakes and micro-tremors and theoretical studies of multiple reflections of waves, as well as from statistical studies of earthquake damage. Roughly speaking, from the results of studies, it is known that there are two kinds of spectral response type peculiar to each ground. That is to say, one type of the spectral response of ground has a predominant peak and another is irregular in shape and has a number of peaks.

The ground of the former type is considered to consist of a single layer, and the semi-empirical formula for the seismic characteristics of ground has already been studied.[1]

The ground of the latter type is considered to consist of plural layers and the present paper mainly deals with the vibrational problem of this type of ground by means of theoretical studies.

In this investigation, for simplicity, we try to examine the multiple reflection, problems of elastic waves in the layer, in which distortional waves of the purely plane type of sinusoidal infinite train are propagated vertically upwards.

The notations used in this paper are as follow: $U$=displacement of primary waves, $U_n$=resultant displacement of each layer, $\rho_n$=density, $v_n$=velocity, $\alpha_n=\rho_n v_n/\rho_{n+1}v_{n+1}$, $H_n$=thickness, $t$=time, $p=2\pi/T$, $T$=period of waves, $f_n=2\pi/L_n$, $L_n$=wave length, $z$=vertical axis, $q_n=pH_n/v_n$, $n$=number of a layer from free surface.

1) K. KANAI, *Bull. Earthq. Res. Inst.*, **35** (1957), 20.
K. KANAI, R. TAKAHASI and H. KAWASUMI, *Proc. World Conference Earthq. Engg.*, June 1956, 31-9, equation (23). *Corrigenda* to the denominator of (23). Read 0.2 for 0.1.

## 2. Preliminary study

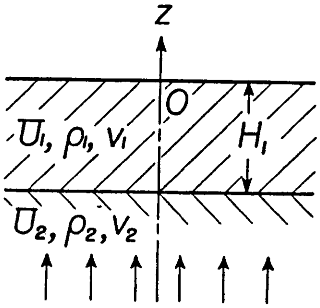


Fig. 1.

Firstly, we shall try to examine the simplest case of a single stratified layer.

If the incident waves in the lower medium be of the type expressed by the following

$$U_0 = \cos(pt - f_2 z)\ , \tag{1}$$

the resultant displacement of the stratum is expressed by[2)]

$$U_1 = \frac{2\cos\left(q_1 \dfrac{x}{H_1}\right)\cos\{pt - \tan^{-1}(\alpha_1 \tan q_1)\}}{\sqrt{\cos^2 q_1 + \alpha_1^2 \sin^2 q_1}}\ . \tag{2}$$

Then, the resultant displacement on the free surface is written by

$$|U_{1_{z=0}}| = \frac{2}{\sqrt{\cos^2 q_1 + \alpha_1^2 \sin^2 q_1}}\ . \tag{2'}$$

The requisite conditions for predominant vibration to appear at the ground surface may be obtained from the following equations:

$$\frac{\partial U_{1_{z=0}}}{\partial q_1} = 0\ , \tag{3}$$

and

$$\left|\frac{\partial^2 U_{1_{z=0}}}{\partial q_1^2}\right|_{q_1 = q_m} < 0\ , \tag{4}$$

in which $q_m$ is the solution of (3).

Substituting (2′) in (3), we obtain

$$(1 - \alpha_1^2)(\cos^2 q_1 + \alpha_1^2 \sin^2 q_1)^{-3/2} \cos q_1 \sin q_1 = 0\ . \tag{3'}$$

The solutions of (3′) become as follow:

$$\cos q_1 = 0\ , \quad \text{i.e.} \quad q_1 = (2m+1)\frac{\pi}{2}\ , \qquad [m = 0, 1, \cdots] \tag{5}$$

and

2) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **13** (1935), 253, Equation (9).

$$\sin q_1=0\ , \quad \text{i.e.} \quad q_1=(n+1)\pi\ . \quad [n=0, 1, \cdots] \tag{6}$$

The left term of (4) becomes as follow :

$$(1-\alpha_1^2)\{\cos^2 q_1(1+\sin^2 q_1)-\alpha_1^2\sin^2 q_1(1+\cos^2 q_1)\}(\cos^2 q_1+\alpha_1^2\sin^2 q_1)^{-5/2}\ . \tag{7}$$

Substituting (5) in (7),

$$\alpha_1^{-3}(\alpha_1^2-1)\ . \tag{8}$$

Then, (8) tells us that the condition of $\alpha_1<1$ satisfies (4). In other words, when $\alpha_1<1$, that is to say, the vibrational impedance of the stratum is less than that of the subjacent medium, the maximum of the vibration amplitude at the free surface appeared at such period as $T=4H_1/(2m+1)v_1$ and the amplitude at the surface in synchronous vibrations becomes $2/\alpha_1(\equiv 2\rho_2 v_2/\rho_1 v_1)$ by substituting (5) in (2).

In the same way, from (4), (6), (7) and (2), we knew that, when the vibrational impedance of the stratum is higher than that of the subjacent medium, the amplitude at the surface becomes maximum at such period as $T=2H_1/(n+1)v_1$ and the maximum amplitude at the surface is twice as large as the value of incident waves amplitude. This case has little meaning in connection with the nature of excitation of vibrations.

Now, taking only the fundamental mode of (3) into consideration, that is to say, the cases of $m=0$ in (5) and $n=0$ in (6), the amplitude distribution in the stratum can be written as shown in Figs. 2 and 3

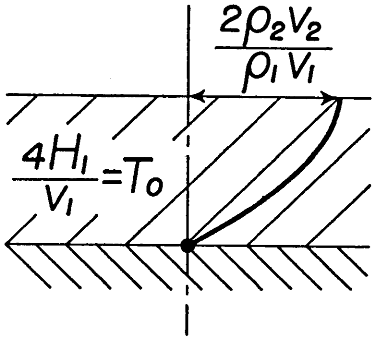


Fig. 2.

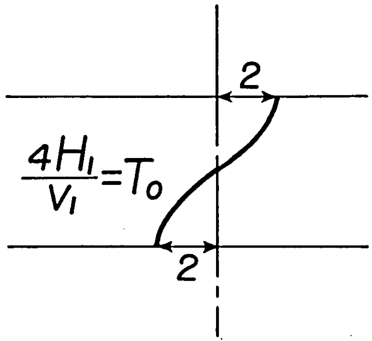


Fig. 3.

by using (2). Figs. 2 and 3 directly tell us the same conclusion which has been drawn from somewhat complicated calculation by using (4). The diagrammatical treatments like this seem to play an important part in the problem of plural stratified layers.

### 3. The case of two stratified layers

If the incident waves in the lowest medium be of the type:

$$U_0=\cos(pt-f_3z)\,, \tag{9}$$

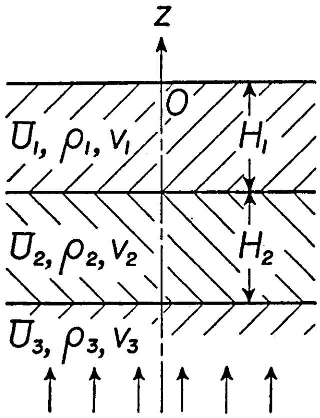

Fig. 4.

the resultant displacements in the each layer are expressed by[3)]

$$U_1=\frac{2}{\Phi}\cos q_1\frac{z}{H_1}\cos\left(pt-\tan^{-1}\frac{Q}{P}\right)\,, \tag{10}$$

$$U_2=\frac{2}{\Phi}\left\{\cos q_1\cos q_2\left(\frac{H_1}{H_2}+\frac{z}{H_2}\right)-\alpha_1\sin q_1\sin q_2\left(\frac{H_1}{H_2}+\frac{z}{H_2}\right)\right\}\cdot\cos\left(pt-\tan^{-1}\frac{Q}{P}\right)\,, \tag{11}$$

where

$$\left.\begin{aligned}&\Phi=\sqrt{P^2+Q^2}\,,\\&P=\cos q_1\cos q_2-\alpha_1\sin q_1\sin q_2\,,\\&Q=\alpha_2(\cos q_1\sin q_2+\alpha_1\sin q_1\cos q_2)\,.\end{aligned}\right\} \tag{12}$$

The requisite conditions for predominant vibration to appear at the ground surface may be obtained from (3) and (4), that is to say, $\partial U_{1_{z=0}}/\partial q_1=0$ and $|\partial^2 U_{1_{z=0}}/\partial q_1^2|_{q_1=q_m}<0$, in which $q_m$ is the solution of the former equation.

Substitution (3) in (10), we obtain the following equation:

$$AB-CD=0\,, \tag{13}$$

in which

$$\left.\begin{aligned}&A=\alpha_1\sin q_1\sin q_2-\cos q_1\cos q_2\,,\\&B=(\alpha_1 a_1+1)\sin q_1\cos q_2+(\alpha_1+a_1)\cos q_1\sin q_2\,,\\&C=(\alpha_1 a_1+1)\sin q_1\sin q_2-(\alpha_1+a_1)\cos q_1\cos q_2\,,\\&D=2\alpha_2(\alpha_1\sin q_1\cos q_2+\cos q_1\sin q_2)\,,\\&a_1=\frac{H_2}{H_1}\cdot\frac{v_1}{v_2}\,.\end{aligned}\right\} \tag{14}$$

3) K. KANAI, *Bull. Earthq. Res. Inst.*, **30** (1952), 31. Special case of $\xi_1=0$, $\xi_2=0$, $\xi_3=0$,

It is easy to find the general solutions of (13) by looking at (14), that is to say, the following conditions make $A$ and $C$ zero simultaneously and then they satisfy (13):

$$\cos q_1=0\,, \qquad \sin q_2=0\,, \tag{15}$$

$$\sin q_1=0\,, \qquad \cos q_2=0\,. \tag{16}$$

By the same way, we can find that the following conditions make $B$ and $D$ zero simultaneously and then they satisfy (13):

$$\cos q_1=0\,, \qquad \cos q_2=0\,, \tag{17}$$

$$\sin q_1=0\,, \qquad \sin q_2=0\,. \tag{18}$$

The conditions (15)-(18) may be rewritten as follows:

$$q_1\left(\equiv\frac{2\pi H_1}{Tv_1}\right)=(2m+1)\frac{\pi}{2}\,, \qquad q_2\left(\equiv\frac{2\pi H_2}{Tv_2}\right)=(n+1)\pi\,, \tag{15$'$}$$

$$q_1=(m+1)\pi\,, \qquad q_2=(2n+1)\frac{\pi}{2}\,, \tag{16$'$}$$

$$q_1=(2m+1)\frac{\pi}{2}\,, \qquad q_2=(2n+1)\frac{\pi}{2}\,, \tag{17$'$}$$

$$q_1=(m+1)\pi\,, \qquad q_2=(n+1)\pi\,. \tag{18$'$}$$

$$[m=0,1,\cdots,\quad n=0,1,\cdots]$$

It is considerably difficult to find mathematically whether the conditions expressed by (15′)-(18′) will satisfy the condition of (4) or not.

Then, we shall try the diagrammatical treatments as in the last part of the former Section, by using the fundamental modes of (15′)-(18′), in order to find the requisite conditions for predominant vibration to appear at the ground surface. Substituting $m=0$ and $n=0$ of (15′)-(18′) in (10) and (11), we obtain the displacement distribution in each layer as shown in Figs. 5-8. Figs. 5-7 show that, when the values of the numerator of the expressions of surface displacement are higher than those of their denominator, the predominant vibration at the ground surface will appear. On the other hand, Fig. 8 show that, this case has little meaning in connection with the nature of excitation of vibrations.

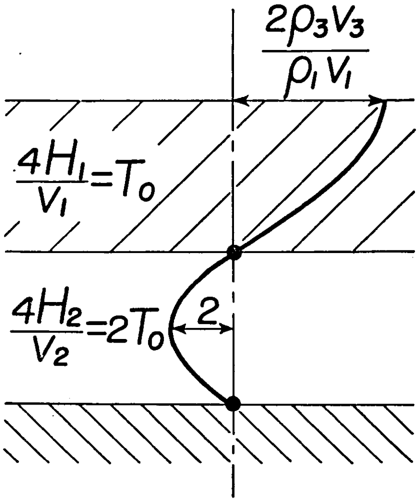


Fig. 5.

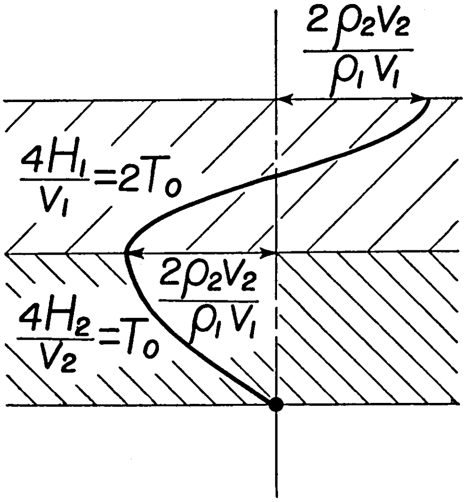


Fig. 6.

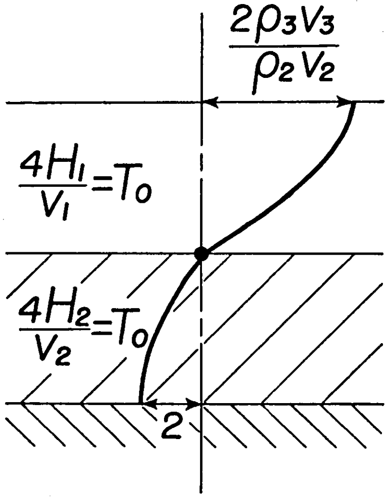


Fig. 7.

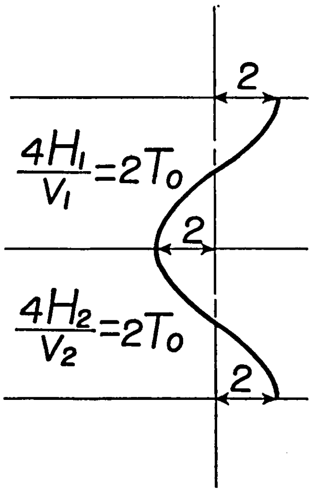


Fig. 8.

The requisite conditions for predominant vibration to be appeared at the ground surface, which may be found from these results, are enumerated as follows.

(i) In the case of $\rho_1 v_1 < \rho_3 v_3$, $T_0 = 4H_1/v_1$ and $H_2/H_1 = 2v_2/v_1$, the predominant amplitude at the surface is $2\rho_3 v_3/\rho_1 v_1$. In this case, the value of $\rho_2 v_2$ itself scarcely affects the surface amplitude. (See Fig. 5.)

(ii) In the case of $\rho_1 v_1 < \rho_2 v_2$, $T_0 = 4H_2/v_2$ and $H_2/H_1 = v_2/2v_1$, the predominant amplitude at the surface is $2\rho_2 v_2/\rho_1 v_1$. In this case, the value of $\rho_3 v_3$ itself has no connection with the surface amplitude. (See Fig. 6.)

(iii) In the case of $\rho_2 v_2 < \rho_3 v_3$, $T_0 = 4H_1/v_1$ and $H_2/H_1 = v_2/v_1$, the predominant amplitude at the surface is $2\rho_3 v_3/\rho_2 v_2$. In this case, the value of $\rho_1 v_1$ itself has no meaning in this problem. (See Fig. 7.)

These results may be explained by wave problem as follow: In general, since the wave reflections at the various boundaries interfere with one another, the spectral response of the amplitude of the surface is very irregular. Except in very unusual circumstances the maximum value of the peak is not large as in the case of the single layer. Otherwise, in some particular circumstances, when all the waves, being reflected at every boundaries and arriving at the free surface, are in the same phase, the surface amplitude grows up to an extremely large value.

It is a noteworthy fact that the rigidity itself of the medium of a layer among three layers scarcely affects the predominant vibration of ground surface. In other words, these results tell us that, it will be provable that the predominant vibration appears at the ground surface, even if a rigid medium exists at the superficial layer or intermediate one.

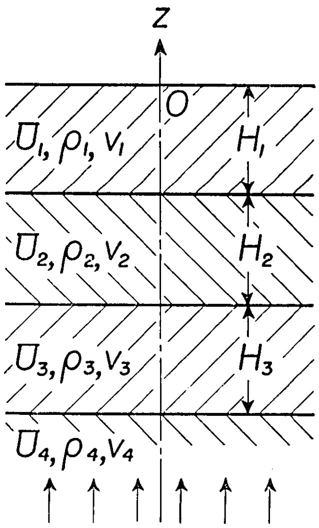

Fig. 9.

## 4. The case of three stratified layers

If the incident waves in the lowest medium be of the type expressed as follows

$$U_0 = \cos(pt - f_4 z)\ , \qquad (19)$$

the resultant displacements in the each layer are expressed by[4)]

$$\left.\begin{aligned} U_1 &= \frac{2}{\Phi}\cos f_1 z \cos\gamma\ , \\ U_2 &= \frac{1}{\Phi}\Big\{(1+\alpha_1)\cos\Big(q_1 - q_2\frac{z+H_1}{H_2}\Big) \\ &\qquad + (1-\alpha_1)\cos\Big(q_1 + q_2\frac{z+H_1}{H_2}\Big)\Big\}\cos\gamma\ , \end{aligned}\right|$$

4) K. KANAI and S. YOSHIZAWA, *Bull. Earthq. Res. Inst.*, **31** (1953), 275.

$$\left.\begin{aligned}U_3=\frac{2}{\Phi}\Big\{&(1+\alpha_1)(1+\alpha_2)\cos\left(q_1+q_2-q_3\frac{z+H_1+H_2}{H_3}\right)\\&+(1-\alpha_1)(1+\alpha_2)\cos\left(q_1-q_2+q_3\frac{z+H_1+H_2}{H_3}\right)\\&+(1-\alpha_1)(1-\alpha_2)\cos\left(q_1-q_2-q_3\frac{z+H_1+H_2}{H_3}\right)\\&+(1+\alpha_1)(1-\alpha_2)\cos\left(q_1+q_2+q_3\frac{z+H_1+H_2}{H_3}\right)\Big\}\cos\gamma\ ,\end{aligned}\right\}\tag{20}$$

where

$$\left.\begin{aligned}&\Phi=\sqrt{P^2+Q^2}\ ,\\&P=\cos q_3(\cos q_1\cos q_2-\alpha_1\sin q_1\sin q_2)\\&\qquad-\alpha_2\sin q_3(\sin q_1\cos q_2+\alpha_1\cos q_1\sin q_2)\ ,\\&Q=\alpha_3\{\sin q_3(\cos q_1\cos q_2-\alpha_1\sin q_1\sin q_2)\\&\qquad+\alpha_2\cos q_3(\cos q_1\sin q_2+\alpha_1\sin q_1\cos q_2)\}\ ,\\&\gamma=pt-f_4(H_1+H_2+H_3)-\tan^{-1}\frac{Q}{P}\ .\end{aligned}\right\}\tag{21}$$

The requisite conditions for predominant vibration to appear at the ground surface may be obtained from (3) and (4), that is to say, $\partial U_{1_{z=0}}/\partial q_1=0$ and $|\partial^2 U_{1_{z=0}}/\partial q_1^2|_{q_1=q_m}<0$, in which $q_m$ are the solution of the former equation. Substituting (3) in (20), we obtain the following equation :

$$AB+CD=0\ ,\tag{22}$$

in which

$$\left.\begin{aligned}&A=\cos q_3(\cos q_1\cos q_2-\alpha_1\sin q_1\sin q_2)\\&\qquad-\alpha_2\sin q_3(\sin q_1\cos q_2+\alpha_1\cos q_1\sin q_2)\ ,\\&B=\sin q_3\{\cos q_1\cos q_2(a_3+\alpha_2+\alpha_1\alpha_2a_2)\\&\qquad-\sin q_1\sin q_2(\alpha_1a_3+\alpha_2a_2+\alpha_1\alpha_2)\}\\&\qquad+\cos q_3\{\sin q_1\cos q_2(1+\alpha_1a_2+\alpha_2)\\&\qquad+\cos q_1\sin q_2(a_2+\alpha_1+\alpha_2)\}\ ,\\&C=\cos q_3\{-(a_3+a_2\alpha_2+\alpha_1\alpha_2)\cos q_1\cos q_2\\&\qquad+(a_3\alpha_1+a_2\alpha_1\alpha_2+\alpha_2)\sin q_1\sin q_2\}\end{aligned}\right.\tag{23}$$

$$\left.\begin{aligned}
&+\sin q_3\{(\alpha_1 a_2+\alpha_1\alpha_2 a_3+1)\sin q_1\cos q_2\\
&\quad+(a_2+\alpha_2 a_3+\alpha_1)\cos q_1\sin q_2\}\ ,\\
D&=\alpha_3\{\sin q_3(\cos q_1\cos q_2-\alpha_1\sin q_1\sin q_2\\
&\quad+\alpha_2\cos q_3(\cos q_1\sin q_2+\alpha_1\sin q_1\cos q_2)\}\ ,\\
a_2&=\frac{H_2}{H_1}\frac{v_1}{v_2}\ ,\qquad a_3=\frac{H_3}{H_1}\frac{v_1}{v_3}\ .
\end{aligned}\right\}$$

It will be able easy to find the general solutions of (22) by looking at (23), that is to say, the following conditions make $A$ and $C$ zero simultaneously and then they satisfy (22):

$$\cos q_1=0\ ,\qquad \cos q_2=0\ ,\qquad \cos q_3=0\ ,\tag{24}$$

$$\cos q_1=0\ ,\qquad \sin q_2=0\ ,\qquad \cos q_3=0\ ,\tag{25}$$

$$\cos q_1=0\ ,\qquad \sin q_2=0\ ,\qquad \sin q_3=0\ ,\tag{26}$$

$$\cos q_1=0\ ,\qquad \cos q_2=0\ ,\qquad \sin q_3=0\ .\tag{27}$$

In the same way, we find that the following conditions make $B$ and $D$ zero simultaneously, and then they satisfy (22):

$$\sin q_1=0\ ,\qquad \cos q_2=0\ ,\qquad \cos q_3=0\ ,\tag{28}$$

$$\sin q_1=0\ ,\qquad \sin q_2=0\ ,\qquad \cos q_3=0\ ,\tag{29}$$

$$\sin q_1=0\ ,\qquad \cos q_2=0\ ,\qquad \sin q_3=0\ ,\tag{30}$$

$$\sin q_1=0\ ,\qquad \sin q_2=0\ ,\qquad \sin q_3=0\ .\tag{31}$$

The conditions (24)-(31) may be rewritten as follow:

$$\left.\begin{aligned}
&q_1\left(=\frac{2\pi H_1}{Tv_1}\right)=(2m+1)\frac{\pi}{2}\ ,\qquad q_2\left(=\frac{2\pi H_2}{Tv_2}\right)=(2n+1)\frac{\pi}{2}\ ,\\
&q_3\left(=\frac{2\pi H_3}{Tv_3}\right)=(2s+1)\frac{\pi}{2}\ .
\end{aligned}\right\}\tag{24$'$}$$

$$q_1=(2m+1)\frac{\pi}{2}\ ,\qquad q_2=(n+1)\pi\ ,\qquad q_3=(2s+1)\frac{\pi}{2}\ ,\tag{25$'$}$$

$$q_1=(2m+1)\frac{\pi}{2}\ ,\qquad q_2=(n+1)\pi\ ,\qquad q_3=(s+1)\pi\ ,\tag{26$'$}$$

$$q_1=(2m+1)\frac{\pi}{2}\ ,\qquad q_2=(2n+1)\frac{\pi}{2}\ ,\qquad q_3=(s+1)\pi\ ,\tag{27$'$}$$

$$q_1=(m+1)\pi\ ,\qquad q_2=(2n+1)\frac{\pi}{2}\ ,\qquad q_3=(2s+1)\frac{\pi}{2}\ , \tag{28'}$$

$$q_1=(m+1)\pi\ ,\qquad q_2=(n+1)\pi\ ,\qquad q_3=(2s+1)\frac{\pi}{2}\ , \tag{29'}$$

$$q_1=(m+1)\pi\ ,\qquad q_2=(2n+1)\frac{\pi}{2}\ ,\qquad q_3=(s+1)\pi\ , \tag{30'}$$

$$q_1=(m+1)\pi\ ,\qquad q_2=(n+1)\pi\ ,\qquad q_3=(s+1)\pi\ . \tag{31'}$$

$$[m=0, 1, \cdots,\quad n=0, 1, \cdots,\quad s=0, 1, \cdots]$$

It is considerably difficult to find mathematically whether the conditions expressed by (24′)-(31′) will satisfy the condition of (4) or not.

Next, we shall try the diagrammatical treatments as in the former Section, by using the fundamental modes of (15′)-(18′), namely, $m=0$, $n=0$ and $s=0$, in order to find the requisite conditions for predominant vibration to appear at the ground surface.

Substituting (24′)-(31′) of $m=0$, $n=0$ and $s=0$ in (20), we obtain the displacement distribution in each layer as shown in Figs. 10–17.

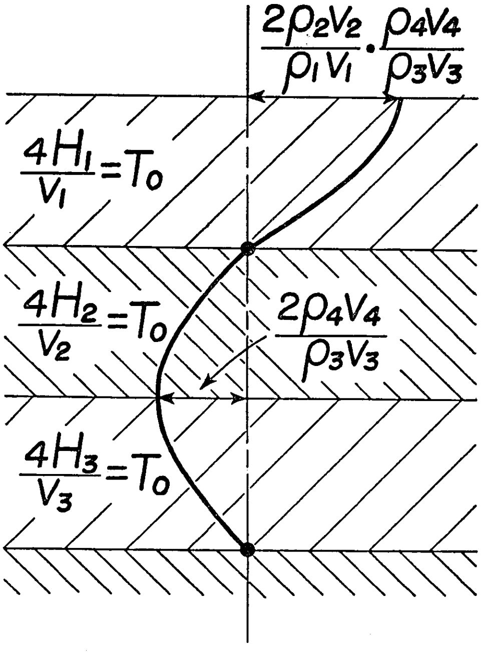


Fig. 10.

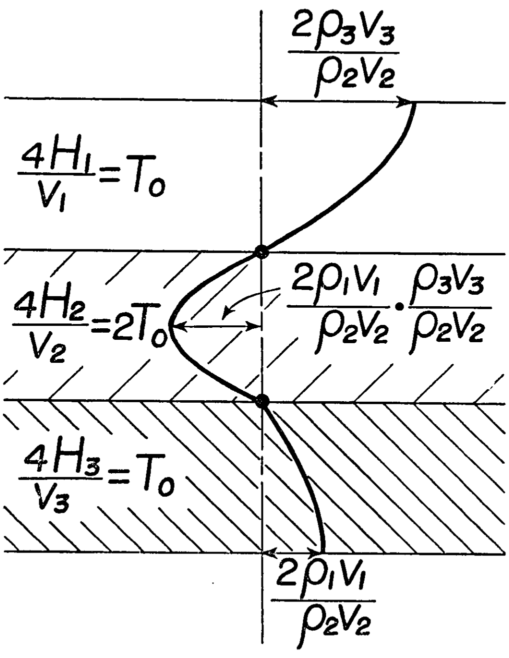


Fig. 11.

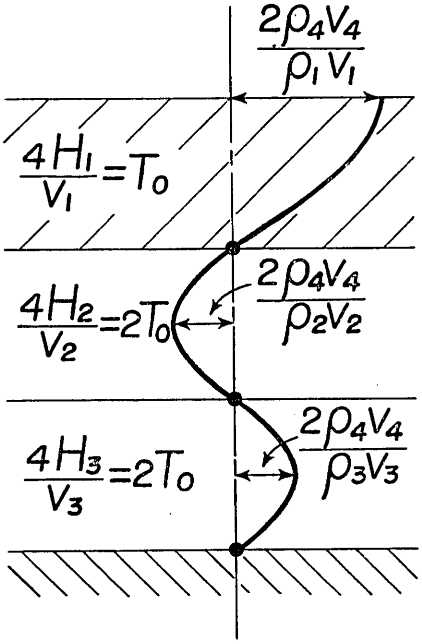


Fig. 12.

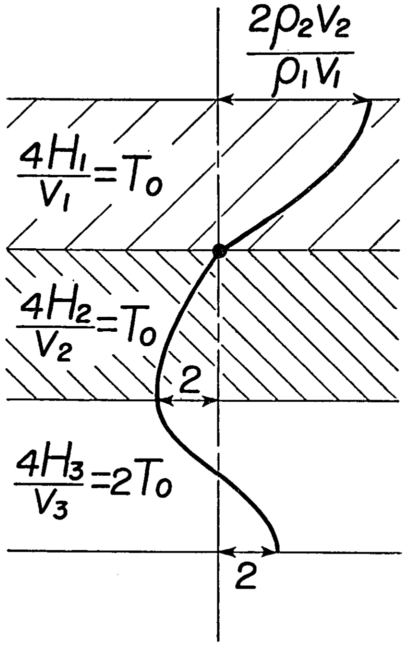


Fig. 13.

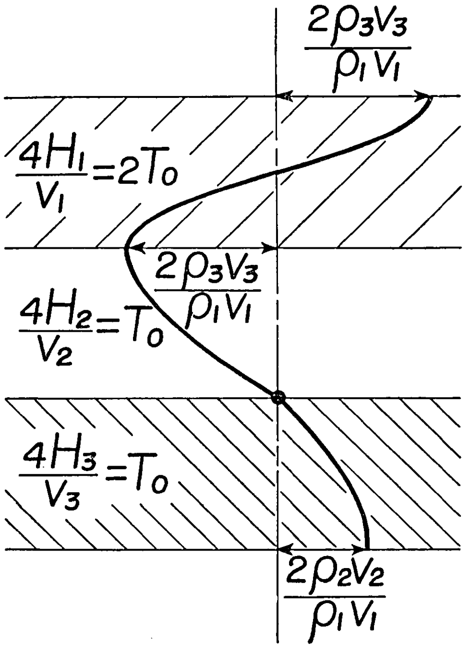


Fig. 14.

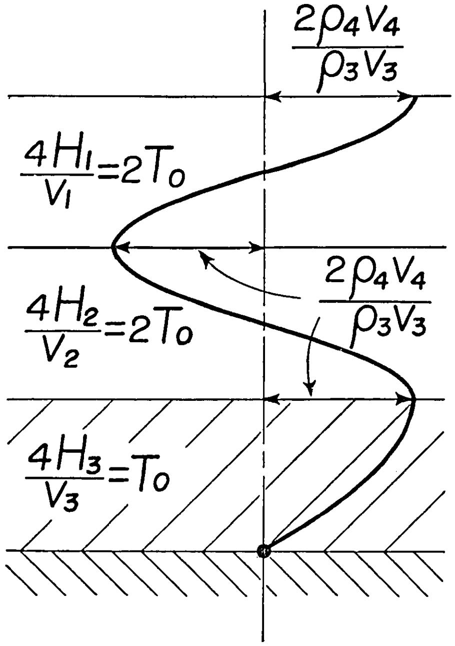


Fig. 15.

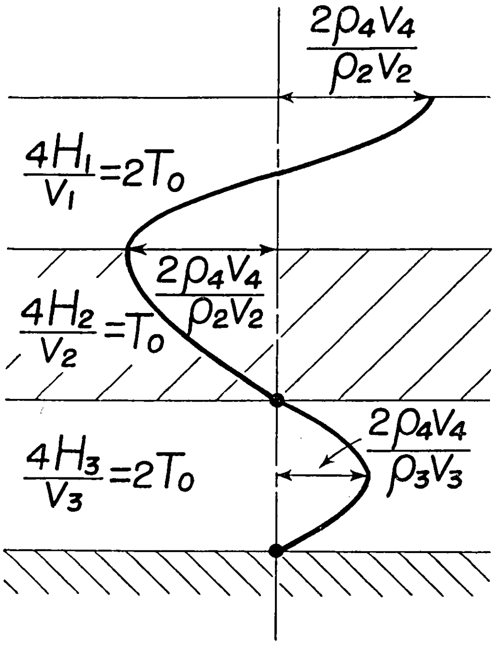


Fig. 16.

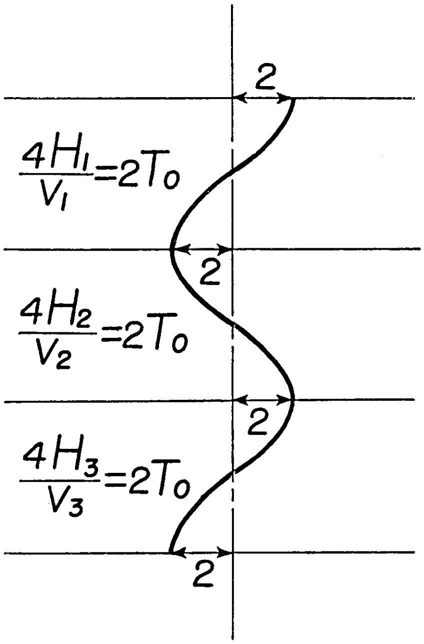


Fig. 17.

Figs. 10-16 show that, when the values of the numerator of the expressions of surface displacement are higher than that of the denominator of them, the predominant vibration at the ground surface will appear. On the other hand, Fig. 17 shows that, this case has little meaning in connection with the nature of excitation of vibrations.

The requisite conditions for predominant vibration to appear at the ground surface, which may be found from these results, are written as follows.

( i ) In the case of $\rho_2v_2>\rho_1v_1$ and $\rho_4v_4>\rho_3v_3$ under the conditions $T_0=4H_1/v_1$, $H_1/v_1=H_2/v_2=H_3/v_3$, the amplitude at the surface will take the value of $2\rho_2v_2/\rho_1v_1\times\rho_4v_4/\rho_3v_3$. (See Fig. 10.)

( ii ) In the case of $\rho_3v_3>\rho_2v_2$ under the conditions $T_0=4H_1/v_1$, $2H_1/v_1=H_2/v_2=2H_3/v_3$, the surface amplitude is $2\rho_3v_3/\rho_2v_2$. In this case, the values of $\rho_1v_1$ and $\rho_4v_4$ themselves have no meaning concerning this problem. (See Fig. 11.)

(iii) In the case of $\rho_4v_4>\rho_1v_1$ under the conditions $T_0=4H_1/v_1$, $2H_1/v_1=H_2/v_2=H_3/v_3$, the surface amplitude is $2\rho_4v_4/\rho_1\rho_1$. The values of $\rho_2v_2$ and $\rho_3v_3$ themselves have no meaning concerning this problem. (See Fig. 12.)

(iv) In the case of $\rho_2 v_2 < \rho_1 v_1$ under the conditions $T_0 = 4H_1/v_1$, $2H_1/v_1 = 2H_2/v_2 = H_3/v_3$, the surface amplitude is $2\rho_2 v_2/\rho_1 v_1$. The values of $\rho_3 v_3$ and $\rho_4 v_4$ themselves have no meaning concerning this problem. (See Fig. 13.)

(v) In the case of $\rho_3 v_3 < \rho_1 v_1$ under the condition $T_0 = 4H_2/v_2$, $H_1/v_1 = 2H_2/v_2 = 2H_3/v_3$, the surface amplitude is $2\rho_3 v_3/\rho_1 v_1$. The values of $\rho_2 v_2$ and $\rho_4 v_4$ themselves have no meaning concerning this problem. (See Fig. 14.)

(vi) In the case of $\rho_4 v_4 < \rho_3 v_3$ under conditions $T_0 = 4H_3/v_3$, $H_1/v_1 = H_2 v_2 = 2H_3/v_3$, the surface amplitude is $2\rho_4 v_4/\rho_3 v_3$. The values of $\rho_1 v_1$ and $\rho_2 v_2$ themselves have no meaning concerning this problem. (See Fig. 15.)

(vii) In the case of $\rho_4 v_4 > \rho_2 v_2$ under the conditions $T_0 = 4H_2/v_2$, $H_1/v_1 = 2H_2/v_2 = H_3/v_3$, the surface amplitude is $2\rho_4 v_4/\rho_2 v_2$. The values of $\rho_1 v_1$ and $\rho_3 v_3$ themselves have no meaning concerning this problem.

It is a noteworthy fact that, in the case of the predominant vibrations appearing at the free surface, the surface amplitude seems to be concerned with the rigidity ratio of two layers, and values of rigidity themselves of the other two layers have no connection with the predominant amplitude. These results tell us again that, it will be provable that the predominant vibration appears at the ground surface, even if one or two rigid media exist among four media. Figs. 5–7 and Figs. 10–16 show that as the number of layers increase, the number of the requisite conditions of the present problem increase. That is to say, we know that as the number of layers increases the case of the predominant vibration of ground become rarer.

It will be seen from the present investigation that the number of cases, in which the period of the predominant vibrations is equal to the natural period of the uppermost layer is larger than half of the number of all possible cases of the predominant vibration.

At all events, the above results of the present investigations closely resemble the features of the spectrogramic diagram of earthquake movements of the ground or micro-tremor observed at several places.[5)]

It may be born in mind that, when the properties of the ground satisfy some particular conditions, it is probable that an extremely large

5) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **12** (1934), 234; **13** (1935), 438; **15** (1937), 526.

F. KISHINOUYE, *ditto*, **29** (1951), 403.

K. KANAI, T. TANAKA and K. OSADA, *Monthly Meeting of the Earthq. Res. Inst.*, (Sept. 1956).

vibration amplitude will appear at the ground surface, even if a rigid medium exists at the superficial layer or intermediate one.

In such cases, discrimination between good or bad ground considered from a vibrational point of view will sometimes differ from the statical point, namely, the bearing power of ground, etc.

The above results are one of the most important factors in the development of earthquake resistant structural design.

## 5. Conclusion

From the present mathematical investigations on the vibration problem of the surface layer, the following has been made clear:

(i) In general, in the case of multiple stratified layers, since the elastic wave reflections at the various boundaries interfere with one another, the spectral response of the amplitude of the surface layer is very irregular and the maximum value of the peak is not so large as in the case of the single layer.

(ii) In some particular circumstances, when all the waves being reflected at every boundary and arriving at the free surface, are in the same phase, the surface amplitudes grows up to an extremely large value.

(iii) It is probable that extremely large vibration amplitude will appear at the ground surface, even if a rigid medium exist in the superficial layer or intermediate one.

(iv) At many times, the predominant period of ground vibration coincides with the natural period of the uppermost layer.

(v) The most important conclusion of the present investigation concerning the development of earthquake resistant structural design is that the discrimination of good or bad ground from the vibrational point of view will sometimes differ from that viewed from the statical point, namely, the bearing power of ground, etc.

## 23. 卓越震動が現われるための地盤の条件

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地盤の震動が或周期で卓越する現象については，多くの験震的並びに数理的な研究が行われて来た．最近，この問題は，耐震工学上で特に注意をひき始めたようである．

自然地震並びに常時微動からみた地盤の震動性状は2つの型に大別できる．即ち 1 つの型は地盤特有な或周期の波がはつきり卓越するものであり，他の1つの型は2つ以上の卓越する周期があり，極端な場合には，その数が多くて卓越周期の言葉があてはまらない．

前者は 1 層から成る表面層の震動問題として一応の説明がつき，これについては験震的研究結果から求めた実験式に数理的検討を加えた半実験式を既に発表した．

本研究は後者の場合，即ち表面層が 2 層以上から成つている場合に，如何なる条件があれば，地表面に特別に大きな震動が誘発されるかをしらべたものである．この論文の計算法は少しも新しいものではなく，20 年も前から順々に出されたものの応用に過ぎない．

地表面の震動が特に大きくなるのは，境界面で反射して来る波が同位相になつて重り合うからであるから，層数が多くなる程，この現象の起こりにくくなる事は容易に想像できる．

しかし，多層の場合でも，各層の剛性と厚さの関係が特別な条件を満足する場合には，各境界面で反射してくる波がすべて同位相になる現象も起り得て，地表面の震動は特別に大きくなる．

しかして，最上層或は中間層に硬い層があつてもこのような条件が存在し得ることがわかつた．

即ち，この事は，震動的にみた地盤の良否と，地耐力等から見たそれとは一致しない場合があり得る事を示唆するものである．

地耐力等からは，かなり良いと推定される地盤が，地震動，常時微動からみた震動性状並びに過去の震害資料からはどうしても良いとは言えない実例がいくつかあるが，これは，恐らく本研究の結果で解釈されるものと考えられる．