1. *Study on Surface Waves XIII. Nomograph for the Phase Velocity of Love-Waves in Doubly Stratified Medium.*

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1. Introduction

It is more than forty years ago that A. E. H. Love discovered the possibility of the existense of surface waves which are constituted by the *SH* waves and are propagated in a stratified medium[1]. This sort of surface wave is now called by his name, and a number of modified forms have been discovered by later authors. One of the most prominent is the same kind of surface waves propagated in a medium with double superficial layer, which is especially important because the practical structure of the crust is today believed to be such in the larger part of the world. These waves, which we will call "Love waves in a doubly stratified medium," were theoretically discovered by R. Stoneley and T. Matuzawa[2] independently, and are now used for the analysis of the seismogram and for the estimation of the thickness of the crust[3].

Thirty years ago, the proof of the existence and the calculation of dispersion curves of these waves deserved to be an independent paper, but today we must calculate many cases giving various values of parameters in the characteristic equation and search for a structure giving a dispersion curve that fits well with the observed one. This is, however, a fairly troublesome task, for the calculation of a dispersion curve requires many hours of numerical calculation work, and we are apt to be satisfied by not a large number of the combination of parameters such as density ratio, velocity ratio, thickness ratio and so on. As a result the progress of investigation in this branch of seismometry is obliged to be slow, and it is desirable to establish a method by which

1) A. E. H. LOVE, "Some Problems of Geodynamics," 1911 (Cambridge).
2) R. STONELEY and E. TILOTSON, *M.N.R.A.S., Geo. Sup.*, **1** (1927), 521.
T. MATUZAWA, *Proc. Phys. Math. Soc. Japan*, [iii] **10** (1928), 25.
3) Papers refered in 2) and Y. SATÔ, *Bull. Earthq. Res. Inst.*, **29** (1951), 435.

we can easily calculate the velocity. The present author has already published several nomographs for the case of a single layer[4], but the ones for the doubly stratified medium have remained undone.

The nomograph presented here is made urged by the above request. Because of the complicated form of the characteristic equation, we cannot make a complete nomograph, but must be satisfied by finding an approximate value of frequency corresponding to some given value of velocity by the method of trial and error. The precision, nevertheless, seems to be satisfactory for our practical work, and the time required to obtain dispersion curves is reduced to about one tenth or one fifth that of the ordinary method.

2. Fundamental equation

As the form of the fundamental characteristic equation was often presented in various papers[5], it will not be necessary to describe here the process of deriving this equation. We will only give the final form

$$D \equiv D^c + D^s = 0 \, ,$$
$$\begin{cases} D^c = \mu_2 s_2 \cos s_1 H_1 (\mu_3' s_3 \cos s_2 H_2 + \mu_2 s_2 \sin s_2 H_2) \, , \\ D^s = \mu_1 s_1 \sin s_1 H_1 (-\mu_3' s_3 \sin s_2 H_2 + \mu_2 s_2 \cos s_2 H_2) \, , \end{cases} \tag{2.1}$$

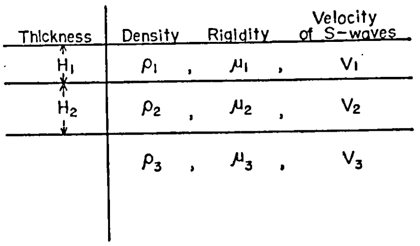

Fig. 1.

where

$s_k = f\sqrt{(V^2/V_k^2 - 1)} = i\sigma_k$,

$\mu_3' = i\mu_3$,

V; the phase velocity of Love-waves ,

f; $2\pi/$(wave length of Love-waves) ,

and other notations are illustrated on Fig. 1.

We modify the above expression for convenience sake, namely dividing the expression (2.1) by $(\mu_2 s_2)^2 \cos s_1 H_1 \cos s_2 H_2$, we have

$$-\frac{\mu_3}{\mu_2}\frac{\sigma_3}{s_2} + \tan s_2 H_2 + \frac{\mu_1}{\mu_2}\frac{s_1}{s_2} \tan s_1 H_1 \left(\frac{\mu_3}{\mu_2}\frac{\sigma_3}{s_2} \tan s_2 H_2 + 1 \right) = 0 \, . \tag{2.2}$$

4) Y. SATÔ, *Bull. Earthq. Res. Inst.*, **31** (1953), 81 and 255. *Journal of Physics of the Earth*, **4** (1956), 41.

5) *e.g.*, Y. SATÔ, *Bull. Earthq. Res. Inst.*, **29** (1951), 435.

Or, introducing new notations the next equation is obtained.

$$XR = \frac{Y-S}{YS+1}, \tag{2.3}$$

where

$$X \equiv \frac{\mu_1}{\mu_2}\frac{s_1}{s_2}, \qquad R \equiv \tan s_1 H_1 ,$$

and

$$Y \equiv \frac{\mu_3}{\mu_2}\frac{\sigma_3}{s_2}, \qquad S \equiv \tan s_2 H_2 .$$

When the velocity of Love-waves is smaller than that of the intermediate layer ($V < V_2$), we put

$$s_2 = i\sigma_2 , \tag{2.4}$$

into (2.2) and obtain

$$\frac{\mu_3}{\mu_2}\frac{\sigma_3}{\sigma_2} + \tanh \sigma_2 H_2 - \frac{\mu_1}{\mu_2}\frac{s_1}{\sigma_2} \tan s_1 H_1\left(\frac{\mu_3}{\mu_2}\frac{\sigma_3}{\sigma_2} \tanh \sigma_2 H_2 + 1\right) = 0 . \tag{2.5}$$

As before, introducing new notations

$$X' \equiv \frac{\mu_1}{\mu_2}\frac{s_1}{\sigma_2},$$

and

$$Y' \equiv \frac{\mu_3}{\mu_2}\frac{\sigma_3}{\sigma_2}, \qquad S' \equiv \tanh \sigma_2 H_2 ,$$

we have

$$X'R = \frac{Y'+S'}{Y'S'+1} . \tag{2.6}$$

Using the equations (2.3) and (2.6) we obtain the relation between the velocity and the period. The details will be described in the next section.

3. Explanation of the figure and directions for use[6]

Case I. $V > V_2$. (Fig. 6.)

1) In Fig. 2, [1] is the axis of V_1/V_2. When some structure[7] of

6) In this section $A(V_1/V_2)$, for example, represents a point A whose ordinate is V_1/V_2.

the crust is given we can at once calculate the value of V_1/V_2, so we can take a point A on this axis corresponding to this value.

2) Next we choose some value of V/V_2 and find a curve α which has the value given above. (Usually we give several values of V/V_2 at once.) α is the curve of s_2/s_1 and we can find this value on the axis-[2] following the arrow line. (Point C.)

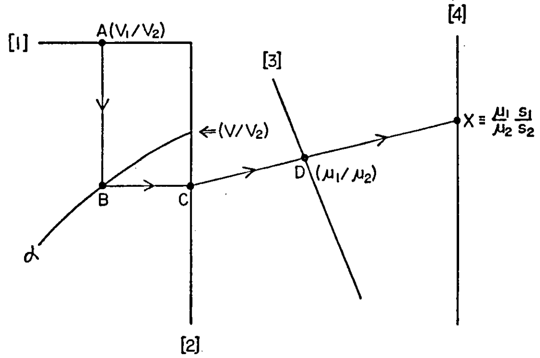

Fig. 2. $A(V_1/V_2)$, for example, means a point A with an ordinate V_1/V_2.[6]

$A(V_1/V_2)$ and $D(\mu_1/\mu_2)$ can be directly obtained from the structure[7] of the medium.

⇨ means to find a curve with a parameter of assigned value.

3) By the right half of the same figure we can easily obtain $X \equiv \frac{\mu_1}{\mu_2}\frac{s_1}{s_2}$ on the axis-[4] taking the value of μ_1/μ_2 calculated from the given structure. We must remember this value X and use it at the last stage of this calculation. As is clear in the above procedure, X is independent with the value of ω.

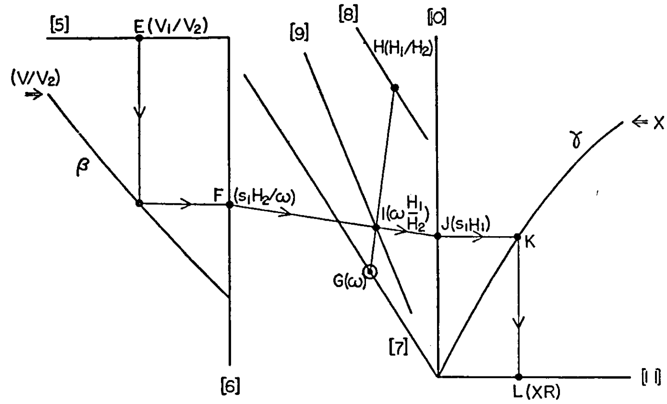

Fig. 3.

$E(V_1/V_2)$ and $H(H_1/H_2)$ can be directly obtained from the structure[7] of the mediunm.

⇨ means to find a curve with a parameter of assigned value.

⊙ is a point tentatively given.

7) In this section the word "structure" means the doubly stratified structure and the material constants as shown in Fig. 1.

4) Just as in 1) we find a point $E(V_1/V_2)$ on [5], and as in 2) we take a curve β. V_1/V_2 and V/V_2 must be equal to those used in the previous stages. (See Fig. 3.)

5) Since β is the curve of s_1H_2/ω[8] we can get this value shown by the point F on the axis-[6]. (We are using H_2 as the unit of length.)

6) We assume some value of ω tentatively and take it on [7]-axis. (Point $G(\omega)$.) On the other hand, $H(H_1/H_2)$ is given by the structure, therefore we can easily determine $I(\omega H_1/H_2)$ on the axis [9].

7) From $F(s_1H_2/\omega)$ and $I(\omega H_1/H_2)$ we get $J(s_1H_1)$ on the axis-[10]. This is simple multiplication.

8) Using a curve having a parameter X, which we have obtained in 3), we can find a point L on the axis-[11], which gives the value XR. The expression (2.3) requires that XR is equal to $Z\equiv(Y-S)/(YS+1)$, which must be calculated in the next steps. The nomographs necessary for them are given just below those which were used for the above calculations.

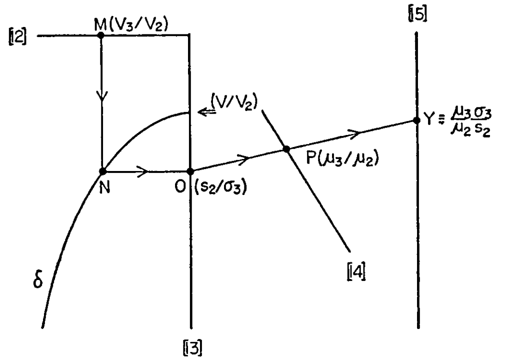

Fig. 4. $M(V_3/V_2)$, for example, means a point M with an ordinate V_3/V_2.

$M(V_3/V_2)$ and $P(\mu_3/\mu_2)$ can be directly obtained from the structure[7] of the medium.

⇨ means to find a curve with a parameter of assigned value.

9) By exactly the same way as in 1)~3) we can obtain Y on the axis-[15]. (See Fig. 4.) We only have to notice that the value of V/V_2 used in this step should be the same as the one used before.

10) Next, on the axis-[16] we take a point Q. Since s_2H_2/ω is a function of V/V_2 only, we only have to find a point with an assigned value of V/V_2 on this axis. (See Fig. 5.)

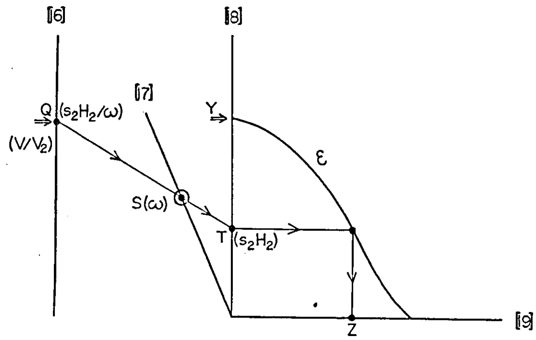

Fig. 5.

⊙ is a point tentatively given.

8) $\omega=(2\pi/T)(H_2/V_2)$ gives the frequency without a dimension.

11) The following processes are very simple. Assuming $S(\omega)$ on the axis-[17], in which ω must be equal to the value used before in $G(\omega)$, we get $T(s_2H_2)$ on the axis-[18].

12) Using Y, which was already obtained in 9), we can get $Z=(Y-S)/(YS+1)$ on the axis-[19] following the arrow line.

If the obtained values of XR and Z are equal, assumed value $\omega=\frac{2\pi}{T}\frac{1}{V_2}H_2$ is the frequency which gives the phase velocity V/V_2. Usually, however, such a circumstance does not occur, and we must assume another value of ω and again try the same procedure until we can get the result that satisfies the equation $XR=Z$.

Case II. $V<V_2$. (Fig. 7.)

In this case we only have to use another sheet of nomograph. (Fig. 7.) The only difference is that

$$X,\ Y,\ Z \text{ and } s_2$$

are replaced by X', Y', Z' and σ_2 respectively.

The range of parameters involved in our nomographs is:

V_1/V_2;		$0.80\sim0.96$,		
V_3/V_2;		$1.04\sim1.25$,		
V/V_2;	$0.82\sim0.98$	$(V<V_2)$,	$1.02\sim1.20$	$(V>V_2)$,
μ_1/μ_2;		$0.30\sim1.00$,		
μ_3/μ_2;	$1.00\sim3.40$	$(V<V_2)$,	$1.00\sim4.00$	$(V>V_2)$.

It takes an hour or less to find several points lying on the same dispersion curve. If the material of the three media are unaltered and only the thickness of the superficial layer changes, time for the calculation will be reduced markedly.

1. 表面波の研究 XIII.

二つの表面層をもつ媒質内を伝はるラブ波の位相速度を求める計算図表

地震研究所　佐　藤　泰　夫

一つの表面層がある媒質を伝はるラブ波については，位相速度及び群速度を求める計算図表を数回にわたつて発表したが,[4] 今回は二重の表面層をもつ媒質を伝はるラブ波についての図表を作つた．計算に必要な時間は数値計算の 1/5 乃至 1/10 にすることができる．精度は実用上十分である．

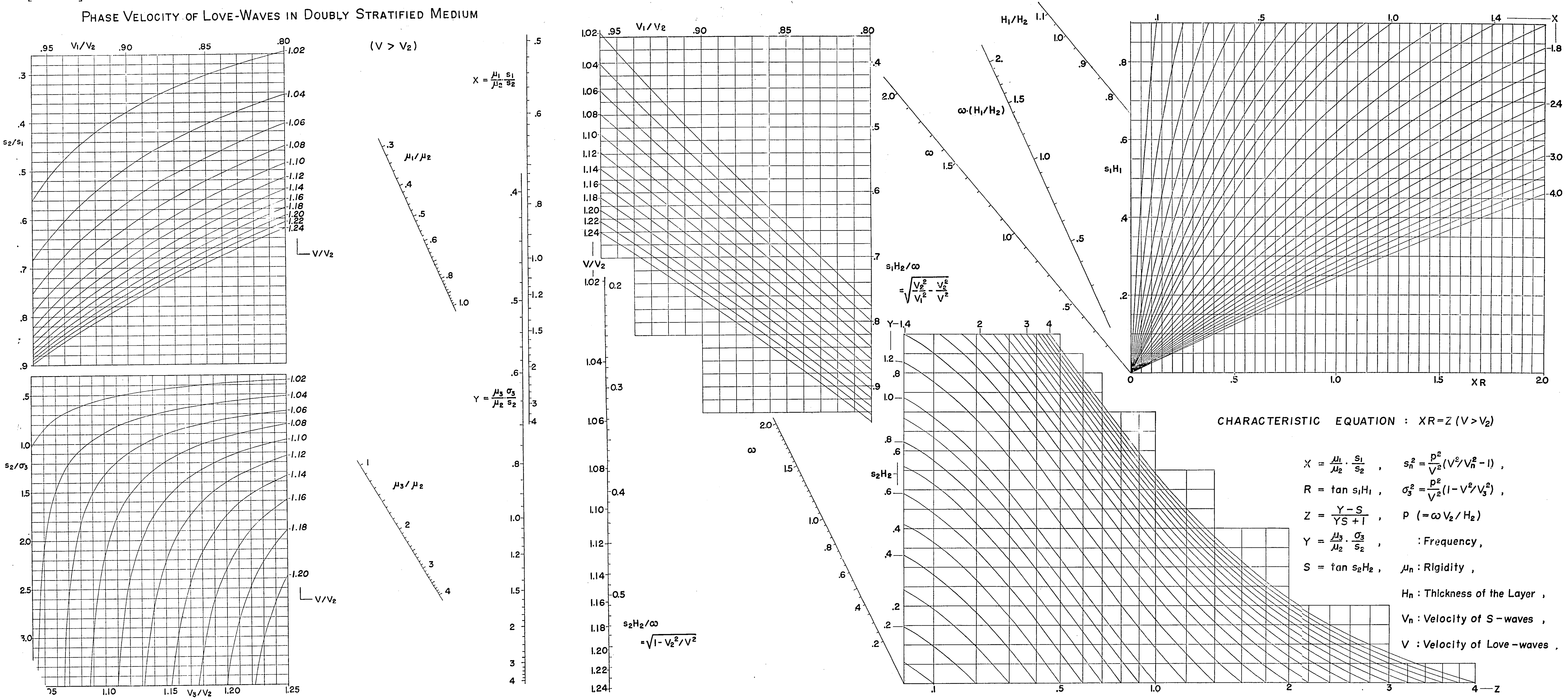

Fig. 6. Nomograph for the Phase Velocity of Love-Waves in Doubly Stratified Medium ($V > V_2$).

[Bull. Earthq. Res. Inst., Vol. XXXV, Pl. 2.]

PHASE VELOCITY OF LOVE-WAVES IN DOUBLY STRATIFIED MEDIUM

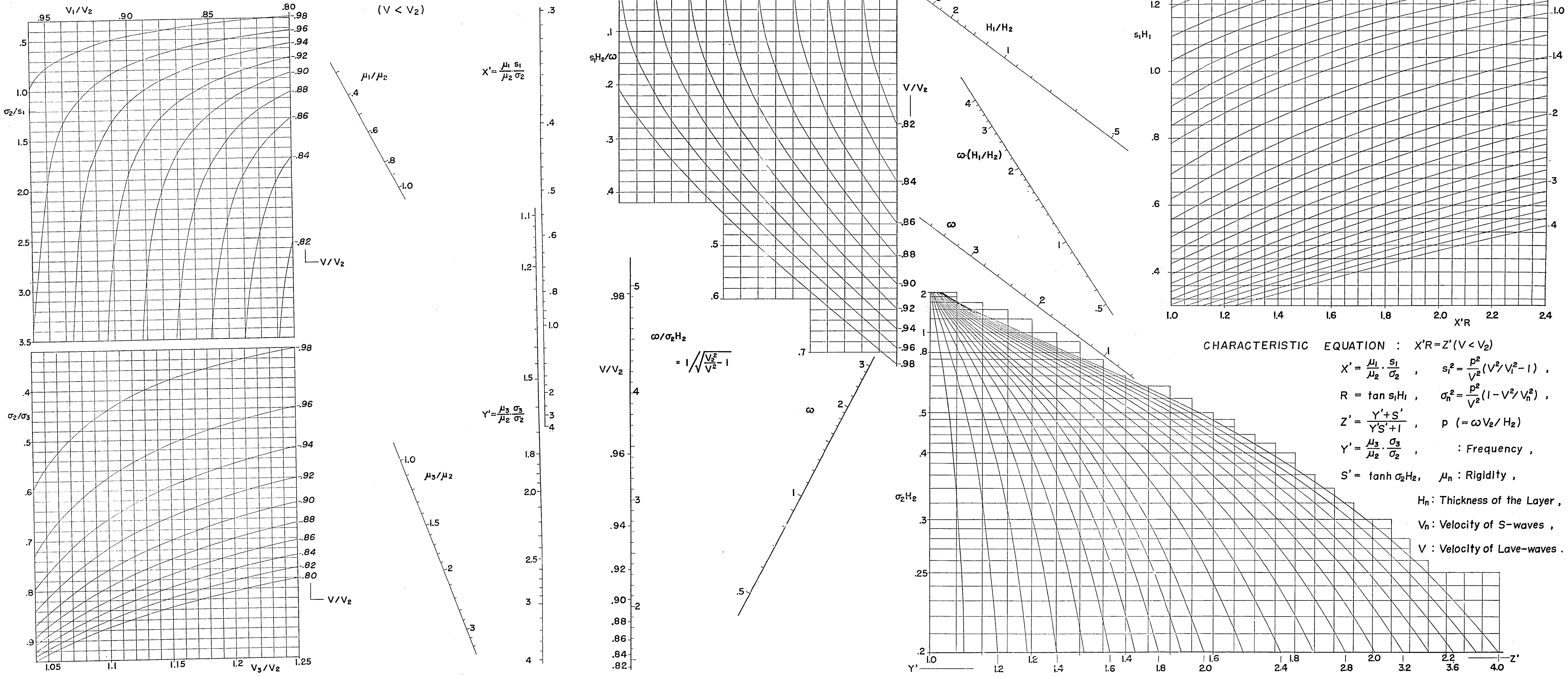

Fig. 7. Nomograph for the Phase Velocity of Love-Waves in Doubly Stratified Medium ($V < V_2$).

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