Multi-Objective Portfolio Optimization and Re-balancing using Genetic Algorithms

遺伝的アルゴリズムを用いた多目的ポートフォリオ最適化とリバランシング

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Abstract

An asset is any financial instrument like shares of companies, bonds, foreign exchange assets and a lot of others. A Portfolio is a collection of these assets held by companies, financial and other institutions and rich individuals. People invest in Portfolios to invest their capital and make profits out of them.

In this work, we discuss in depth the problem of Portfolio Optimization through the application of Evolutionary Algorithms, Genetic Algorithms in particular. Portfolio Optimization can be considered a resource allocation problem, where the capital to be invested is the resource and it is invested in various assets held in the Portfolio. It is mainly based on the Modern Portfolio Theory (MPT) proposed by Harry Markowitz in 1952. The main idea is that by diversification, holding various kinds of assets, the total risk associated with the Portfolio can be reduced (the specific risk) while maintaining the target expected return.

A lot of researchers have proposed various optimizing techniques to find an optimized Portfolio for different markets. The MPT was an extremely significant work in this field. However, it has a lot of restrictions and assumptions in it. If we can reduce/remove those restrictions that do not hold in real world, the problem becomes a very complicated one. At the same time, the optimized Portfolio evolved like this is much more realistic and can be practically used.

There have been a number of researches proposing various numerical techniques to solve this problem. We are going to concentrate on Genetic Algorithms in this work, which are random search meta-heuristics. Genetic Algorithms have proved time and again, to work well with these kinds of problems.

In this work, we have proposed some new points, which we believe will overcome the weak points of previous researches. The point of new mutation operator that optimizes each and every weight present in the Portfolio has been proposed, in addition to the concept of Traded Volumes which is basically an indirect way of incorporating the impact of news on the financial markets. The results achieved are motivating and the approach, we believe, will be used as a reference for future related works to develop Portfolios in real world.

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Table of Contents

List of Figures

List of Tables

Chapter 1
1: Introduction

Many people must have heard of Portfolios and Portfolio Management problems, but I am sure a lot of them don't really know what a Portfolio means or what Portfolio Optimization means.

A Portfolio Optimization problem is a problem that can be linked to a real life problems easily. Consider a man who took out some cash from an ATM. He was going somewhere on a tour. On his way to the destination, he was robbed of his wallet and thus, he lost all his money, credit cards, his license and some other important stuff. He told this story to one of his friends the other day, and his friend gave him some important piece of advice: "When you carry a lot of cash, never carry it in the wallet". What he wanted to say is that he could keep some of his cash in the wallet, some in his pant, some in his bag etc. If he had done that, he could have saved at least some money even in the case when he got robbed. By keeping his cash splitting in three different places, he could have lessened his risk of losing all the cash in the case of a robbery.

You can basically try minimizing your risk by splitting your cash, by diversifying the places where you put your cash. This is one of the basic principles of Portfolio Optimization. And you can get rich with it too.

There used to be days when Portfolio Management was something, only for institutions, companies, rich individuals etc. With the coming of the age of Internet and computers and laptops, even I pads etc., in the past decade and a half, the trading of financial assets in more limited and is available to many more people including normal salary men. Many people even use it as their source of income. You get much more news about the financial markets on the Television. There are hundreds (Thousands?) of websites providing people with the knowledge and predictions about the markets.

In this environment, the interest on financial research, and its development outside the academic circles of economists and mathematicians, has also flourished. The improved computer power allows the calculation of a greater amount of data, and with much more precision and speed. This leads to the development of computational tools for the aid of the financial trader. More accurate methods for prediction of future trends (Nikolaev and Iba, 2001), the simulation of market behavior (Subramanian et al., 2006), and more recently, the automation of simpler tasks in the financial market (Jiang and Szeto (2003), Aranha et al. (2007)), are some of the examples.

In this work, we address the problem of Financial Portfolio and its Optimization and re-balancing by means of Genetic Algorithms. This problem of Portfolio selection, Portfolio Optimization and the re-balancing of the Optimized Portfolio have gathered a lot of attention, especially by the researchers working in the Evolutionary Algorithms field. Although, this problem has been around for a while now, it still has a lot of scope for betterment. In this work, we will talk about the basic principles involved in the Portfolio Optimization problem, the recent and some other important works being done about this problem. Finally, we will talk about our new approach to some of the problems involved in this field, and compare our results with some other approaches.

1.1 A Portfolio

A Portfolio basically means a collection. It can be a collection of any sort. A Portfolio can be a collection of photos/pictures or a collection of assets, which is a Financial Portfolio.

A Financial Portfolio is, thus, a collection of assets like stocks, bonds, foreign exchange assets and lots more. These investments not only help the companies and organizations, but also help the investors investing in these companies and organizations. Therefore, a lot of people are managing their personal Portfolios as secondary and primary sources of income as well. Investing in securities such as shares, debentures and bonds can be profitable and is exciting as well. Where on one hand it can be profitable, on the other hand it involves a great deal of risk as well, if not properly managed. Since markets keep changing every minute and every second, investing all of the investment an individual holds is rarely a good idea. At the same time, different securities (shares, bonds, foreign exchange assets etc.) show different behaviors towards the changing markets and also towards various kinds of news. Also, different securities have different kinds of characteristics like liquidity, dividends etc., to name a few. Therefore, diversity in the Portfolio not only means the assets from different companies but also from different industries. An investment strategy that takes into account all these characteristics is necessary to minimize the risk associated with the Portfolio and also maximize its predicted returns.

What we talked above is about the long term Portfolios management, like for 1 year or more. For short term Portfolios however, the traders might be more interested in short term profits and therefore they might concentrate on a few securities rather than trying to invest in a more diverse Portfolio.

For long term Portfolios, the diversification reduces the risk associated to it to a great extent, and thus makes the investment more rewarding. If made well, Portfolios can even work in the hard market conditions like the recession of 2008, although it is very hard.

1.2 Portfolio Management

An investor considering investing in securities is faced with a lot of problems. Some of these are as follows:

1) Choosing the number of assets he should invest in, since both buying and selling of investments at any point of time needs transaction costs to be paid.
2) Choosing the assets themselves. Which assets to invest in. Since there are thousands of assets available in the market, it can be a cumbersome job to select the assets for the Portfolio.
3) Suppose you have 1 million US dollars to invest. You have decided the number of assets you want to invest in, and you have also decided which assets to invest in. Then the biggest problem is: how much of those 1 million dollars you should invest in each asset. The configuration of the Portfolio plays one of the most important roles in predicting the amount of risk involved with the Portfolio and also the expected return of the same.
4) Once you have decided upon the Portfolio, you also need to change it with the changes in the market, since the risk return characteristics of any individual asset also changes the characteristics of the Portfolio it is a part of.

These are some of the many important problems involved with the investment strategy.

Therefore, since all the investors expect to get good returns from their Portfolios at the end of the day, the return realized has to be measured accurately and the performance of the Portfolio has to be evaluated periodically.

It is evident that Portfolio Management problem compromises of all the processes involved in the creation and maintenance of an investment Portfolio. It deals specifically with the assets analysis, Portfolio analysis, Portfolio selection, Portfolio evaluation and also Portfolio revision, which we call the Re-balancing of the Portfolio. In one line, it is a complex process, which if worked out well can result in more attractive rewards and lesser risk.

1.3 Financial Industry Jargon

Before we go any further, we will have a quick look at the financial industry jargon. Although the financial industry jargon is huge, we will look only at the jargon that we have used in this dissertation, and will also make the further reading and understanding more comfortable. (Note: This Financial Industry Jargon is in no particular order).

Asset (Securities): A resource with economic value that an individual, corporation or country owns or controls with the expectation that it will provide future benefit.

Derivative Securities: These are those financial instruments whose values depend on those of other assets.

Diversification: Diversification is the process of investing a portfolio across different asset classes in varying proportions depending on various issues like risk tolerance, time for which the investor wants to invest, expected return from that Portfolio in that specified time period.. While diversification does not assure or guarantee better performance and cannot eliminate the risk of investment losses, it does to a great extent reduce the risk involved while investing. Assets can be anything from equity, fixed income, foreign exchange assets etc.

Financial Engineering: Financial engineering is a process that utilizes existing financial instruments to create a new and enhanced product of some type. Just about any combination of financial instruments and products can be used in financial engineering. The process may involve a simple union between two products, or make use of several different products to create a new product that provides benefits that none of the other instruments could manage on their own.

Markowitz Portfolio Theory: This is the base model for most of the Portfolio selection and Portfolio Optimization problems. The Markowitz Portfolio Theory or Modern Portfolio Theory (MPT) says that each asset available in the market has a Specific Risk and a Systematic Risk associated with it. It also says that, by the method of diversification of the Portfolio, the specific risk can be cancelled out and thus the total risk associated with the Portfolio. We will talk about this theory in detail in later chapters.

Portfolio: A Portfolio is a collection of assets like stocks, bonds, and foreign exchange assets etc, held by organizations, institutions and rich individuals etc. to make profits through investments.

Risk: Risk, also called volatility, is a measure of how likely an asset will show a certain amount of return in the future. There are many ways to calculate the risk of an asset, each of them focusing on different aspects of the risk. In the Markowitz model, the risk of an asset is measured by the variance of its past returns.

Return: ROA tells you what earnings were generated from invested capital (assets). ROA for public companies can vary substantially and will be highly dependent on the industry. This is why when using ROA as a comparative measure, it is best to compare it against a company's previous ROA numbers or the ROA of a similar company.

Specific Risk: Specific risk is also called Firm Specific Risk. Firm specific risk is related to the individual risk involved with a particular firm in a particular industry. For example, for an oil company operating in Iraq they have a firm specific risk relating to political instability versus a company that operates in India, Japan or China. Let us look at one more example: if a rice wholesaler has only one supplier and the supplier goes bankrupt, this could greatly impact the wholesaler's sales, profits, and other operations. Firm Specific Risks are difficult to quantify and thus qualitative analysis is needed to determine behaviors and predicting their future. However, according to the MPT, Firm Specific Risks can be reduced or cancelled out by diversification.

Systematic Risk: Systematic Risk is also called the Market Risk. This is the risk inherent to the entire market. Interest rates, recession and wars all represent sources of systematic risk because they affect the entire market and cannot be avoided through diversification. This type of risk affects a broader range of securities. Systematic risk can be mitigated only by being hedged. One of the good examples for this kind of risk will be the recession of 2008. That kind of recession not only affects the whole market in the broad sense, it is also difficult to generate profits during that time by means of diversification (not impossible though).

1.4 Optimization Methods and Genetic Algorithms

The Modern Portfolio Theory was proposed by Harry Markowitz in 1952 in an article and in 1959 through a book. This theory attempts to maximize Portfolio's expected return for a given amount of Portfolio risk and vice versa, by carefully choosing the contents of the Portfolio and assigning proper weights to the assets included in the Portfolio. MPT proposes the concept of diversification in Portfolio selection. This model has already been solved in theory, which proves that the optimal Portfolio is the market Portfolio, which includes all the assets in the market weighted carefully and according to capitalization.

While on one hand, MPT forms the basis of all (most of them?) the modern Portfolios, it however, has a lot of assumptions included in it. Some of the important assumptions included are as follows:

1) All investors are risk averse
2) All investors have access to the same information at the same time
3) MPT doesn't take into consideration the transaction costs
4) It doesn't care about the constraints like LOTS, trading volume limits etc.

And many more assumptions. As soon as we remove these restrictions, the model becomes largely complex and thus, obviously, it becomes harder to find an optimal Portfolio.

We have seen a lot of works solving this Portfolio problem using numerical methods like Quadratic Programming (Ammar 2007). While methods like Quadratic Programming are very good in dealing with optimization problems searching for minima or maxima, they however, have this problem of becoming too complex as the number of assets in the Portfolio increases. It grows exponentially with the size of the data.

While in cases where the number of assets involved (less than 20, for example) is relatively, they might work well, in cases with assets number increasing 20 or 30, it is hard to solve this problem by these methods. Here, the use of search heuristics and evolutionary algorithms (Like Genetic Algorithms) can come in handy.

Genetic algorithms are a part of evolutionary computing, which is a rapidly growing area of artificial intelligence and is widely used in a lot of problems involving optimization.

Genetic algorithms are inspired by Darwin's theory about evolution and therefore, optimized solutions to a problem solved by genetic algorithms are evolved in several stages. Algorithm is started with a set of solutions and the solutions are represented by chromosomes. This set of possible solutions is called population. Solutions from one population are taken and used to form a new population. The new solutions are called offspring or children/kids. This is motivated by a hope, that the new population will be better than the old one. Solutions which are selected to form new solutions are selected according to their fitness - the more suitable they are the more chances they have to reproduce (There are several selection methods/strategies, depending on the type of problem). These stages are then continuously repeated until some end condition is met or the optimum is achieved.

The idea of Evolutionary Computation has been around for more than 50 years now. The first book on Genetic Algorithms was first published in 1975. Genetic Algorithms thus, has been around for a long time and has matured with time. However, it started in the first of 1990's that researchers started using Genetic Algorithms in various fields.

Genetic Algorithms are also very convenient to use. Once the system is built, it adapts to various situations very easily. For example, the system we built for the dataset of DowJones30 assets, we used it for the NASDAQ100 assets dataset as well, with very minor changes in the system. Also, the changes in the parameters are unnecessary (in our case). Moreover, for example, if you think that the number of generations you specified in the system is not optimizing the solution, just a small change in the number of generations will do the trick (most probably). Also, it is easier to play with them. Once the system is ready, you can try changing the parameters one by one to notice the changes in the results with the change in the parameter values.

All these features make the Evolutionary Algorithms (Genetic Algorithms) very easy to use and adaptable to various kinds of problems, including the problem we are dealing with, The Portfolio Optimization.

1.5 Related Works

The Modern Portfolio Theory proposed in 1952 still forms the basis of the Portfolio Optimization problems, with some more additions to it. However, the use of Evolutionary Algorithms, Genetic Algorithms in particular, for this problem is not old. Most of the serious work of using Genetic Algorithms for the Portfolio Optimization problem started after the year 2000s. So it will not be wrong to say that this approach is still in an early stage of development and motivates up for our work. Moreover, many works restrict themselves to single scenarios as proposed by Markowitz. We discuss some of the related works and researches in the section below.

An early reference on Portfolio Optimization with Multi-Objective Evolutionary Algorithms (MOEA) was proposed by Vedarajan *et. al* (1997). Here in this work, the group introduces the use of Non-Dominated Sorting Genetic Algorithm (NSGA) to optimize investment Portfolios as an alternative to quadratic programming. Moreover, in addition to the objectives of increasing expected returns and decreasing the total risk of the Portfolio, transactions costs have also been considered here.

Since Modern Portfolio Theory considers the Portfolio Optimization problem as a static problem, it ignores a lot of constraints that need to be implemented while trying to solve the problem. For example, Chang *et. al* (2000) introduces the constraints like the limits on the number of assets as well as on the weights assigned to them. They also use various meta-heuristics like taboo search, Genetic Algorithms and simulated annealing for the problem. Similar work was also proposed by Busetti (2000) which uses taboo search and Genetic Algorithms with the cardinality constraints and also the transaction costs.

Usually arrays are used for the representation of the assets included in the Portfolios. While the binary valued arrays are/can be used to represent the presence or absence of an asset in the Portfolios, the real valued arrays can be used directly to represent the Portfolio, with the value of each array element representing the weight of assets in the Portfolio. Lipinski *et al.* (2007), Lin and Gen (2007) and Hochreiter (2007) have used the real valued Portfolio representation in their recent works. Real valued array representations directly solve both the problems, the availability of an asset in the Portfolio and their assigned weights, in one go.

In a more interesting work, Streichert *et. al* (2003) proposed a Portfolio selection problem using cardinality constraints using Non-Dominated Sorting Genetic Algorithm (NSGA) and Evolutionary strategies. The interesting part is the comparison of different representations of solutions like binary, gray binary and real valued arrays which they have performed.

Apart from binary and real-valued array representations, there are works which propose the indirect representations as well. For example, Werner and Fogarti (2002) proposed the use of Genetic Programming (GP) to generate some rules that calculate the values for the Portfolio weights rather than directly optimizing them. However, the work has not been continued.

Aranha and Iba (2009) have used the use of Tree-based Genetic Algorithms (TGA) for the same Portfolio Optimization problem, in a very interesting work. In this work, they tried to represent a Portfolio as the process of dividing the capital among multiple assets emphasizing on finding the relationships among the different assets. They could achieve motivational results as they also found the relationship between the assets.

While most of the related works use the historical data to implement their proposed algorithms and test the results, some works have also use artificial markets data. Subramanian *et. al* (2006) proposed the use of artificial market composed by many agents to perform the Portfolio selection. The main motive was to analyze the effect of buying or selling large amounts of desired assets under the presence of all the agents that influence the market. However, since the data is artificial, its applications in the real market are still doubtful.

The most common use of estimation of expected returns is moving averages, weighted moving averages etc. But we also see some other proposed methods. For example, Werner and Fogarti (2002) use least squares optimization method to model the value of the return for the assets included in the Portfolio. Azzini and Tettamanzi (2006), on the other hand, use meural networks with weights calibrated by evolutionary computing to calculate the future price of stocks. Similarly, some other methods for the risk estimation have also been used. Lipinski *et. al* (2007) and Subramanian *et. al* (2006) use the semi-variance for the risk estimation to reinforce the value of positive work.

As we look into the fitness functions used, the Sharpe Ratio comes out to be the most widely used and apt one (Yan and Clack (2007), Subramanian *et. al* (2006) and few more). By incorporating Sharpe Ratio, we reduce the burden of having to use two different fitness functions for increasing the expected return and decreasing the risk of the Portfolio. However some other works have proposed different fitness functions. For example, Hochreiter (2007) and Lin *et. al* (2005) have used a fitness function as shown below in equation 1.1:

$$\mathrm{P(x)} = \propto \sigma_P + (1 - \alpha) R_P \qquad 1.1$$

Here, σ_P and R_P are the risk and expected return of the Portfolio respectively, while $\propto$ is the weight parameter. However, the addition of an extra parameter in the system and the addition of risk and return are the disadvantages in this proposed work. Is it really possible to add risk and return in the fitness function? It obviously doesn't sound that wise.

In other works, Mukerjee *et. al* (2002) utilize NSGA-2 to implement a decision-making multi criteria model used in the risk/return negotiation by a bank loan manager. In a recent work, Radhakrishnan (2007) proposes a Multi-Objective Evolutionary Optimization technique on Portfolios.

All the methods and techniques used in our work will be discussed in a later chapter.

1.6 Outline of the Thesis

In Chapter 2, we will give a brief introduction about Modern Finance. We will talk about Financial Engineering and Computation. We will discuss about the financial markets and also the Markowitz's Modern Portfolio

Theory, on which our problem of Portfolio Optimization is based. Then, we will discuss the mathematical model of the problem.

In Chapter 3, we will discuss about what is good about the Evolutionary Algorithms, Genetic Algorithms in particular, for the problems like ours. How they work and what are the main steps in their working will be discussed as well.

Chapter 4 firstly explains about the dynamic behavior of the financial markets. Then we talk about our work, the model of our system, the new points we have included in our research, and how we are going to use the Genetic Algorithms for this problem. Re-balancing concept will be discussed as well.

In Chapter 5, we will show the results we have achieved through various methods and the results will be compared within themselves and with the indices (the datasets). We will also analyze the results.

In Chapter 6, we will finally conclude our work. The points that have met our expectations and that did not, will be discussed.

Chapter 7 proposes some works related to this research that can be tried in the future. We also talk about some of the features we wanted to work on but could not due to the unavailability of the data.

Chapter 2

2: Modern Finance: Financial Engineering and Computation

Modern finance began in the 1950s with the breakthrough of Markowitz's Modern Portfolio theory, The Sharpe Ratio concept, and Mossin's Capital Asset Pricing Model in the 1960's. These became the quantitative model for measuring risk. Another influence of research on Portfolio practices came with the efficient market hypothesis by Samuelson-Fama in the 1960s.

At present, we have such a wide variety of financial products in the market that some of them can even dazzle the knowledgeable of the field. Organizations, institutions, rich individuals, and even common salary men, can trade options, futures, stock index options and innumerous other financial products, in addition to the traditional products like stocks (equities/shares) and bonds. These customers, even invest in complex derivative securities (which sometimes, they don't even understand properly) to manage their Portfolios and financial risks. All these complex financial products are the fruits of Financial Engineering.

However, there has been progress not only in the field of Financial Engineering but also in the field of Computer Technology. In other words, in modern times, they go hand in hand and one cannot think of either of them without the other. All the modern investment banks and other financial institutions use sophisticated models and software for most of their businesses including trading.

However, the point to be noted is that, all the input for the sophisticated models and software is not 100% accurate and there are always some kinds of assumptions in them. Software is also prone to bugs and the results obtained by them might not work as well in real-life as the results from them promise to. Nonetheless, computer programs are great and have a significant advantage over the human errors and mistakes, in addition to all the calculations and work they can do, which is in some cases almost impossible by humans.

2.1 Financial Markets and the Portfolio Problem

Any society needs liquidity in the capital market for smooth functioning and it also improves its welfare through investments (an example would be the growth in economies of India and China after they opened their markets for the foreign companies and investors). Businesses need capital from investors when you don't have enough capital to invest by yourself, or the amount is too big to take a loan from some bank. Governments need funds to invest in public services. That is why, the liquidity of capital from borrowers to savers and vice versa is really important and the financial markets help achieving that liquidity.

Well, if you have enough money, you might think "What is the need to invest?? I can just keep my money with myself". Here comes the concept of "Time Value of Money".

2.1.1 Time Value of Money

Let's explain this with an example. Suppose there is a situation where you won a prize and you have two options to receive your prize money. Option 1: Take 10 thousand JPY now, Option 2: Take 10 thousand JPY in 3 years. Any matured person would take the money now rather than receiving it in 3 years, even if he/she doesn't know the reason why. It just feels right. It just feels right to have the money now rather than later. But why is it so?? I mean, if you think logically, 10 thousand JPY bill today will still be the same value one year later. However, you can do something with this money. If you have 10 thousand JPY today, you can invest it and earn interest over it over the 3 years. But if you receive it in 3 years, that is the future value. Fig. 2.1 explains Time Value of Money.

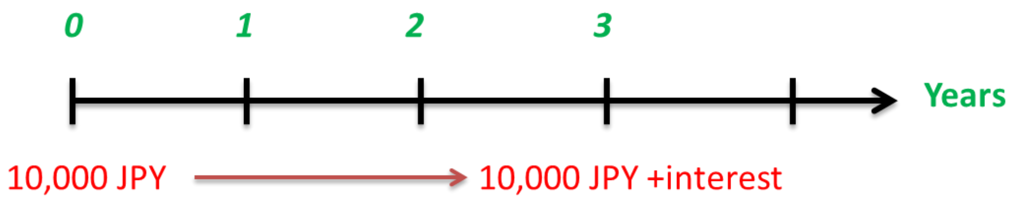

Fig. 2.1: Time Value of Money

The future value of money from the present value can be calculated by the following equation 2.1:

$$\textit{Future Value} = \textit{Original Amount} * (1 + \textit{interest rate/period})^{\textit{Number of Periods}}$$
$$= P*(1+i)^n \tag{2.1}$$

This is what the basic concept of **Time Value of Money** is and this is what gives birth to "**Portfolio Management**".

2.1.2 Price Volatility

In Financial markets, the prices of assets are dynamic and keep on changing every day, every minute and even every second. This variation in the prices of assets is termed as *Volatility*. Therefore, since prices of assets keep on changing, thus, so is the returns associated with them and thus is the risk associated with them. Volatility is thus used, in particular, to express the risk associated with the financial instruments over a period of time. It can be either expressed in absolute numbers like X JPY or it can be expressed as a fraction of the mean in % (like 7.5% etc.).

There has been a lot of research regarding the calculation of volatility and how to speculate it, but it would not be wrong to state that, till date, there is no research that can accurately explain the movement of prices of assets, and thus the volatility associated to them.

Rather than a theoretical term of Finance, it is a very important concept for investors. If there is a great deal of fluctuation in the prices of an asset, it is harder for investors to think of investing in that asset. In other words, that asset has a bigger risk associated with it. On the other hand, price volatility also provides with a chance to investors for making fast-easy money. Although it might be hard, but if speculated correctly, buying the assets with comparatively larger volatility, when the prices of those assets are less than they should be in the market, and selling those appropriate times when the prices go sharply up (even for a short period), the investors can make profits.

However, all said, investing all the capital the investor has, in just one asset can be really risky. Therefore, it is always wise to invest in a diversified Portfolio, which in-turn has lesser risk. Fig. 2.2 shows how fast the prices of an asset changes every day, for a publicly traded company.

Fig. 2.2: Volatility in prices of a publicly traded company
(Source: Google Finance)

Here, we see that, while for short term investments investing in just a single asset might provide you with some returns; it has a significant amount of risk associated with it. Therefore, it is always better to invest in a diversified Portfolio with various kinds of assets, having comparatively smaller amount of risk. This again, calls for dealing with Portfolio Management problem.

2.2 Markowitz Model: Modern Portfolio Theory (MPT)

The Modern Portfolio Theory was introduced by Harry Markowitz in 1952. Even after 60 years, it is still one of the most important and most influential economic theories developed in the field of Finance and still forms the basis of most of the Portfolio Management strategies. While on one hand, having Portfolios with 0 risks is impossible to achieve, MPT says that the specific risk of the Portfolio san be can be cancelled through diversification. Diversification, specific risk and systematic risks are explained in the following sections.

2.2.1 Diversification

Diversification in Finance basically means that the risk associated with the whole Portfolio as a whole can be significantly reduced by investing in a variety of assets. It is a way of spreading the investment risk by investing in several categories of assets, not only including the assets of different companies but of different industries.

The concept is that when one company goes down or one industry goes down, not all others go down along with them. Some of them still perform better than the others. You can even find articles on the internet about the investors who still made profits during the time of recession of 2008. One example would be the bonds of stable economies, which tend to perform better than equities of companies during the period when the market is not performing so well. According to the MPT, there are two types of risks associated with the Portfolios, systematic and specific risks, and the specific risk of the assets can be cancelled by the mode of diversification.

2.2.2 Specific and Systematic Risks

Systematic risk is called the market risk and cannot be diversified away. Interest rates recessions are some of the examples of systematic risk.

Specific risk/unsystematic risk are the risk associated with specific companies or specific securities. This, according to the MPT, can be diversified by including the assets from different companies and different industries.

In a well-diversified Portfolio, the risk of each asset contributes a little to the risk of the Portfolio as a whole. It is the covariance between the individual assets that finally contributes to the risk of the Portfolio as a whole. Therefore, it is always a better strategy to hold a well-diversified Portfolio rather than investing all of the capital in a single asset.

2.2.3 Modern Portfolio Theory: The Mathematical Model

Under the Modern Portfolio Theory, the Portfolio return is the sum of weighted returns of the assets included in the Portfolio. Moreover, the risk associated with the Portfolio is the covariance of all the assets included in the Portfolio.

Mathematically, after all the assets to be included in the Portfolio are selected, they are given appropriate weights such that the expected return of the Portfolio is maximized and the risk minimized.

Let's consider a market with n assets available for trading. Let the Portfolio be P and w_i the weight of each asset available in the Portfolio. Then we have two restrictions for the weights of the assets in the Portfolio:

$$\sum_{i=1}^{n} w_i = 1 \qquad 2.2$$

$$1 > w_i \geq 0 \qquad 2.3$$

The first equation says that, the sum of all the weights of the asset available in the portfolio should be equal to 1. The second equation says that the weight of each asset should be equal to or greater than zero and less than 1. The second equation also says that short-selling (selling the assets that the investor has borrowed) is not allowed. However, Yuh-Duah-Lyu, 2002, have proved that Portfolios with and without short selling are equivalent.

The total return of the Portfolio P is calculated by the weighted sum of the returns of all the assets available in the Portfolio:

$$R_p = \sum_{i=1}^{n} w_i \, r_i \qquad 2.4$$

Where R_p is the total return of the Portfolio and r_i is the return of each asset.

Similarly, the risk associated with the Portfolio is the covariance of all the assets included in the Portfolio:

$$\sigma_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \, w_j \sigma_{ij} \qquad 2.5$$

Here, σ_{ii}=σ_i^2 is the variance of r_i, and σ_{ij} is the covariance between r_i and r_j.

The equation 2.2.3.4 can also be written as following:

$$\sigma_p = \sum_i^n w_i^2 \, \sigma_i^2 + \sum_{i \neq j} w_i \, w_j \sigma_{ij} \qquad 2.6$$

As you can see, the equation has two parts on the right hand side. The first part is the risk component of the Portfolio composed by the risk of the individual assets. This is the specific/unsystematic risk that we talked about earlier. The second part, however, is the covariance of the returns of the composing assets. This part automatically becomes 0, if the correlation between the composing assets of the Portfolio is 0.

The first part, as the MPT says, can be reduced significantly by diversification. Let us assume that the each asset in the Portfolio has a weight 1/n, i.e. $w_i = \frac{1}{n}$ and that covariance between each asset is 0, i.e. σ_{ij}=0. So, the total risk of the Portfolio is:

$$\sigma_p = \frac{\sum \sigma_i^2}{n^2}$$

The mean of the risks is:

$$\bar{\sigma} = \frac{\sum \sigma_i}{n}$$

Let us make the risk of all the composing assets maximum, we get:

$$\frac{\sum \sigma_{max}^2}{n^2} \leq \frac{\sum \sigma_{max}}{n}$$

$$\frac{n\sigma_{max}^2}{n^2} \leq \frac{n\sigma_{max}}{n}$$

$$\frac{\sigma_{max}^2}{n} \leq \sigma_{max}$$

We can notice that the specific risk approaches 0 with the increase in the number of assets (n) in the Portfolio.

2.2.4 The Efficient Frontier

Let's look at the Fig. 2.3.

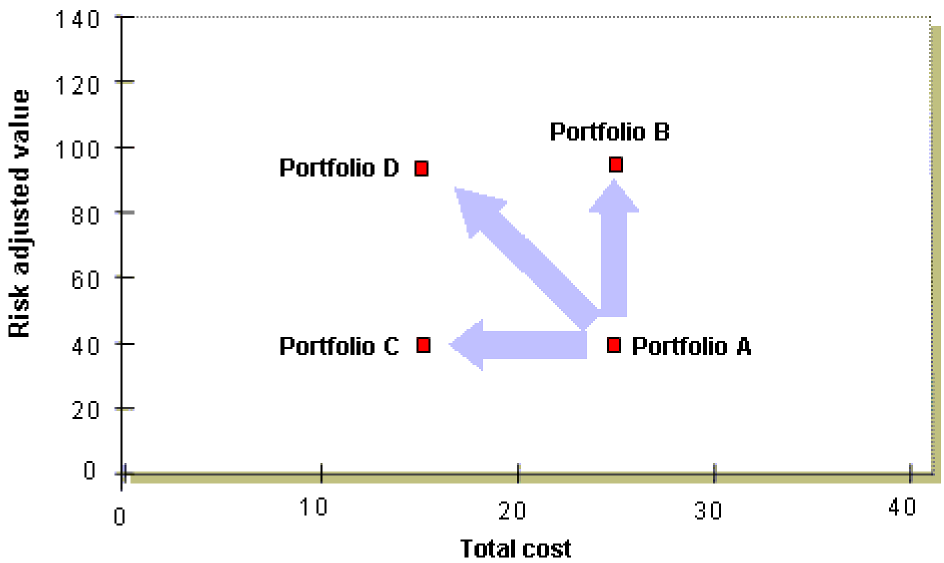

Fig. 2.3: Scenario of four Portfolios (http://www.prioritysystem.com)

In Fig. 2.3, if we compare all the Portfolios A, B, C, D, we find that the Portfolio D is the best amongst them all. Between C and D, both have the same total cost but D provides better risk adjusted value. Between D and B, both have the same risk adjusted value, however, D comes with a smaller cost and so on.

So, Harry Markowitz and other economists came up with this concept called

“The Efficient Frontier”. The Efficient Frontier is a Portfolio (collection of assets) which gives the best possible expected return for a given value of risk associated with that Portfolio. Let us look at Fig. 2.4. Here, the X-axis is the risk associated with the Portfolio defined by the standard deviation, and the Y-axis is the expected return of the Portfolio. Let us assume that each point in the region of the hyperbola is a possible Portfolio and this set of points of possible Portfolios is called a feasible set. In the region covered by hyperbola, each Portfolio for a constant risk, as we move upwards has a better expected return. Similarly, each Portfolio for a constant expected return, as we move

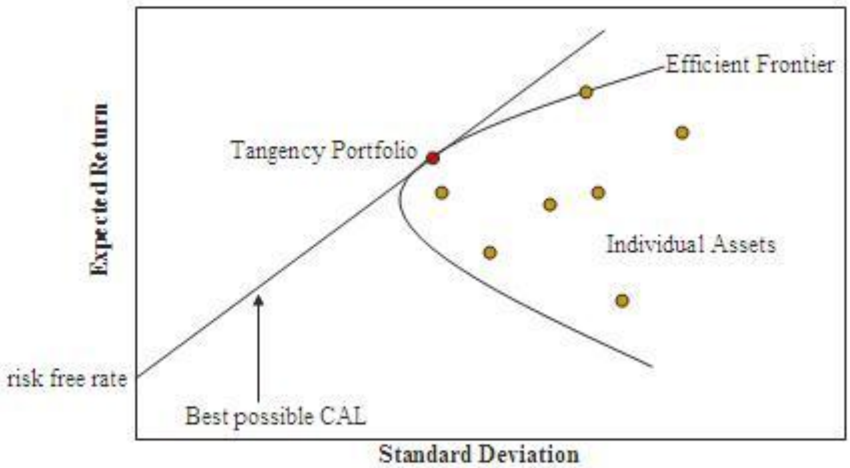

Fig. 2.4: The Efficient Frontier (en.wikipedia.org)

leftwards has a smaller risk associated with it. In other words, Portfolios towards the left and upwards are better than others. Therefore, for a given risk/return combination represented this way is called efficient, if there exists no other Portfolio that provides with a higher return for the same amount of risk and vice versa.

The combination of all these efficient sets is known as the efficient frontier and all the Portfolios falling on this efficient frontier are called efficient Portfolios. A Portfolio is called efficient if and only if no other feasible Portfolio exists that improves at least one of the two optimization criteria without worsening the other.

Moreover, there are some assets called risk-free assets. These assets usually do not exist in real life. These are the assets whose deviation on the returns is 0 and are more or less theoretical / ideal. When a risk-free asset is introduced in a Portfolio, the change in return is linearly related to the change in risk as the weights of the assets in the Portfolio vary. In this case, the tangent to the efficient frontier now becomes the new efficient frontier (Fig. 3). In other words, it is the tangent to the hyperbola with the highest Sharpe ratio (explained in later sections).

2.2.5 Sharpe Ratio

Sharpe Ratio is a very important and very widely use tool to compare the Portfolios. Think of a situation, where one Portfolio has an expected return of 100% over a year. It sounds great, cool and like so many more similar adjectives. But if you don't know how much is the risk associated with it, it has no meaning right? What if, while the Portfolio having an expected return of 100% has a risk of 200% associated with it? Then surely, you would not like to hold such a Portfolio.

Therefore, after having calculated the values of risk and return associated with a Portfolio, we need to have a way to compare them. This is when the Sharpe Ratio comes in handy. The Sharpe Ratio is calculated as given in the following equation 2.7:

$$S_r = \frac{R_P - R_{Riskless}}{\sigma_P} \qquad 2.7$$

Here, S_r is the Sharpe Ratio, R_P is the expected Return of the Portfolio, $R_{Riskless}$ is the return on a riskless asset (usually, relatively short-term bonds of stable governments which theoretically has 0 risk and a low return rate) and σ_P is the total risk associated with the Portfolio.

The Sharpe Ratio expresses the trade-off between the risk and the expected return for a Portfolio. In other words, it calculates the ratio of the extra expected return in comparison with the return of the riskless asset over the risk of the Portfolio. In this way, the Sharpe Ratio gives us a way to compare different Portfolios. A higher Sharpe Ratio indicates that that Portfolio provides with a higher expected return for a smaller amount of risk associated with the Portfolio. Therefore, obviously, the Portfolios with higher values of the Sharpe ratios are more desirable and in-demand.

2.3 Mean Variance Analysis of Risk and Return

We have seen that, although it has some limitations, The MPT still forms the basis of most of the Portfolio Optimization problems. In order to effectively use this model for our problem, we need to extract some relevant data from the data like historical prices, through some numerical methods. For example, from the historical prices, we need to extract the expected returns and the risk associated with the Portfolio, which are the two most important technical measures.

Expected return estimation is important to make more realistic estimation. Similarly, the estimation of the risk based on the past variance of the assets is important too. More closer and accurate estimations will help us evolve a Portfolio which is more realistic and more feasible in real life. While there numerous different ways of calculating these two technical measures, we will talk about the ones most widely used and also used in our work.

2.3.1 Return Analysis

While there are numerous ways of calculating the expected returns, there are very few technical indicators as popular as moving averages. Moving averages also come in different forms, but their underlying motive is the same. They help the traders track the fluctuations in prices and expected returns of the financial assets in the market.

So, let’s look at the simplest form of moving averages, which are Simple Moving Averages (SMA). The SMAs are simply put the arithmetic mean of a given set of values, which in our case would be the prices of the financial assets. So how do we use them?

1) Firstly, the expected return for a short period of time is calculated.
2) Simple Moving Averages are taken.

Usually, the closing price of the asset is used. It is used as follows:

$$R_{day} = \frac{C.P._{t} - C.P._{t-1}}{C.P._{t-1}} \qquad 2.8$$

Here, R_{day} is the expected return per day, $C.P._t$ is the closing price of the asset on the day t and $C.P._{t-1}$ is the closing price of the asset on the day (t -1). Once we have calculated the expected return per day, we take the SMAs over the desired period of time as given below in equation 2.9:

$$R_t = \frac{R_1 + R_2 + R_3 + \cdots + R_n}{N} \qquad 2.9$$

Here, R_t is the expected return for a time period t and $R_1 + R_2 + R_3 + \cdots + R_n$ are the returns on time periods 1, 2, 3… n.

Note that, it is not compulsory to estimate daily returns for all return calculations. Weekly and monthly return estimations are widely used as well (and then SMAs are calculated for 1 year etc.).

Since SMAs use past (historical) data for return estimations, it produces a time lag in the estimations. This is one of the problems with SMAs.

To rectify this problem, weighted moving averages and exponential moving averages can be used. Here, the weights can be assigned to appropriate elements in the calculations. For example, you can highly weight the recent data, in the calculation, and give relatively smaller weight to the comparatively older data.

2.3.2 Risk Analysis

Risk is the deviation of the expected returns. It can be expressed as a chance that returns will not be as expected by calculations. Risk is also calculated from time series of historical market prices of financial assets. Risk, although mostly, expressed in terms of annualized terms, it can also expressed in monthly and weekly terms or for several years. So, after having calculated the returns, the risk estimation tells us how much of swings can observed in the expected returns over some period of time. The variance (risk) is defined as follows:

$$\overline{\sigma_R^2} = \frac{1}{N-1}\sum_{i=1}^{N}(R_i - \bar{R}) \qquad 2.10$$

Here, R_i is the expected return at time i and $\bar{R}$ is the average expected return for the time period (mobbing average, any kind of).

Again, the generalized risk for time T years is expressed as follows:

$$\sigma_T = \sigma\sqrt{T} \qquad 2.11$$

Once we have the risk of an asset, we then need to calculate the total risk associated with the Portfolio. It is calculated as the weighted covariance between all the assets included in the Portfolio, and shown in the equation below:

$$\mathrm{Cov}(R_a, R_b) = \frac{\sum(R_a^i - \overline{R_a})(R_b^i - \overline{R_b})}{N-1} \qquad 2.12$$

Here, $\overline{R_a}$ and $\overline{R_b}$ are the moving averages of the returns of assets a and b respectively. This equation is for two assets, a and b. However, as the number of assets in the Portfolio increase, so does the calculation of covariance. The calculations really become complex very fast, making the system computationally expensive.

2.4 Dynamic Market and Conclusions

We know that the market is very dynamic and it keeps changing all the time. For example, the prices of asset publicly traded in the market keep changing every day, every hour and every second. This makes the problem very dynamic, unlike the MPT which considers the Portfolio Optimization problem as a static one.

Apart from these price changes, there are several real-life constraints that make this problem very much dynamic. While the MPT can be solved by optimizing techniques like Quadratic Programming, the inclusion of these real life constraints makes it complex to solve this problem with such techniques. Let us look at some of these constraints.

One of them is the large number of assets available in the market. Another one is the limits on the number of assets to be included in the Portfolio. The selection of assets for the Portfolio from the market, which has 1000s of assets available, is a difficult task. Once, you have selected the assets to review for the Portfolio, the next problem is how many assets should be included in the Portfolio. Too few assets in the Portfolio might higher the risk in the Portfolio. On the other hand, too many assets in the Portfolio might lead too expensive transaction costs when you change the Portfolio over time.

Another important constrain is the LOTS constraint. It says that no asset is infinitely divisible. For example, suppose you have 100 JPY to invest in two assets A and B. One share of A is 40 JPY and each share of B is 30 JPY. Now let us assume that our system optimized the Portfolio and the weights assigned to each A and B is 0.5 each. This means that you have 50 JPY to invest in A and 50 JPY to invest in B. In case of A, you can buy one share and you are left with 10 JPY. In case of B, you can again buy one share and you are left with 20 JPY. So, in total, you actually invested only 70 JPY, while your system gave results for all 100 JPY invested. This is one of the constraints that need to be included in the problem, to make the Portfolio more close to real-life.

There are some more constraints related to the volumes of assets traded, but since they usually don't affect the market so much, we will not go in to those details.

One of the most important constraints (according to us) is the trading costs. What are trading costs? Let us explain this with some example. Suppose, our system has evolved a good Portfolio and shows that it will give good returns (expected) with a comparatively low risk associated to it. However, we just talked that the markets are very dynamic and the prices of the assets keep changing all the time. This dynamic behavior of the market calls for the Re-balancing of the Portfolio. Re-balancing means changing the Portfolio in a way that the expected returns of the Portfolio are maintained and the risk of the Portfolio doesn't increase either. However, both buying and selling of assets cost money, given to the traders who complete the deal for us. These costs are called transaction costs and should be included in the Portfolio Optimization problems to bring the results close to real-life.

Therefore, while MPT forms the basis of the Portfolio Optimization problems, it has some problems and a lot of assumptions in it. To make the problem more realistic, there is a need to include constraints as possible in the problem. While including all the constraints in the problem might not be possible, some of them are more important like the transaction costs. Moreover, with the increase in the number of assets included in the Portfolio, the calculations in the problem become too complex to be solved by optimization techniques like Quadratic Programming. Here the heuristics, especially meta-heuristics like Evolutionary Algorithms have shown better results and motivated us to use them to try to solve this problem.

Chapter 3

3: Evolutionary Algorithms

In this chapter, we will talk about Evolutionary Algorithms (EAs), the roles that they play in specific problems, and their main components.

While there are different variants of Evolutionary Algorithms, the basic idea or the concept behind all of them is the same: A population is created, in the environment pressure the concept of "the survival of the fittest" leads to the rise in the fitness of the population as a whole. The main steps in any Evolutionary Algorithm are as follows:

1) So, we start with a utility function that has to be maximized or minimized.
2) A random population is created, which has nothing to do with the fitness of the utility function, at this point of time. This randomly created population is the set of candidate solutions. All the candidate solutions in the population are called individuals.
3) Then, these individuals are passed through the utility / fitness function, to check their fitness: the higher the fitness, better it is.
4) Next, some selections strategies are used to select the candidate solutions which are passed through some recombination operators. There is no hard and fast rule of how to select the candidates in this step. The strategy may depend upon the type of problem, the Evolutionary Algorithms are trying to optimize.
5) On the candidates selected in step 4) some recombination operators are used to try to increase the fitness of the candidate solutions. These recombination operators are usually of two types: the first one is called crossover and the second one is called the mutation operator (although there can be some other operators too).
6) In mutation, the values of the candidate solutions are changed randomly or through some specific strategy. The main motive is to increase the fitness of the individuals.
7) Next is the crossover operator (Note: mutation operator can be used before or after crossover). For this operator to be used, two candidate solutions are selected through some strategy. These two individuals are

called parents. Then, of these two parents, a new candidate solution is created which has the properties of both the parents. This newly formed candidate solution is called a child. Again, the main motive here too, as in case of mutation, is to increase the fitness of the candidate solutions.

8) Seeding is sometimes used and sometimes not. This is more of an optional step. This is taking some best individuals from the current population to the next population.
9) These steps are repeated until some stopping criterion is met.

All these steps together, lead to the improvement of the fitness of the entire population through a number of generations. A basic algorithm is shown in Fig. 3.1 below:

```
Start
        Initialize population;
        Evaluate;
        Repeat (Termination Condition[s]) Do
                Select Parents;
                Mutation operator;
                Crossover operator;
                Evaluate;
                Seed;
                Jump to next generation;
End
```

Fig. 3.1: A basic Evolutionary Algorithm

It can be seen that these algorithms fall under the category of trial-and-test algorithms. The utility function represents a heuristic estimation of solution quality and the search process is driven by the variation and the selection and recombination operators.

The Genetic Algorithms also fall under the category of Evolutionary Algorithms. These were proposed later in 1975 (J.H. Holland (1975), Jong (1975)). Genetic Programming also falls under the category of EAs, but in this work we concentrate mainly on Genetic Algorithms.

However, Evolutionary Algorithms have some limitations too. The EAs guide the population towards a goal through the fitness function.

However, think when the problem has two fitness functions. And what if those fitness functions are independent of each other or even contrary? In these cases, for the EAs to work properly and accurately, we need to define a new fitness function which basically optimizes both the fitness functions, which might be a difficult job to do.

Our problem of Portfolio Optimization is one such problem, where we are trying to address three problems at once. We are trying to minimize the risk associated with the Portfolio, maximizing the total expected returns of the Portfolio and also trying to minimize the transaction costs when we Re-balance the Portfolio.

In the following sections, we will discuss about the Genetic Algorithms in detail. We will also discuss a number of components, procedures and operators in detail.

3.1 Genetic Algorithms

Genetic Algorithms is a meta-heuristic technique for optimization problems. These are a part of the family of Evolutionary Algorithms. Below described notations will be used in the sections from on.

1) Search Space (S)
2) Population (P)
3) Candidate Solutions (individuals) (s)
4) A utility/fitness function (f(s))
5) Mutation Operator (m(s))
6) Crossover Operator (c(s1, s2))
7) Parents (P1, P2)
8) Offspring (k)

So, as we discussed before, the population (P) is composed of a number of individuals, and all of those individuals are the candidate solutions for the problem. How to represent the candidate solutions depends on the type of the problem you are trying to deal with. For example, for the problem of Portfolio Optimization, since the weights of the assets have to be real valued numbers, array representation is considered an apt choice.

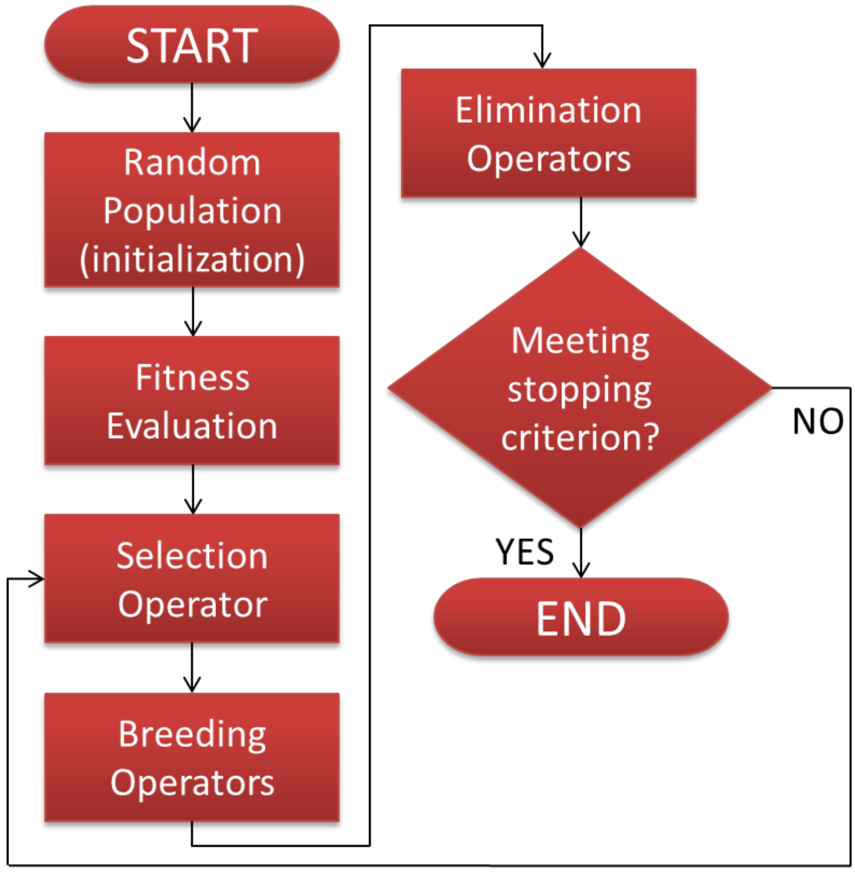

Fig. 3.2: A basic Genetic Algorithm flowchart

For some other type of problems, binary representation string of bits might be a good idea. The genome representation is one of the most widely used representations. A basic Genetic Algorithm is shown in Fig. 3.2 above. Genetic Algorithms have proved their strength in a variety of fields and have shown motivating results with the problem of Portfolio Optimization as well. Better selection, mutation and crossover techniques can yield better results and thus, more extensive experimentation is required.

3.1.1 Fitness

In Genetic Algorithms, fitness evaluation is the next step after the random initial population is formed. The function of the fitness function is to check how good or bad the individuals in the population are i.e. if they are close to the goal or not. Fitness value is computed for each individual in the population. The fitness evaluation gives us an idea about how far we are from the goal. It also helps us in deciding a selection strategy for the individuals, who will in subsequent steps, go through rigorous mutation and crossover operators. The function which is used to evaluate the fitness of the individuals in the population is called a fitness function or a utility function. In case of our Portfolio Optimization problem, for example, it is the Sharpe Ratio.

3.1.2 Selection Strategies

After the fitness evaluation is performed, the next step is the selection. Selection strategy performs the "survival" role for certain selected individuals in Genetic Algorithms. Selection is the genetic operator that chooses some individuals based on some strategy for inclusion in the next generation population or to undergo mutation and/or crossover operators. The main motive is to take the best individuals forward or to give birth to better offspring in order to reach the goal of the problem. However, always choosing the best individuals for mating (recombination operators) can sometimes lead to premature convergence i.e. reaching local minima/maxima. Therefore, out of some of the most common techniques discussed below, the selection operator should be chosen very carefully according to the problem you are dealing with.

1) **Rank based selection:** In this selection technique, first, all of the population is evaluated by the fitness function. After that, all the individuals are ranked based on their fitness value. The best one can be ranked 1 or n, if n is the size of the population. Then, either the best ones or the worst individuals are selected to through the recombination operators. However, this method often leads to slower convergence if the best /worst individuals do not differ too much from other individuals.
2) **Roulette Wheel Selection:** This is another popular selection strategy. In this selection operator, the chance for an individual to get selected is proportional to its fitness or rank. Usually a proportion of the wheel is assigned to each of the possible selection based on their fitness value by dividing the fitness of a selection by the total fitness of all the selections, thereby normalizing them to 1. Then a random selection is made similar

to how the roulette wheel is rotated by generating a random number between 0 and the total fitness.

3) **Tournament Selection:** Here, initially, a relatively large number of individuals are selected through the use of roulette wheel selection. After that, the best individuals are selected from the already selected subset. This method of selection adds to the pressure of being selected for further steps. Since it breaks the search space into several subsets, it can in this way speed up the processing of the Genetic Algorithms.
4) **Elitism:** This is basically not a selection strategy but can be discussed in this section. When we create new individuals (children), we have a probability of losing the best individuals in the process. Elitism simply copies the best individuals from the current to the next generation. Elitism has shown to be very effective and can increase the performance of the Genetic Algorithms because it prevents the best individuals to get lost, and thus decreasing the computational costs.

Although, these are some of the common selection techniques, other techniques exist as well and the new ones can even be formulated depending upon the type of problem being dealt.

3.1.3 Mutation Operator

As we discussed earlier, the GAs are basically used in optimization problems, where the goal is to reach minima or maxima, depending on the type of problem. Mutation operator performs much of exploration work in the search space of the problem. Mutation is a genetic operator which alters one or gene values in a chromosome from its initial value. Through this way, the Genetic Algorithms may arrive at better solutions than it had before. Mutation operator prevents the population from getting stuck at local minima and is thus an important for exploring the search space for reaching the global minima/maxima. Mutation operator can be used before or after the crossover operator in the recombination steps. It occurs according to users' defined values/probabilities, in real valued chromosomes. If the chromosome to be mutated is a binary one, the genes are just flipped from 0 to 1 and vice versa. In real valued chromosomes, care should be taken not to set the defined mutation probability too high as it can just lead to random search rather than achieving better results. Some of the commonly used mutation techniques are discussed below:

1) **Flip Bit mutation operator:** This is basically used in binary chromosomes. The values are from 0 to 1 and vice versa. In Portfolio problems, binary arrays (chromosomes) are used to show the presence or absence of an asset in the final Portfolio, where 0 denotes the absence and 1 the presence.
2) **Boundary mutation operator:** Here, the upper and lower bounds for the genes are first set, randomly or by using some specific strategy. Then, the mutation operator changes the value of the selected gene by upper bound or the lower bound, chosen randomly.
3) **Uniform mutation operator:** Here as well, the lower and the upper bound values of the genes are decided. For example, in the Portfolio problem all the weights of the assets selected in the Portfolio should be greater or equal to 0 (lower bound) and less than 1 (upper bound). The value of the selected gene is then altered to some value between the upper and lower bounds, by using a random number generator function.

Although we have discussed some of the widely used mutation operators, some specific mutation operators can be built for the problem you are trying to deal with. An example of a flip bit mutation operator and a uniform mutation operator is shown in Fig. 3.3 below.

Binary (index) array example	1	0	1	0	1
	1	0	0	0	1

Weight array example	0.35	0.25	0.13	0.22	0.05
	0.35	0.21	0.13	0.22	0.09

Fig. 3.3: Examples of Flip bit and Uniform Mutation Operators

3.1.4 Crossover Operator

As opposed to the exploration of the search space performed by the mutation operators, the crossover operator performs the exploitation task of the search space. As can be inferred from the name itself, the crossover operator combines or merges the information held in two chromosomes (parents in this case) to produce a new chromosome with the features of both of the parents. This newly made chromosome is often called an offspring or a child. Similar to the mutation operator, the crossover is also a stochastic operator. In other words, which parts of the two parents are combined, which method or way is used to combine them can be either random or problem specific. But the main motive is same for all the crossover techniques: using the desirable features of two individuals, produce a new and improved individual which possesses the features of both those parents. Below we discuss some of the most commonly used crossover operator techniques:

1) **One Point Crossover Operator:** In this crossover technique, a single point is selected for both the parents. After that, all the date beyond that point in the chromosomes (parents) is swapped to produce a child.
2) **K-Point Crossover Operator:** This is similar to the one point crossover technique. However, here, rather than selecting just a single point, k points are selected in the parents. Next the child/offspring is composed by alternately copying the elements from the parents for all of the k points. This is basically a one point crossover operator when k is 1.
3) **Uniform Crossover Operator:** In this case, genes are randomly copied from the first or the second parent, depending on some probability. This probability is called a mixing ratio. So, if the mixing ratio is 0.5, the offspring will have around half of the genes from the first parent and the rest from the second parent.
4) **The Intermediate Crossover:** This crossover basically takes the average of the genes of two parents to produce an offspring. However, rather than taking a simple mean, it can be changed a bit as shown in the equation below:

$$\mathrm{C} = \propto P_1 + (1 - \propto) P_2 \qquad 3.1$$

Here, C is the child, P_1 is the first parent and P_2 is the second parent. ∝ is a chosen weight, to decide which parent contributes how much to the child.

There are some crossover operators, but we will not go into those details. Since, crossover operators influences what kind of information is kept from which parent, and they also decide what kind of offspring is produced. Therefore, the choice of the crossover operators is very important to exploit the search space of the problem, and should be very carefully chosen. Again, crossover can be either performed before or after the use of mutation operators. An example of a K-point crossover is shown in figure below:

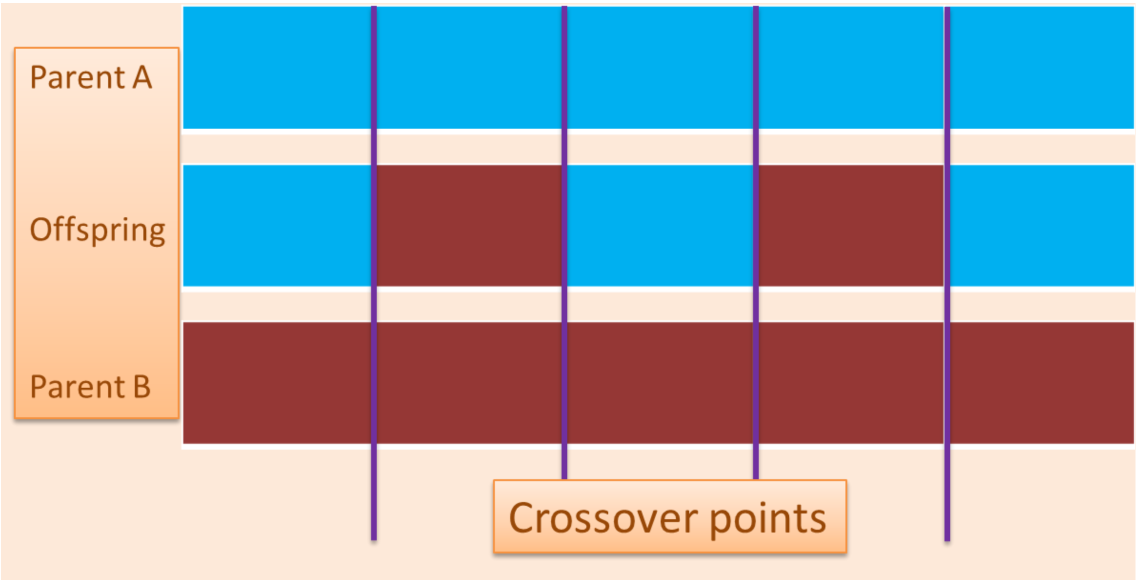

Fig. 3.4: An example of a K-point crossover operator

3.2 Conclusion

After all the steps mentioned above, some elimination of the worst individuals in the population takes place. All these steps are then repeated until some stopping criterion is met or the goal is achieved.

So we discussed how the Evolutionary Algorithms, Genetic Algorithms in particular, work. They work through various steps by moving towards the goal, the optimization of something, slowly. It should be noted that the choice of the fitness function also decides the quality and accuracy of results. A bad fitness function might not produce desirable results. For example, in our Portfolio Optimization problem, since our main aim is to maximize the expected returns of the Portfolio and minimize the risk with associated with it, rather than using two objective functions, the use of single Sharpe Ratio as the fitness functions might produce better results.

Chapter 4
4: Our Work: Multi Objective Portfolio Optimization and Re-balancing

A Portfolio Optimization problem can be considered a resource allocation problem. Here the resource to be allocated is the money available to be invested and it is to be allocated to the assets included in the Portfolio.

While on one hand we have included some of the good approaches from some previous works, on the other hand, we have also introduced some new features in our system. The main ones are

1) The proposal of the new "*Greedy Co-ordinate Ascent Mutation Operator*" and

2) The "*Concept of Traded Volumes as a way to include the impact of news on the markets*".

These two new features will be discussed in the later sections of this chapter.

Our goal is to develop a Portfolio Management strategy that not only works well in normal market conditions but also during the difficult times like the recession of 2008.

Therefore we have chosen NASDAQ100 assets and the Dow Jones Industrial Average 30 assets as our data sets, since the American markets were the worst hit by the recession of 2008. We have performed a series of simulations by different approaches to achieve better results and also to test the reliability of the results.

In this chapter, we will explain the details of our Portfolio model, and then we will explain about the optimization system we have developed for sound Portfolio evolution and maintenance. We will be explaining the model in sequential steps that we have adopted to do our simulations.

4.1 The Portfolio Model

We can start with an assumption of a market where there are N assets available for consideration for the Portfolio. We have to first evolve a Portfolio which maximizes the expected return for the Portfolio and also minimizes the risk of it. Then we also have to Re-balance the Portfolio for several future scenarios since the market is dynamic and keeps changing all the time. The main motive of the Re-balancing step is to keep the expected return maintained to a certain level and maintain the risk as low as possible.

Therefore, we will have several Portfolios at different times t. Let the Portfolios be P_t at times t. The weight assigned to the assets is the proportion of the capital allocated to that particular asset to be invested. Since all the assets included in the Portfolio have to have weights, let w_i represent the weight of the i^{th} asset in the Portfolio. Since the total sum of all the weights of the assets in the Portfolio cannot exceed 1 (where 1 represents the total capital available for investment), we have two restrictions for the weights as given below:

$$0 \leq w_i < 1 \tag{4.1}$$

$$\sum_{i=0}^{n} w_i = 1 \tag{4.2}$$

Here, n is the number of assets included in the final Portfolio. The equations say that the weight of each asset should be less than 1 and greater than or equal to 0 (in case the weight is 1, it is a Portfolio with a single asset), and that the sum of all the weights of the assets should be equal to 1.

4.1.1 Expected Return and Risk

Expected returns: maximizing these is one of the goals of our work. The return of each asset is calculated as shown in the following equation:

$$R_{day} = \frac{C.P._t - C.P._{t-1}}{C.P._{t-1}} \tag{4.3}$$

Here, R_{day} is the expected return per day, $C.P._t$ is the closing price of the asset on the day t and $C.P._{t-1}$ is the closing price of the asset on the day $(t$ -1). Once we have calculated the expected return per day, we take the SMAs over the desired period of time as given below:

$$R_t = \frac{R_1 + R_2 + R_3 + \cdots + R_n}{N} \qquad 4.4$$

Here, R_t is the expected return for a time period t and $R_1 + R_2 + R_3 + \cdots + R_n$ are the returns on time periods 1, 2, 3... n (which can be per day, per month etc.).

Once we have the returns for each asset, next we have to calculate the returns for the Portfolio as a whole. The total return of the Portfolio is calculated as shown in the following equation:

$$R_p = \sum_{i=1}^{N} w_i r_i \qquad 4.5$$

Where R_p is the total return of the Portfolio and r_i is the return of each asset, for all the N assets.

The variance of the return of an asset gives us the risk for that asset. It is calculated over a period of time T. Moreover, the total risk associated with the Portfolio is the covariance of all the composing assets of the Portfolio and is given as shown in the following equation:

$$\sigma_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \qquad 4.6$$

Here, σ_{ii}=σ_i^2 is the variance of r_i, and σ_{ij} is the covariance between r_i and r_j.

The equation 4.7 can also be written as following:

$$\sigma_p = \sum_i^n w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \sigma_{ij} \qquad 4.7$$

4.1.2 Dynamic Market Behavior: The Concept of Traded Volumes and the Impact of News

The MPT considers the Portfolio Optimization technique to be static. It says that the past or future of the market does not influence the Portfolio. However, the change in the prices of assets every second, every minute, the transaction costs, volume limits and other constraints and thus, the need of it to be Re-balanced makes it a very much dynamic problem. Moreover, the people interested in financial markets must be aware of the fact that the news released in the markets also has a strong impact on them. Therefore there is a need to include the concept of news in the Portfolio Optimization problem. But how is that possible, with too much news available online, on television and newspapers. Including all the available news in the problem is almost impossible. However, it might be possible through some indirect method. It is possible through the inclusion of Traded Volumes Concept. Traded Volumes is a measure of how much an asset was traded in the market on a particular day. This data is usually available online for publicly traded companies. So, how to use these Traded Volumes as a means to include the impact of news on the markets? For explaining this let us look at one example through the figure below.

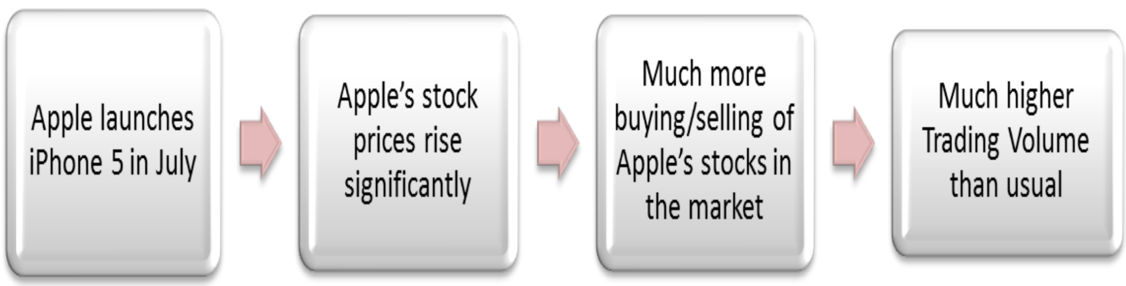

Fig. 4.1: The concept of Traded Volumes and the impact of News

Let us assume a situation where Apple launches a new iPhone, iPhone 5, in the market in July 2012. As soon as the news is released to the market, people will start reviewing the product (sometimes even before the launch). If the new iPhone is better than the competitors, there is a chance for Apple to increase its share in the Smartphone market. This means that Apple will be profitable in the coming days. So, the investors start buying more of Apple's shares than usual trading days, to sell them later at a higher price and make profits. Thus, there is more trading of Apple's shares in the market which means higher volume of shares traded in a single day than usual trading days.

However, if the new iPhone does not meet the expectations of the consumers, there is a chance of Apple losing some of its market share in the smartphone market. So people with a panic start selling the Apple's shares to avoid losing their money. Again, this means higher volume of shares traded in a single day than usual trading days (Fig. 4.2).

Fig. 4.2: The results of news in the market leading to unusual Traded Volumes

Therefore, we can conclude that higher volumes of shares traded in a single day is an indicator of good or a bad trend waiting in the times ahead. If it is bad trend or a good trend can be verified on the next of unusual Traded Volumes. If it is a bad trend, the prices of that particular asset will fall the following day or they will rise in case of a good trend.

Therefore, the inclusion of Traded Volumes Concept can be indirectly thought of as an inclusion of the impact of news on the Portfolio. However, there is a time lag, since we need at least one day for the verification weather it is a good or a bad trend.

So, we include this concept in our work. How is it included? So,

1) Firstly, we calculate the average of Traded Volumes, V_t for a particular time period (a month for example).
2) Then we calculate the Volume Ratio, V_R , as we call it. Volume Ratio can be defined as follows:

$$V_R = \frac{V_d}{V_t} \qquad 4.8$$

Here, V_d is the Traded Volume on a particular day.
So once we have the Volume Ratio, we multiply it with the return on that particular day for that particular asset, for all the assets. This way, we can see the impact of news directly in the returns of the assets. If the good trend is awaiting the asset, the returns will be inflated by the Volume Ratio and deflated in case of a bad trend waiting.

The use of Traded Volumes also provides us very important information: When to Re-balance the Portfolio. From the Volume Ratio, as soon as we get the hint of a trend coming in the coming days, we can Re-balance the Portfolio accordingly to achieve better results. In case of a bad trend, we can sell the Particular asset if it is included in the Portfolio, to avoid making losses, or we can buy some of the asset in case of a good trend to make better profits.

4.2 Genetic Algorithms

We have used Genetic Algorithms for our Portfolio Optimization problem. GAs will be used to basically assign the weights to the assets included in the Portfolio and optimize them. The details are discussed below.

4.2.1 How to represent a Portfolio?

To represent the Portfolio, we have used two arrays. The first one is the binary array, where 0 indicates the absence of the asset from the Portfolio and a 1 represent the presence of the asset in the Portfolio. The second array is a real valued array. Here, all the real values are between 0 and 1, and each value is the weight assigned to that particular asset by the Genetic Algorithm through several steps. Both the arrays are composed of N elements, where N is the total number of assets under consideration to be included in the Portfolio.

The reason to include the binary array is to check if the inclusion of a particular asset in the Portfolio is worth it or not, in a faster way. If we just use real-valued array, it might need more number of generations to reduce the weights of assets to 0 through crossover and mutation to check their importance in the Portfolio. On the other hand, the use of binary array fastens the process.

In the initial population formulation, the arrays are formed using a function that generates random numbers. Both the restrictions regarding the weights of the assets and the Portfolio were incorporated. For generating the binary array, the random number generating function was again used with a 70% probability of generating a 1 and the remaining 30% probability for generating a 0. Both these arrays will be used by the crossover operator in the later steps. However, mutation operator is being used only with the real-valued array.

4.2.2 Fitness Function

Two fitness measures have been used in our work:

1) The Sharpe Ratio
2) The Transaction Costs function

The Sharpe Ratio is the main fitness function, since transaction costs alone cannot lead to an optimized Portfolio. Transaction costs have been used to compare the Portfolio before and after the Re-balancing and the better one is kept.

The Sharpe Ratio has an advantage over using two different fitness functions, one for maximizing the expected returns and the other for minimizing the risk. The first advantage is that it reduces two objectives to one, by incorporating both the objectives in a single function. Second is that it tells us how much the expected return should increase, if we increase the risk of the Portfolio, by leveraging it with the riskless asset.

The Sharpe Ratio is basically the excess expected return to the riskless asset over the risk of the Portfolio and is defined as shown in the following equation:

$$S_r = \frac{R_P - R_{Riskless}}{\sigma_P} \qquad 4.9$$

Here, S_r is the Sharpe Ratio, R_P is the expected Return of the Portfolio, $R_{Riskless}$ is the return on a riskless asset (usually, relatively short-term bonds of stable governments which theoretically has 0 risk and a low return rate) and σ_P is the total risk associated with the Portfolio.

Obviously, the higher the value of Sharpe Ratio the better the Portfolio is. Although a good Sharpe Ratio value depends on the kind of assets, market the investor wants to invest in; still the following values are good for comparison with the evolved Portfolios:

1) Sharpe Ratio $\leq$ 1: Not good. It means that the risk with the Portfolio might be too high for whatever return it promises.
2) 2>Sharpe Ratio $>$ 1: Might be a good Portfolio depending on the market.
3) Sharpe Ratio $>$ 2: These are usually considered good.
4) Sharpe Ratio $>$ 2: These are usually considered excellent for investments.

While using the transaction costs as a fitness measure, the candidate Portfolios from the current scenario is compared to the best Portfolio of the previous scenario. Here, the one with the lowest transaction cost is the winner and is kept as the best Portfolio. Each scenario represents a time period after which the Portfolio is Re-balanced. In our works, the Portfolio is Re-balanced every last day of the month.

4.2.3 Selection

Selection is the process of selecting some individuals from the population for the recombination operator and the mutation operator. As we have discussed earlier, there are a number of ways of performing the selection process.

We have chosen Deterministic Tournament Selection (DTS) as our selection method. The DTS has mainly two advantages over the methods. The first one is that it selection pressure can be adjusted by adjusting the number of participants in the tournament. For example, the tournament size can be made to be 5 individuals or 50 individuals depending on the type of problem, and the pressure of selection necessary. The second one is that, since it breaks down the selection process into a number of subsets, it lead to parallel implementation and thus leads to faster processing.

We have also incorporated the Elite strategy. This means that without going through the crossover or mutation operator, some of the best solutions from the present generation are carried forward to the next generation.

4.2.4 Crossover Operator

As discussed earlier, the crossover operator does more of the exploitation of the search space. In the selection process, 5 individuals are selected 100 times in every generation through the Deterministic Tournament Selection. Out of those 5 individuals, the top are sent through the crossover operator, for both the real-valued array and the binary array. Whenever required, normalization is carried out as well.

Here, the K-point crossover is used. In each crossover step, the new child is produced by incorporating half the genes from both the parents.

4.2.5 Greedy Coordinate Ascent Mutation Operator

As we discussed earlier, the mutation operator does more of the exploration work in the search space. One of the new points that we have included in these research, apart from the Traded Volumes concept, is the proposal of Greedy Coordinate Ascent Mutation Operator.

Looking at the normal mutation operators, we felt that there is too much trial and testing in them. For example, after the individual for going through the mutation operator is selected, its elements are changed very randomly. However, the Portfolio Optimization problem calls for the proper optimization of the weights of each and every asset present in the Portfolio. So we came up with this idea. The Greedy Coordinate Ascent Mutation Operator does the following with an 'n' assets' vector:

1) In the first step, the individual is mutated through the normal mutation operator. Some element is chosen randomly and then its value is changed and the individual evaluated.
2) Then, in the next process, the weight of the first element in the array is optimized, keeping all the other (n-1) assets' weights constant.
3) The weight is moved in both positive and negative directions (weight is increased and decreased) and the array is then normalized.
4) The best of the three, the original, the increased weight or the decreased weight, is evaluated by sending the vector through the fitness function.
5) Next, the second weight is optimized, while keeping the first optimized element and the rest (n-2) weights constant.
6) Steps from 2) through 5) are repeated for all the 'n' elements in the array.

The advantage of using this mutation operator, as you may notice, is that each and every element in the array is optimized one by one. This not only reduces the chance of much trial and testing, but also gives better results. The mutation operator has only been used for the real-valued arrays. The Fig. 4.3 on the next page shows how this mutation operator works, through arrays.

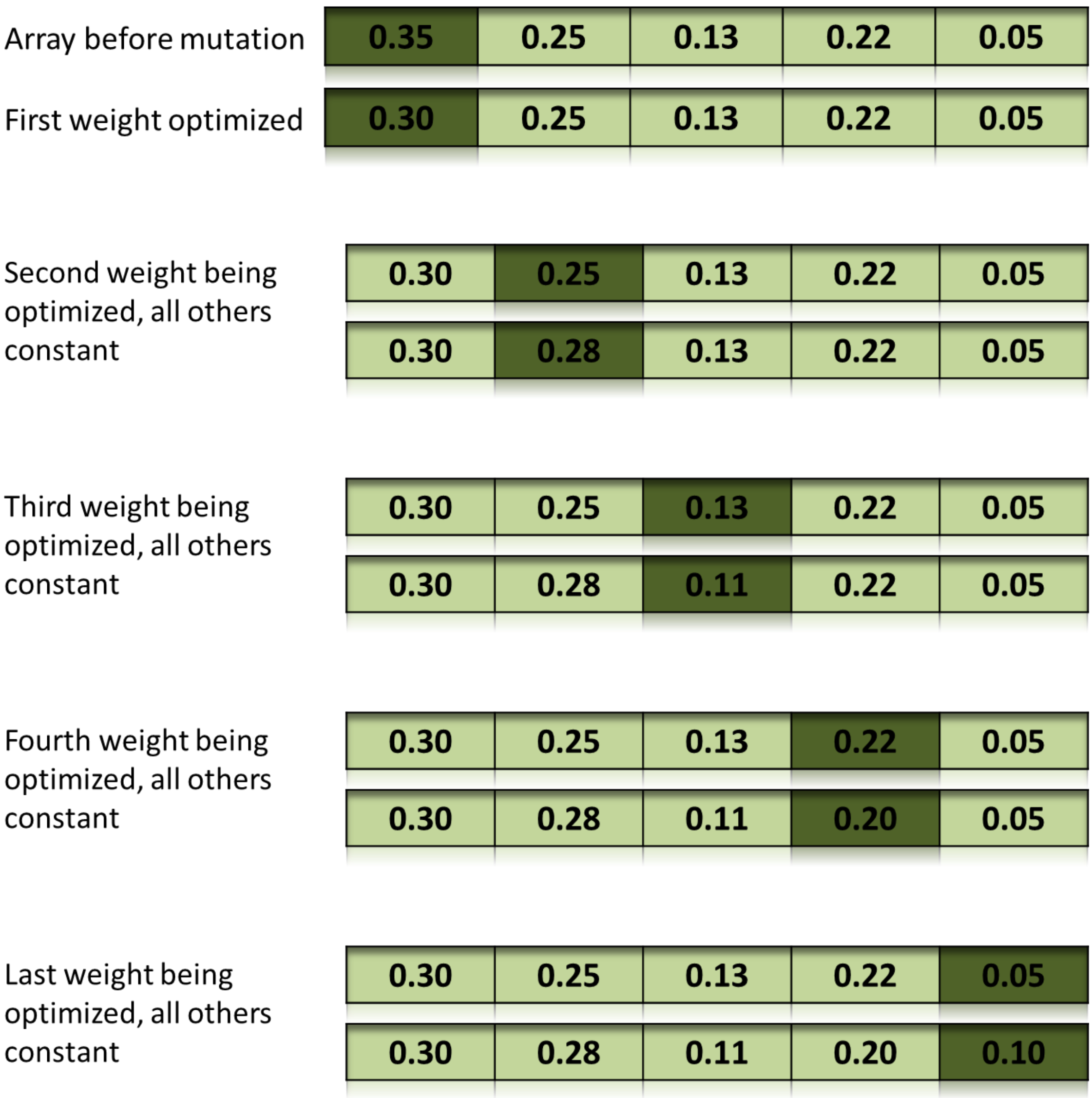

Fig. 4.3: The working of Greedy Coordinate Ascent Mutation Operator (Note that weights are not normalized in this example figure; however, they are optimized in experimental simulations)

One more problem that we wanted to address through this newly introduced mutation operator is the problem of the presence of ***insignificant weights*** in the Portfolios.

So what are insignificant weights? Consider a Portfolio, where the assets under consideration to be included in the Portfolio are 50. Let us say, we used simple Genetic Algorithms and evolved a Portfolio. This evolved Portfolio consists of 15 assets, i.e. it has included 15 assets out of the 50 under consideration into the Portfolio. Now that is ok. However, 10 assets present in the Portfolio have weights which are negligible, let us say something like 0.00023. One example of a Portfolio with insignificant weights is shown in the figure below.

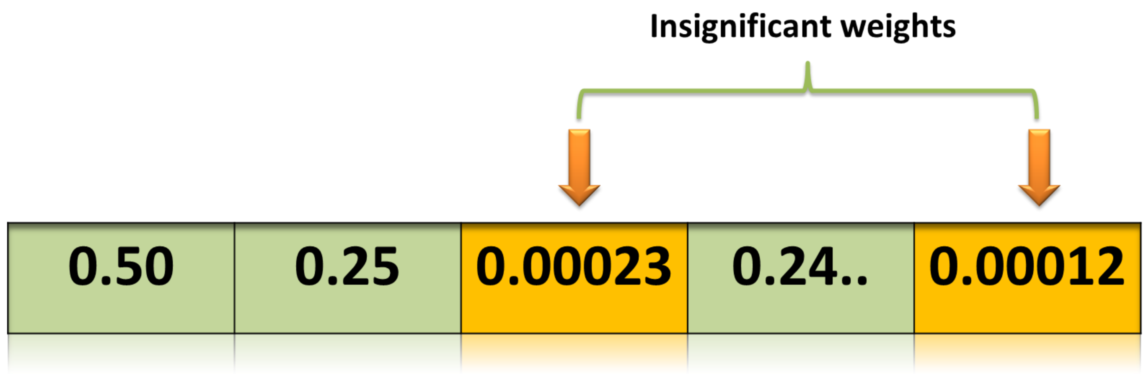

Fig. 4.4: An example of a Portfolio with assets with insignificant weights

As we can notice, the assets in orange color have insignificant weights as compared to the other weights present in light green color. Their contribution in increasing the expected returns of the Portfolio or lowering the risk of the Portfolio as a whole might be very small or even negligible. However, when we Re-balance the Portfolio, and the new Portfolio suggest us to sell those assets, we have to pay a fee, the transaction costs, which is usually higher than their contribution in making the Portfolio more profitable. Therefore, rather than keeping those assets with insignificant weights in the Portfolio, it might be a better strategy to get rid of them in the first place to avoid paying transaction costs at a later stage to sell them at some point. This is an important issue, which needs to be addressed and we hope to achieve better results in this regard, by the use of our new Greedy Coordinate Ascent Mutation Operator.

4.3 Re-balancing

All the sections of the Chapter 2 till now, are necessary to evolve a Portfolio with the historical data available. However, since markets are dynamic, we need to change the Portfolio according to the changes in the market, to keep the expected returns high and the risk associated as low as possible.

Re-balancing does not come for free though. Changing the Portfolio from time to time means selling and buying the assets from time to time. Selling of assets in the market also costs money, as does the buying of assets. Therefore, if we just keep changing the Portfolio too much, we might lose a big fraction of our capital we had in the beginning to invest. But if we don't change the Portfolio at appropriate times, we might also end up with huge losses with the investment.

So, Re-balancing means we have to pay the transaction costs to the brokers. We need to minimize these transaction costs in order to make decent profits by investments. Therefore, at this point of time, minimizing the transaction costs become our new objective in this problem, as shown in the figure below.

Change In Expected Return In The Portfolio > Transaction Costs

Fig. 4.5: New Objective Function while Re-balancing the Portfolio

Although there have been various methods for calculating the transaction costs, like Aranha and Iba (2006, 2009) used Euclidean distance of the weights as the transaction costs, we are to use more realistic and close to real life approach. In our work, the transaction costs are calculated as given in the following equations:

$$\mathrm{T.C.} = Cost_{fixed}\,, if\ 0 < T_i < T_{min} \qquad 4.10$$

$$\mathrm{T.C.} = T_i * \delta_c\,, if\ T_i > T_{min} \qquad 4.11$$

Here, T.C. are the transaction costs, $Cost_{fixed}$ is some fixed cost, T_i is the amount of i[th] asset traded (bought or sold), T_{min} is some fixed minimum amount of asset, δ_c is some cost parameter.

So, according to these equations, some fixed cost is to be paid to brokers if the deal, amount of some asset to be bought or sold, is smaller than some fixed minimum level of trade. The second equation in simple terms means, that some percentage of the deal amount is charged as transaction costs.

We have also incorporated the concept of *Seeding* in the Re-balancing part. In seeding, the first random population is generated or initialized as usual. However, the best solutions of the population of the scenario at time (t-1) are also included in the population of scenario at time t. This way, we tilt the solution of scenario at time t towards the individuals coming from the previous scenario. This way, we can try to keep the transaction costs to a minimum level.

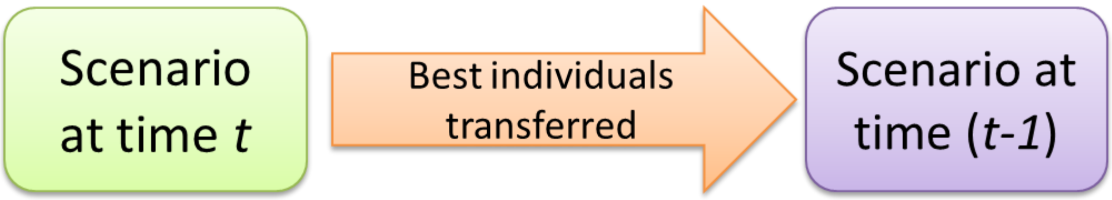

Fig. 4.6: The concept of Seeding

Chapter 5
5: Simulation Framework and Results

We will explain the main framework for the simulations and discuss the results achieved, in this Chapter.

To compare the results of our Portfolio with the inclusion of the new "Greedy Coordinate Ascent Mutation Operator" and the Traded Volumes Concept, with that of Portfolios without them, a number of simulations were carried out. We also compare both of our results with those of simple Genetic Algorithms, without our new mutation operator and without the Traded Volumes Concept, and with the indices.

There were two main goals of these simulations:

1) Since we concentrate mainly on the time period of 2008, when the recession was affecting the financial markets to a great extent, the first goal is to verify how our Portfolio performs in these difficult times, as compared to the indices of the datasets used and also the results of the simple Genetic Algorithms.
2) The second goal was to evaluate the impact of the two new points included by us in our work. The first one is the newly proposed mutation operator. The second was to check the impact of the inclusion of the concept of Traded Volumes.

Regarding the results, apart from achieving better results in terms of higher expected returns and lower risk associated with the Portfolio as a whole, there was one more problem we wanted to address: the problem of availability of insignificant weights in the Portfolio, as we discussed in the earlier Chapter (Chapter 4).

The results achieved by our Portfolio through various simulations are very motivating. They outperform the indices and the results from simple Genetic Algorithms. However, there are still minor problems that need to be addressed to make the Portfolio Optimization even better. The results and these problems are discussed in the following sections of this Chapter.

5.1 Datasets

We have used two datasets to check our algorithm and our system. The two datasets are:

1) NASDAQ100 assets
2) Dow Jones Industrial Average 30 assets

Both of these datasets and the data related to them are available online and can be downloaded from websites like “Yahoo Finance” or “Google Finance”.

The reasons for choosing these datasets are mainly three.

1) The first reason is that we wanted to check our algorithm with real historical data. The simulation with artificial data does not have reliability of providing the same good results in the real markets.
2) The recession of 2008 affected the financial markets significantly. Although, the markets were hit throughout the world, it would not be wrong to say that the American financial markets were the worst hit, out of all of them. Therefore, we wanted to perform the simulations with some American market indices.
3) Lastly, we wanted to check the performance of our algorithm with a relatively smaller dataset and the other with a relatively larger dataset. So, where Dow Jones Industrial Average 30 has 30 assets in it and is a relatively smaller dataset, NASDAQ100 with 100 assets, a relatively broader index is a relatively bigger dataset. Therefore, we have the chance of verifying our system with both the scenarios.

In the Dow Jones Industrial Average 30 assets, 29 assets have been taken in for simulations to generate the Portfolio, due to unavailability of data of one of the composing assets. In the simulations of NASDAQ100 assets, all the 100 assets have been included for the simulations.

The historical data for both the datasets was downloaded and daily returns have been used for the calculation of estimation of expected returns and risk of the Portfolio. For the calculation of daily returns, the daily closing price of all the assets is used. Traded volumes were used as provided in the historical data downloaded.

5.1.1 NASDAQ100 Assets Index

The NASDAQ-100 started its operations on the 31st of January 1985 under the NASDAQ stock exchange, which is the biggest stock exchange in the world. NASDAQ100 is a stock market index of the largest non-financial companies listed on the NASDAQ stock exchange. There are main points which differentiate NASDAQ100 from Dow Jones Industrial Average and the S&P 500 index, given as follows:

1) The first one is that the companies' weights in the index are based on their market capitalizations (with certain exceptions), and only contains non-financial companies
2) Secondly, it also includes companies based outside the United States of America.

So there are two main reasons to do the simulations with the NASDAQ100 assets' historical prices data. The first one is that, it is a relatively medium-big dataset and we can check how our algorithm and the system work with such a dataset, i.e. if the size of the dataset affects the working of the system in any way. Second is, that since it contains the biggest of the non-financial companies only, we can check and confirm the effectiveness of our system on such companies during the recession of 2008, whether we can generate good results with the worst hit companies or not.

The dataset used for generating the Portfolio (evolving the Portfolio with the training data) is used from January 2007 till December 2007, 1 whole year's historical prices data. After the Portfolio is evolved, it is Re-balanced for the next 18 scenarios, i.e. it is Re-balanced starting from January 2008 till June 2009. The reason for this is that we want to check the effectiveness of our system mainly on the data for the recession time of 2008, when most of the big companies were generating tremendous amounts of losses: whether our system can still produce profits from that data.

For the NASDAQ100 assets, which as the name suggests has 100 assets; all the assets have been used for both the training part (evolving the Portfolio) and for the Re-balancing as well. How the NASDAQ100 index performed during that period is shown in figure below:

Fig. 5.1: The NASDAQ100 index from January 2007 ~ June 2009

As we can notice from the Fig. 5.1.1.1, the index starts performing extremely poor from the time around January February 2008 and the drop continues till March 2009. The period from September 2008 ~ March 2009 is the worst performing.

5.1.2 Dow Jones Industrial Average 30 Assets Index

Dow Jones Industrial Average (DJIA) index was founded on May 26 in 1896. It is composed of 30 large and publicly traded companies, all based in the United States of America. The Dow Jones Industrial Average is a price-weighted index. There are two main reasons that we decided to do our simulations with the Dow Jones Industrial Average 30 assets' index:

1) To check and verify if our system works in the same fashion with datasets, irrelevant of their seizes (since NASDAQ100 is a relatively bigger dataset)

2) Since DJIA consists of only the American publicly traded companies, we can concentrate more on the American market scenario that was in the recession times of 2008.

Same as in case of NASDAQ100 index, the dataset used for generating the Portfolio (evolving the Portfolio with the training data) is used from January 2007 till December 2007, 1 whole year's historical prices data. After the Portfolio is evolved, it is Re-balanced for the next 18 scenarios, i.e. it is Re-balanced starting from January 2008 till June 2009. The DJIA 30 index has 30 assets in it. However, due to the unavailability of the full data for one of the assets' historical prices, we have used 29 assets for our simulations.

How the Dow Jones Industrial Average 30 index performed during that period is shown in figure below:

Fig. 5.2: The DJIA 30 index from January 2007 ~ June 2009

As we can notice from the Fig. 5.2, the index has several ups and downs. It starts performing really poor around September 2008. The drop continues and reaches the worst level around March 2009.

One very important point to be noted, apart from the index average, in the Fig. 5.1.1.1 and Fig. 5.1.2.1 is about the traded volumes. The blue candles at the bottom of the figures represent the traded volumes. It may be noted that the volumes change in accordance with some trends. For example, the time around September 2008 is accompanied by unusual volumes (if we comp-are them with the volumes during the times before that). Therefore, suggesting that our proposed Traded Volumes Concept might work well.

5.2 Parameters

Parameters values are important in Evolutionary techniques such as Genetic Algorithms, as they decide the speed of convergence to minima and also the results, to some extent, of the experiments. Apart from the parameters used in almost all simple Genetic Algorithms (here simple GAs refer to GAs without our newly proposed Greedy Coordinate Ascent Mutation Operator and without the Traded Volumes Concept, and some minor changes in our new algorithm), we will also note down the values of some other Parameters used. The parameters values are as shown in the table below and the meaning of the parameters is explained below the table:

Table 1: Values for different parameters used

Parameter	Parameter Value for NASDAQ100 index	Parameter Value for DJIA 30 index
Number of Generations	200	500
Number of Individuals	500	500
Tournament Size	5	5
Number of Tournaments per Generation	100	100
Mutation Ratio	1(100%)	1 (100%)
Mutation Perturbation Value	0.005	0.005
Crossover Ratio (for both binary and real-valued array)	0.5	0.5
Elite Size	100	100
Seeding Size	100	100

In Table 1, “Tournament Size” represents the number of individuals selected to participate in the tournament and the “Number of Tournaments per Generation” represents the number of times that tournament takes place in each generation. “Mutation Ratio” is the chance of an element of an array to get mutated (1 represents that each element is mutated). “Mutation Perturbation Value” represents the value with which each element is perturbed, in both positive and negative directions, i.e. the value is added to the current value and the value is subtracted from the current value and finally the best out of the three (current value, decreased value, subtracted value of weight) is kept. The top two individuals in every tournament are used to pass through the crossover operator and the newly born child is then mutated. “Crossover Ratio” represents how many elements from each parent are used to produce the child. Thus, the Crossover Ratio of 0.5 means that 50% of genes are taken from each parent to generate an offspring.

These parameters were used to run each simulation several times to validate the results. The results achieved were same each time the simulation was run indicating that our proposed method is giving the best possible results our system can produce.

Simple Genetic Algorithms simulations, however, generated different results on many simulation occasions. Therefore, the average of the results produced by the simple GA method was used to compare them with our Portfolio results and the indices' performance for the same period.

5.3 Results

Here, in this section we are going to present the results that we have achieved through our simulations. The main goal of the experiments is to explore the Re-balancing ability of our Portfolio. We want to check how our “Greedy Coordinate Ascent Mutation Operator” works and whether we can achieve good profits even during the recession period of 2008. We also want to check the ability of our Portfolio towards the reduction of assets with insignificant weights. We want to check if our new mutation operator works on this problem well or not. Also, we want to check the impact of news on Portfolio, whether our newly proposed method of Traded Volumes increases the performance of our Portfolio or not. The results are shown below:

5.3.1 Dow Jones Industrial Average 30 Index Results and Analysis

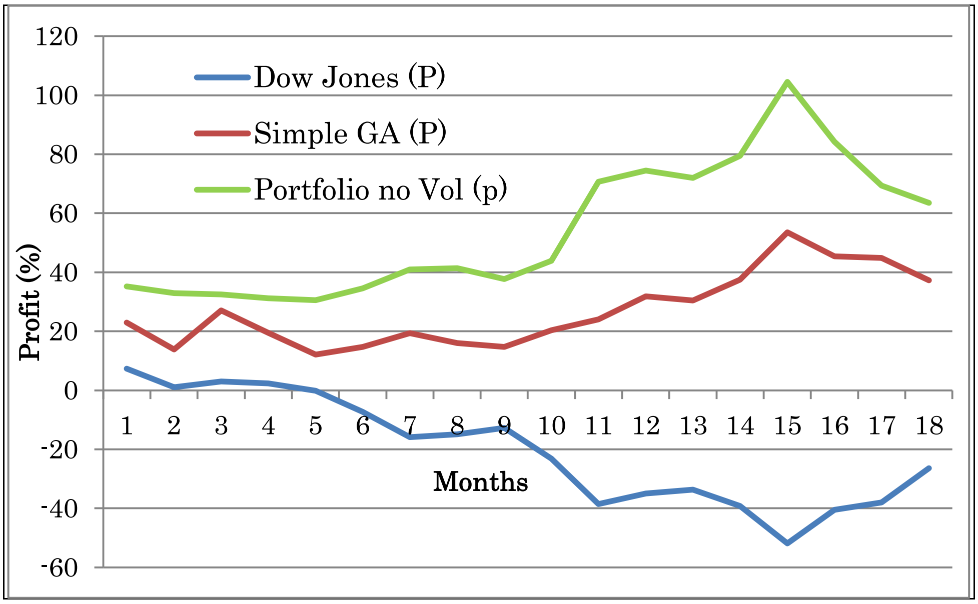

Fig. 5.3: Results of profits given by Simple GA and Portfolio without Traded Volumes: To compare the performance of our mutation operator

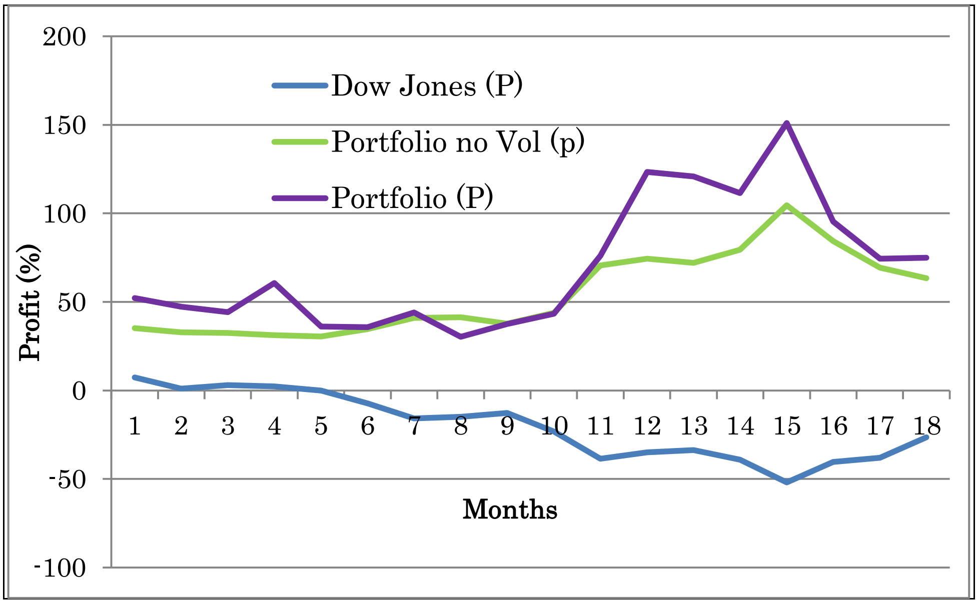

Fig. 5.4: Results of profits given by Portfolio without Traded Volumes and Portfolio with them: To analyze the performance of Traded Volumes Concept

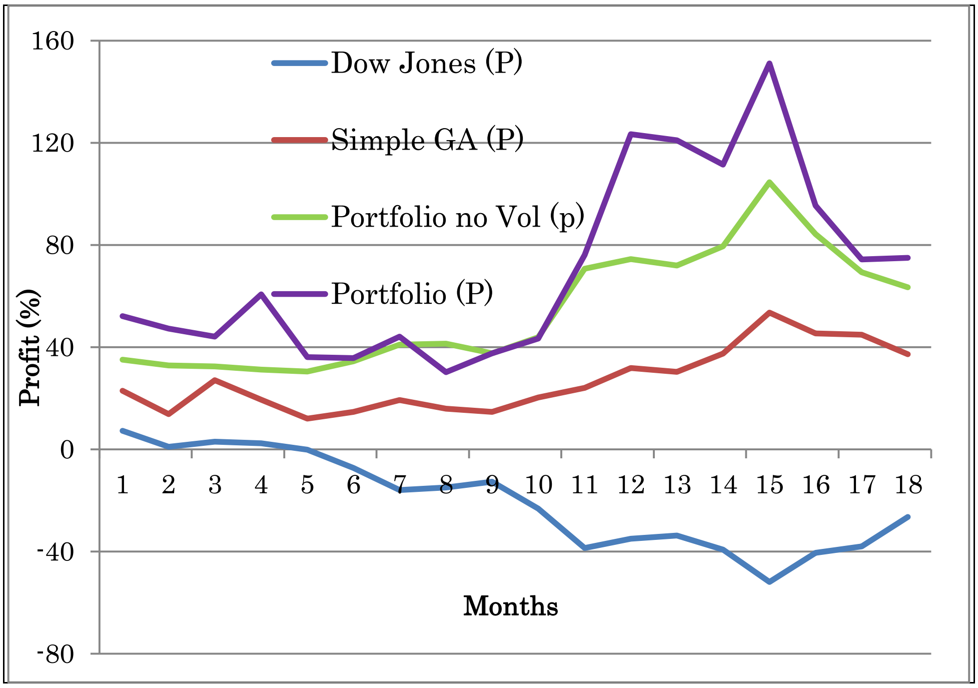

Fig. 5.5: Full experiment results of profits by the index, Simple GA, Portfolio without Traded Volumes and Portfolio with them

Fig. 5.3 through 5.5 show the expected results achieved by three methods namely: Simple GA, Portfolio no Vol and Portfolio compared with the index itself. Here, Portfolio no Vol represent our algorithmic system with the new mutation operator and other techniques and strategies that we have incorporated in our system. Portfolio represents Portfolio no Vol with also the Traded Volumes Concept included.

As we can notice in Fig. 5.3 both Simple GA and the Portfolio no Vol outperform the index. The index starts producing negative profits, i.e. losses around May of 2008, when the recession started to spread on a larger basis. The index goes on to produce losses for the entire period under consideration till June 2009. Simple GA and our Portfolio no Vol starts on a slow note, though with profits even in that period. The momentum shown by both the methods is pretty much similar to each other except for few months. However, Portfolio no Vol outperforms the Simple GA method. Portfolio no Vol is relatively stable till around October 2008 until which it shows the profits of around 40% on the investment. After that, it shows much better profits and even goes to result in profits of more than 100% for a single month. This indicates that our “Greedy Coordinate Ascent Mutation Operator” works extremely well over the normal mutation operator, which though performs well too, and produced motivating results.

Now let us compare the results of the Fig. 5.4. The figure lets us compare the performance of the Traded Volumes Concept. Here, the Portfolio represents our system with all the new points included, including the Traded Volume Concept (Portfolio = Portfolio no Vol + Traded Volumes Concept). Although the difference is not significant, but the Portfolio produces better results than the Portfolio no Vol. For a single month, Portfolio no Vol produces better profits than the Portfolio. However that can be because of fixed Re-balancing dates that we had chosen for Re-balancing the Portfolios. We have re-balanced the Portfolios on the last working day of each month. Therefore, the trend of the prices of the assets, which is usually indicated by the volumes of trade of each asset, has not been fully utilized. However, apart from that single month, August 2008, Portfolio has performed well.

Fig. 5.5 lets us compare all the three methods with the index. Where all the three methods have outperformed the index, by producing profits even when the index was producing losses, the Portfolio's is the best among all.

However, profits alone cannot be used to judge the performance of any system in the Portfolio Optimization problem. Risks need to be taken in consideration as well. This is because, for example, any Portfolio that promises 50% profits cannot be regarded as good if it has a risk of 200% associated with it. So, let us check the results of Sharpe Ratios, which was our main fitness function, to compare the final results of DJIA 30 index. The results are shown in Fig. 5.6 through Fig.5.8 below.

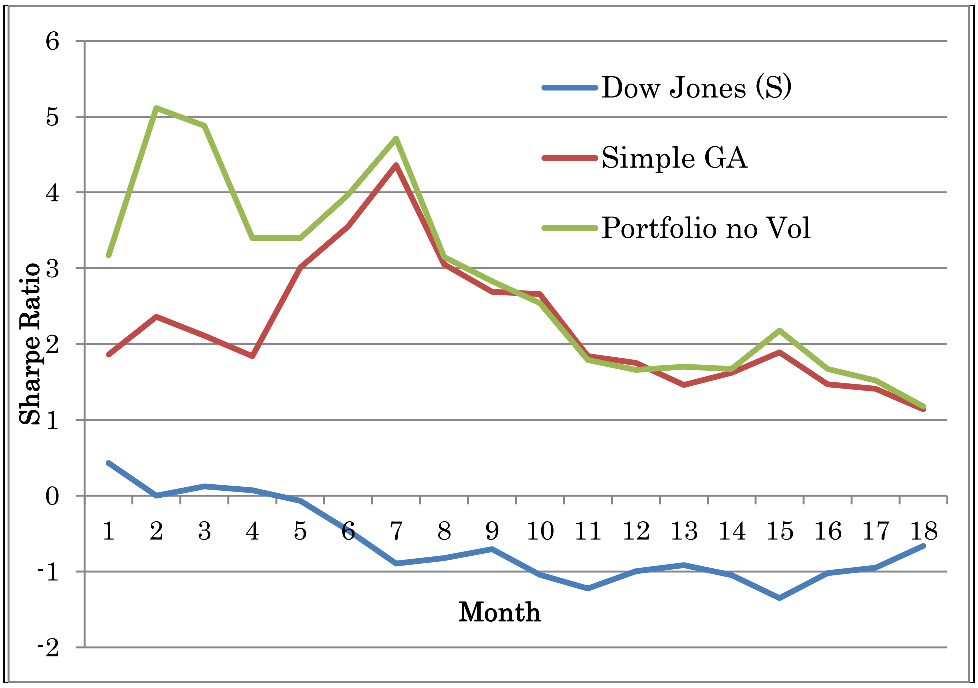

Fig. 5.6: Results of Sharpe Ratio given by Simple GA and Portfolio without Traded Volumes: To compare the performance of our mutation operator

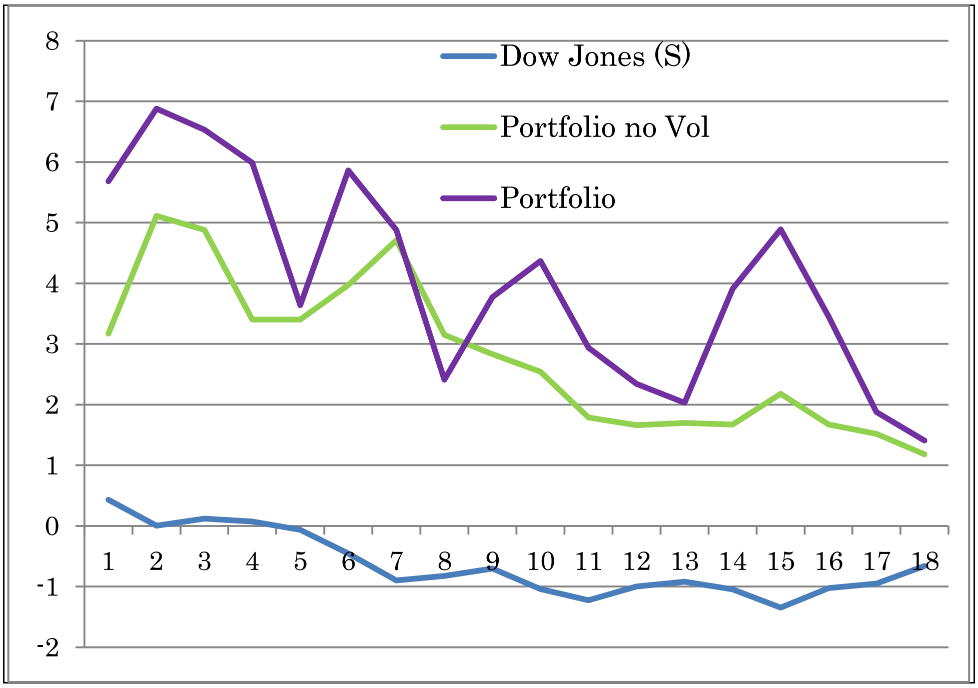

Fig. 5.7: Results of Sharpe Ratio given by Portfolio without Traded Volumes and Portfolio with them: To analyze the performance of Traded Volumes Concept

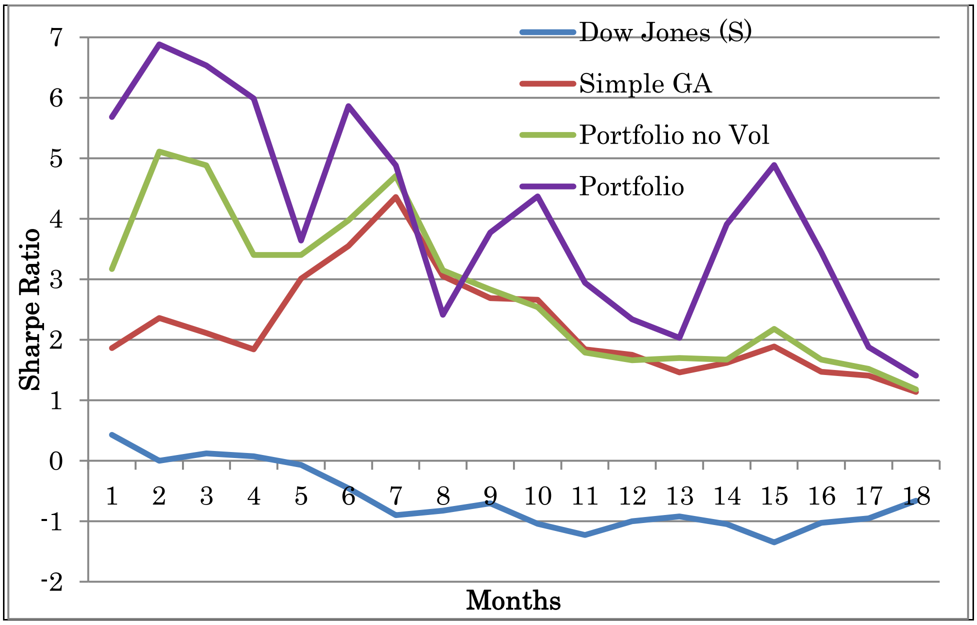

Fig. 5.8: Full experiment results of Sharpe Ratio by the index, Simple GA, Portfolio without Traded Volumes and Portfolio with them

Fig. 5.6 lets us compare the Simple GA and Portfolio no Vol with the index. We can notice that the results are not significantly different, especially after July 2008. Both the methods outperform the index though.

Fig. 5.7 lets us compare the Portfolio and the Portfolio no Vol. We can notice that the Portfolio works much better producing much better Sharpe Ratios from month to month, except for the month of August 2008, where the profits were bad too. This suggests that the Traded Volumes Concept meets the expectations to some extent. The problem of concern is the lack of stability. We can see a lot of ups and downs in the results. However, the reason for that might be the instability in the volumes of assets traded everyday during the period, and since we have incorporated that element in our system, it is visible in the results. However, the other possible reason for the instability can be, as we talked earlier in this section, the need for better Re-balancing timing strategies.

Overall, in the Fig. 5.8, all the three methods, Simple GA, Portfolio no Vol and the Portfolio outperforms the index. The Portfolio comes out as the winner, referring to the facts that not only have we produced good expected returns through the investment in our Portfolio; we have also succeeded in keeping the risk at a low level.

5.3.2 NASDAQ100 Index Results and Analysis

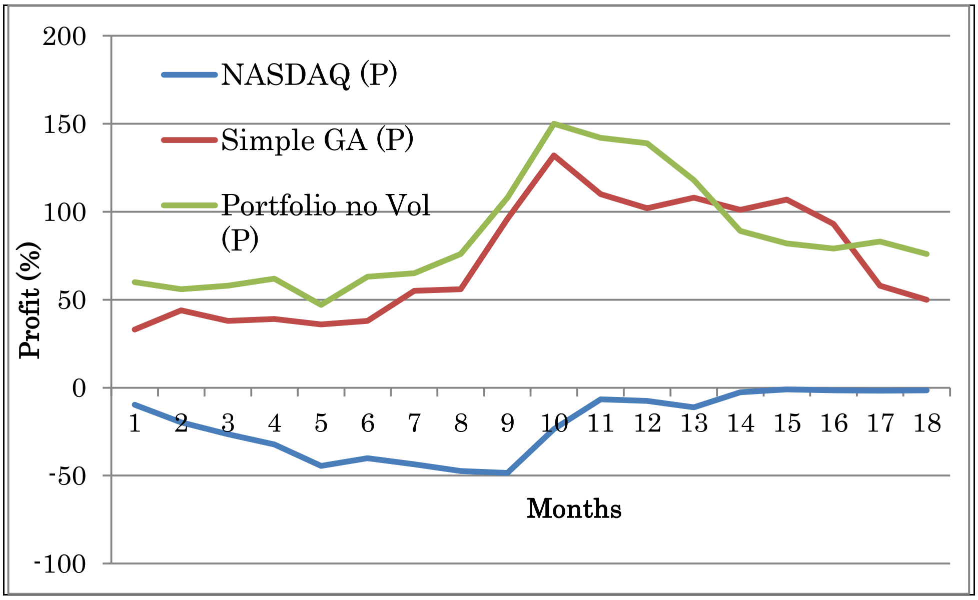

Fig. 5.9: Results of profits given by Simple GA and Portfolio without Traded Volumes: To compare the performance of our mutation operator

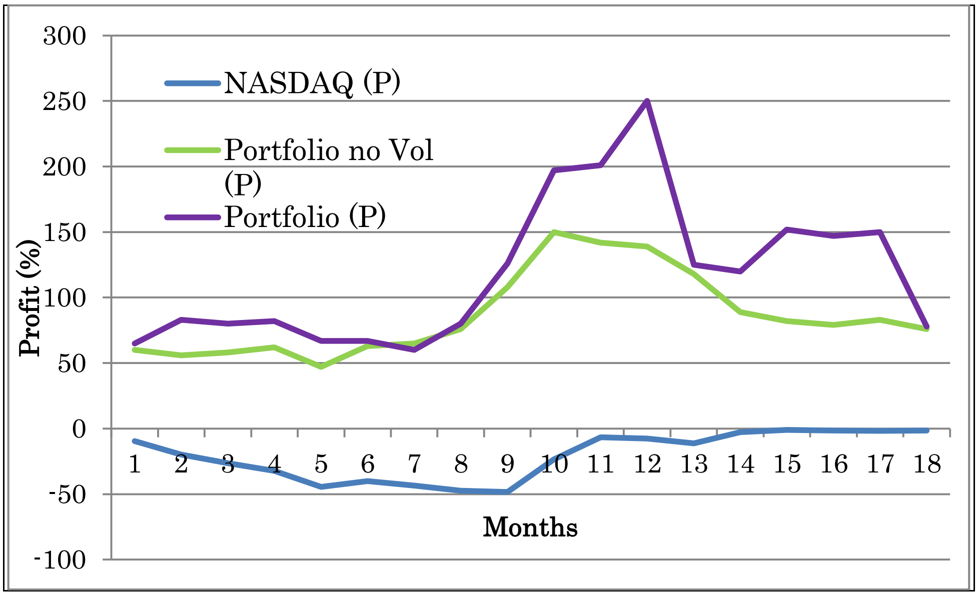

Fig. 5.10: Results of profits given by Portfolio without Traded Volumes and Portfolio with them: To analyze the performance of Traded Volumes Concept

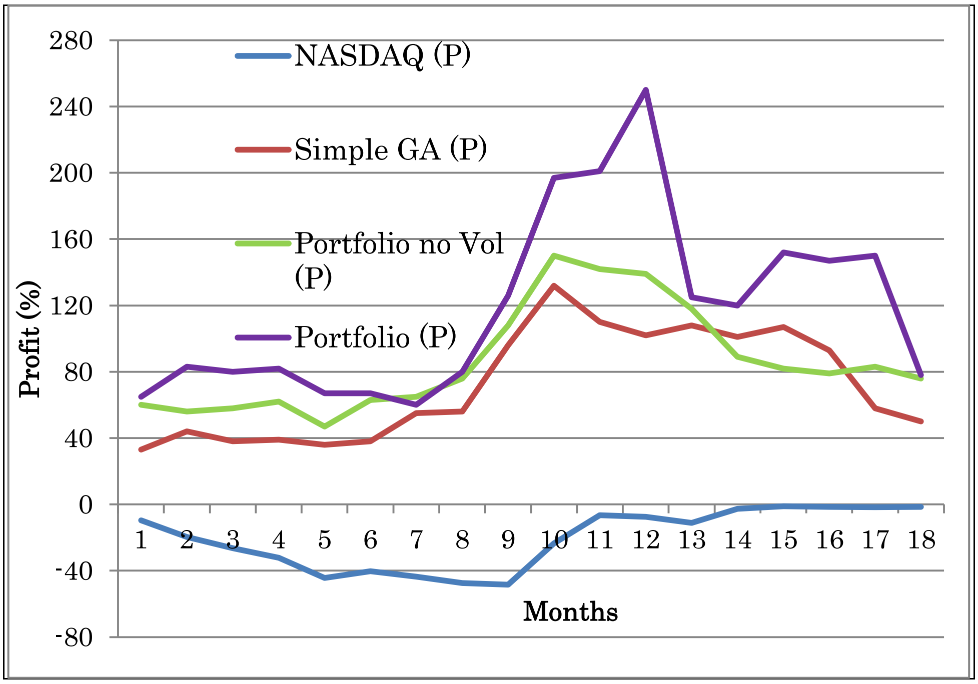

Fig. 5.11: Full experiment results of profits by the index, Simple GA, Portfolio without Traded Volumes and Portfolio with them

Now that we have compared the results of the simulations with a smaller dataset Portfolio, DJIA 30 index, let us look at the results with a relatively larger dataset, i.e. NASDAQ100 assets index.

Fig. 5.9 through Fig. 5.11 shows us the results of profits achieved with the Portfolios evolved from this dataset. Fig. 5.9 lets us compare the profits of Simple GA and the Portfolio no Vol. Both the methods outperform the index, which starts showing negative profits from January 2008 and goes on with that trend till around February 2009. After that, till the entire period under consideration, i.e. June 2009, the index shows the profits around 0%. The Simple GA and the Portfolio no Vol starts on a profit good note, with Simple GA underperforming the Portfolio no Vol with an extent of around 20% ~ 30%. The momentum of both the methods is similar except for the time period around March 2009, where Simple GA has produced better results. Overall, the new mutation operator has performed well with better profits, with a good stability in it.

Now let us look at the Fig.5.10. The figure lets us assess the performance of the Traded Volume Concept. We can notice that both the methods outperform the index and the results are extremely good, with the

Portfolio even reaching a level of around 250% profits on the investment. The Portfolio has produced much better profits at points, however that trend is not stable and there are months when the profits from the two methods are comparable. Again, as we talked before in this section, the instability seen in the profits of the Portfolio is a definite problem that needs to be addressed. However, if we look from an investors' point of view, as long as we get profits of this level (more than 60% for the entire period, especially during recession) with minimum risks, the problem doesn't seem to be big.

Fig. 5.11 shows the results of all the three methods together, with the index.

Now let us look at the Sharpe Ratios results from the NASDAQ100 dataset, to have a better picture at the Portfolio, taking into consideration the risk levels associated with them. The results are shown below.

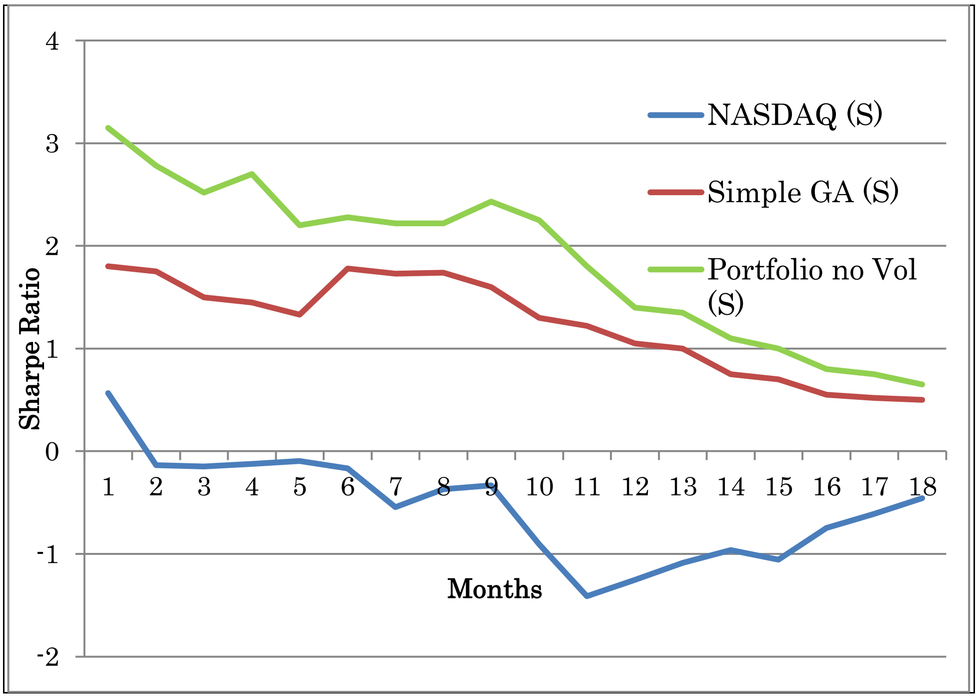

Fig. 5.12: Results of Sharpe Ratio given by Simple GA and Portfolio without Traded Volumes: To compare the performance of our mutation operator

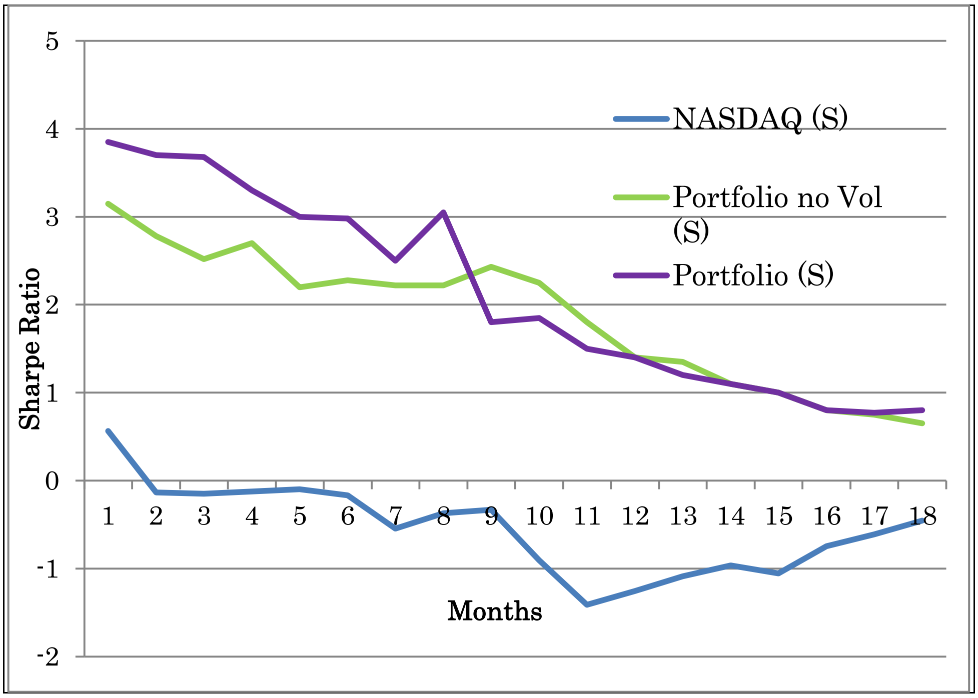

Fig. 5.13: Results of Sharpe Ratio given by Portfolio without Traded Volumes and Portfolio with them: To analyze the performance of Traded Volumes Concept

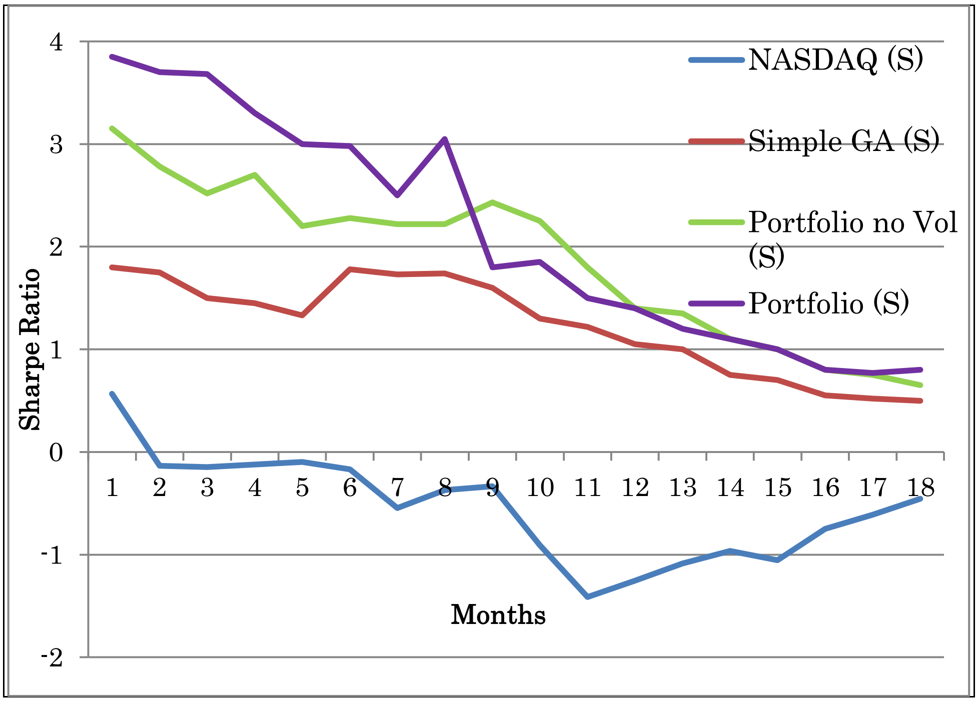

Fig. 5.14: Full experiment results of Sharpe Ratio by the index, Simple GA, Portfolio without Traded Volumes and Portfolio with them

Fig. 5.12 lets us compare the Simple GA and the Portfolio no Vol results with those of the index. Firstly, the index shows negative Sharpe Ratio for almost the entire period. The Simple GA and the Portfolio no Vol again have the same kind of momentum, with Portfolio no Vol producing better results. The results are relatively smoother than the results of DJIA 30 dataset. However, the trend is of a declining nature and the Sharpe Ratio goes on a decrease for the entire period. The Sharpe Ratio goes below 1 towards the end of the period, suggesting that the risk level is pretty high in that level. This trend is understandable, since the non-financial companies took a very long time to recover from the recession of 2008 and performed really bad during the entire season. However, mutation operator proposed by us has worked better than the usual mutation operator, although the difference is not very significant.

Fig. 5.13 lets us compare the performance of the Traded Volumes Concept on the relatively bigger dataset, the NASDAQ100. Here, in the first 8 months of the period, the Portfolio performs better than the Portfolio no Vol. This can be because of the significant changes in the trading volumes of the assets in the dataset during that period. However, after September 2008 the effect is not visible and the results of the methods are almost comparable.

Fig. 5.14 compares the results of all the above mentioned methods with the index, in one graph. All the three methods have outperformed the index, producing good Sharpe Ratios. Portfolio and the Portfolio no Vol works better than the Simple GA, although the trends in the results are similar. Portfolio especially works better than the Simple GA. However, the results of Portfolio and the Portfolio no Vol. are comparable after some months.

5.3.3 Results related to Insignificant Weights

The Tables below show the results attained for the two datasets, DJIA 30 assets index and the NASDAQ100 assets index. The tables show how many significant weights and how many insignificant weights are present in the Portfolios evolved from the first 1 year (January 2007 ~ December 2007) training datasets, by various methods. Here, in for this comparison, all the assets with weights less than 0.001 are considered insignificant.

Table 2: Comparison of number of assets with significant and insignificant weights for the DJIA 30 index dataset

DJIA 30 Assets	***Number of assets with significant weights***	***Number of assets with insignificant weights***
Simple GA	10	14
Portfolio no Vol	5	0
Portfolio	5	0

Table 3: Comparison of number of assets with significant and insignificant weights for the NASDAQ100 index dataset

NASDAQ100 Assets	***Number of assets with significant weights***	***Number of assets with insignificant weights***
Simple GA	21	23
Portfolio no Vol	14	0
Portfolio	12	0

Looking at both the Tables 2 and 3, we can easily notice that with Simple GA, the number of assets with insignificant weights is pretty large, as compare to the number of assets in the indices. Moreover, the Portfolio and the Portfolio no Vol produce neat and clean results with the number of assets with insignificant weights equal to 0, in both the cases. From these results, we can infer that the reason for 0 assets with insignificant weights, in the latter two cases, is the "Greedy Coordinate Ascent Mutation Operator". However, we cannot ignore the effect of Traded Volumes Concept completely, if we look at the table with results for NASDAQ100 assets. Here, Portfolio no Vol has 14 assets with significant weights, two more than those present in the Portfolio.

Chapter 6
6: Conclusions

We have discussed the problem of Portfolio Optimization and Re-balancing using Genetic Algorithms. This problem can be considered a resource allocation problem with the money to be invested as the resource to be allocated to the assets. This problem has mainly two ultimate aims: One, to maximize the expected return of the Portfolio and the other to try to minimize the risk associated with the whole Portfolio. The problem of reducing the number of assets with insignificant weights becomes more important during the Re-balancing of the Portfolio.

We have presented mainly two new points in our work, apart from few other minor changes in the algorithm. These two points are: the proposal of new "Greedy Coordinate Ascent Mutation Operator" and the "Traded Volumes Concept". WE have used a more realistic approach for calculating the transaction costs during Re-balancing of the Portfolio, which is used by traders and brokers in their daily jobs.

We have chosen the recession period's historical prices of assets for two datasets, the Dow Jones Industrial Average 30 index and the NASDAQ100 index. The main aim for the experiments was to check the effects of the two new points we have proposed on our algorithm and to verify their performance during the hard times like the recession of 2008. We wanted to see if we can still produce profits and good Sharpe Ratios in conditions when the whole market suffers.

Looking at the results from the simulations, we can conclude that the new mutation operator has met the expectations to an extent. The problem of assets with insignificant weights is solved, which is very motivating. With the help of this new mutation operator, we have also been able to achieve better results than the Simple Genetic Algorithms. The results are motivating here too.

However, the inclusion of the Traded Volumes Concept has not produced satisfactory results. The concept works from time to time, and there are some times when it doesn't show any improvement over the Portfolio no Vol. However, the main reason for this is, as we thought, the timing of Re-balancing the Portfolio. The reason for including the Traded Volumes Concept was to look at the trends that are visible when looking at daily volumes of assets traded. Due to a lack of time, we have not been able to exploit this strategy fully. We have Re-balanced the Portfolio every last working day of the month. However, we still strongly believe in this concept (Traded Volumes Concept). If we can do the Re-balancing whenever we find a new trend through the Traded Volumes, we hope that we will be able to achieve much better results.

We hope that this work will be seen as a reference, mainly for the Traded Volumes Concept and hope that it will be used in usual investment strategies.

Chapter 7
7: Future Works

Our results from both the datasets, DJIA 30 and the NASDAQ100 index are motivating and suggests that this work can be given attention for further works related to the problem of Portfolio Optimization and Re-balancing.

There are some points, which if given attention, can prove useful in increasing the performance of this system and also bring it closer to real life applications.

The first one is the timing for the Re-balancing of the Portfolio. We have included the Traded Volumes Concept in out Portfolio Optimization system. The volumes of the assets traded everyday change significantly before and during the coming of a significant trend. If the Portfolio is Re-balanced properly, according to this information for the upcoming trends, we strongly believe that the performance of this system will increase significantly.

To bring it close to the real world, LOTS constraint needs to be implemented too. The LOTS constraint says that any asset cannot be divided infinitely. But the results we receive through our method obviously divide the assets to any extent and thus, this constraint needs to be implemented.

The last one is a very interesting approach that we wanted to implement in our system, but could not due to the unavailability of data online. The idea is that instead of using historical prices for the various calculations and simulations, if we could use the real amount of profits or losses made by the assets for a time period, we can better predict the performance of those assets and thus investment strategies. This sounds like a very interesting idea but calls for some kind of collaborative research with certain company that possesses this cant of data.

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