10. Analysis of Dispersed Surface Waves by means of Fourier Transform III.

Analysis of Practical Seismogram of South Atlantic Earthquake.

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1. Introduction

In a previous paper[1] we analysed the record of the explosion that took place on an ice sheet covering the surface of Lake Haruna. The result was pretty satisfactory and the dispersion curve, spectrum of the waves, and the thickness of the ice plate were estimated with success. Moreover, by use of the above data the movement near the origin was synthesized[2], which showed a plausible feature, although some problems still remain unsolved.

Now, in this paper, we will take up a practical seismogram which was presented by James T. Wilson[3] in his study of the South Atlantic Earthquake of August 28, 1933.

2. Formulas for the analysis

The method of analysis is quite similar to that of the former case, namely the spectrum of the wave $F(p\,;r)$ is given by

$$F(p\,;r)^2=c(p\,;r)^2+s(p\,;r)^2=\frac{r_0}{r}F(p\,;r_0)^2\,,$$

$$c(p\,;r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;r)\cos(p\tau)d\tau\,,$$

$$s(p\,;r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;r)\sin(p\tau)d\tau\,. \qquad \cdots\cdots(2.1)$$

1) Y. SATÔ, "Analysis of Dispersed Surface Waves by means of Fourier Transform I," *Bull. Earthq. Res. Inst.*, **33** (1955), 33.

2) Y. SATÔ, *ibid.*, **34** (1956), 9.

3) James T. WILSON, "The Love Waves of the South Atlantic Earthquake of August 28, 1933," *Bull. Seism. Soc. Amer.*, **30** (1940), 213.

where $f(\tau\,;r)$ is the movement observed at the point of epicentral distance r.

Velocity is given by the formula

$$V(p)=p(r-r_0)/[\text{ARCTAN}\{s(p\,;r)/c(p\,;r)\}-\beta(p)+2n\pi]\,. \quad (2.2)$$

If we can neglect pr_0 and $\beta(p)$

$$V(p)=pr/[\text{ARCTAN}\{s(p\,;r)/c(p\,;r)\}+2n\pi]\,. \quad \cdots(2.2)'$$

If two seismograms to which the dispersion formulas are common are available the following expression is preferable

$$V(p)=\frac{p(r^{(1)}-r^{(2)})}{\text{ARCTAN}\left\{\dfrac{s(p\,;r^{(1)})}{c(p\,;r^{(1)})}\right\}-\text{ARCTAN}\left\{\dfrac{s(p\,;r^{(2)})}{c(p\,;r^{(2)})}\right\}+2m\pi}\,. \quad (2.3)$$

These expressions are given in the first part of this investigation[4].

3. Data

As stated above, the data were taken from J. T. Wilson's paper[5]. There are more than twenty seismograms in this paper, which have all

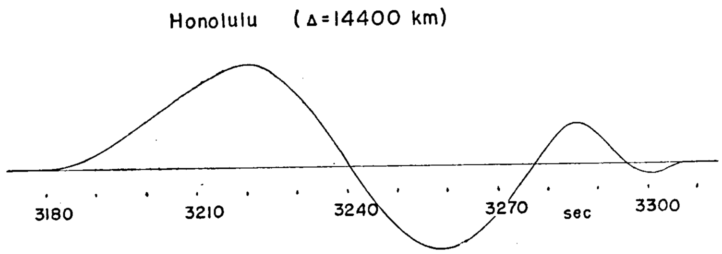

Fig. 1 a.

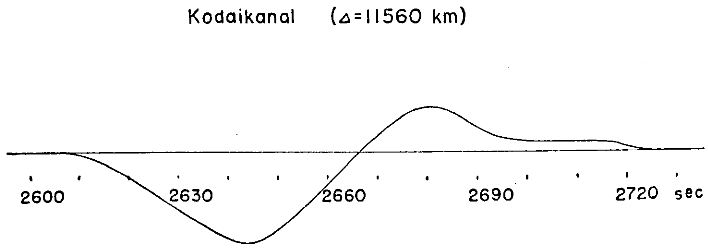

Fig. 1 b.

4) *loc. cit.*, 1). cf. (3.4), (3.6) and (3.9).
5) *loc. cit.*, 3).

Table I.

Honolulu (r=14,400 km)				Kodaikanal (r=11,560 km)			
t (sec)	$f(t;r)$	t (sec)	$f(t;r)$	t (sec)	$f(t;r)$	t (sec)	$f(t;r)$
3180	0	3255	-31.6	2604	0	2679	16.9
81.5	0.3	56.5	-32.7	05.5	0	80.5	17.6
83	0.9	58	-33.4	07	0	82	17.8
	1.4		-33.5		-0.1		17.4
86	2.2	61	-33.0	10	-0.4	85	16.5
	3.5		-31.7		-1.2		15.1
89	4.9	64	-30.0	13	-2.3	88	12.7
	6.2		-28.0		-3.6		10.8
92	7.6	67	-25.5	16	-5.0	91	9.7
	9.4		-22.9		-6.4		8.1
3195	11.2	3270	-20.0	2619	-7.9	2694	7.0
	13.3		-16.6		-9.7		6.1
98	15.3	73	-13.0	22	-11.5	97	5.6
	17.4		-9.1		-13.5		5.1
3201	19.6	76	-5.0	25	-15.5	2700	4.8
	21.8		-0.6		-17.3		4.5
04	24.0	79	3.5	28	-19.2	03	4.3
	26.2		7.5		-21.1		4.2
07	28.4	82	11.0	31	-23.0	06	4.1
	30.4		14.3		-24.9		4·1
3210	32.3	3285	16.0	2634	-26.9	2709	4.1
	34.1		16.3		-28.7		4.0
13	35.8	88	15.3	37	-30.4	12	4.0
	37.2		13.2		-32.2		4.0
16	38.9	91	10.5	40	-34.0	15	4.0
	39.6		7.4		-35.4		3.8
19	40.3	94	3.7	43	-36.3	18	3.3
	40.9		0.9		-36.8		2.8
22	40.8	97	-1.5	46	-36.4	21	2.0
	40.0		-3.0		-35.4		1.3
3225	38.5	3300	-4.0	2649	-33.8	2624	0.7
	36.2		-3.7		-31.7		0.4
28	33.6	03	-3.0	52	-29.4	27	0.2
	30.9		-1.8		-26.9		0
31	27.8	06	-1.0	55	-24.2	2630	0
	24.4		-0.4		-21.4		
34	20.4	09	-0.1	58	-18.5		
	16.3		0		-15.5		
37	12.0	3312	0	61	-12.5		
	7.4				-9.4		
3240	2.5			2664	-6.4		
	-2.1				-3.5		
43	-6.6			67	-0.5		
	-10.7				2.0		
46	-14.6			70	5.0		
	-18.1				6.7		
49	-21.4			73	9.8		
	-24.4				11.9		
52	-27.1			76	13.9		
3253.5	-29.8			2677.5	15.8		

been integrated and the true ground motions are given. Of these many seismograms, two records of Honolulu and Kodaikanal, which seemed not to have been dispersed so much, were chosen. We selected these data because we are specially interested in the existence of a heterogeneous medium upon which the surface waves propagate without showing any dispersion. Sometimes we can find such cases and a typical example has been given by A. Imamura[6].

The graphical and numerical data are given in Fig. 1 and Table I. These are a faithful reproduction of the original waves, only the ordinate having arbitrary units.

4. Result of the analysis

Result of the analysis is given in the following figures and tables.

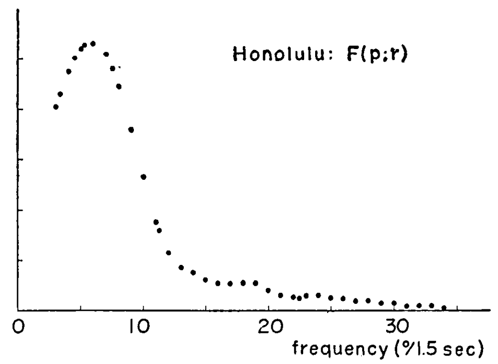

Fig. 2 a.

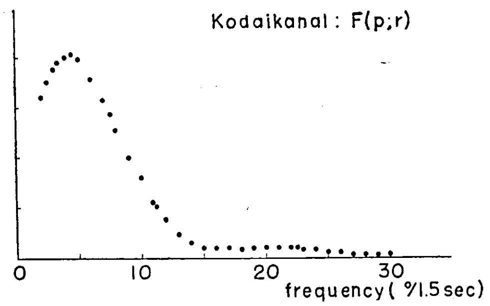

Fig. 2 b.

The spectrum of the curve observed at Honolulu is shown in Fig. 2a and Table II, while that of Kodaikana is in Fig. 2b and the same table. These two figures, which should both give the same curve, namely the spectrum near the origin, resemble each other to a satisfactory extent.

Next, we calculated the phase velocity. At first a simple formula was employed and neglecting the phase angle we obtained $V(p)$. Since the epicentral distances are very large, this neglect seems to cause no fatal error.

The results are shown in Figs. 3a and 3b. Because of the many valuedness of the inverse trigonometric function

6) A. IMAMURA, *Practical and Theoretical Seismology* (Maruzen, 1937), p. 293.

Table II.

p (°/1.5 sec)	T (sec)	$F\ (p;r)$		$\mathrm{A{\scriptsize RCTAN}}\left\{\frac{s(p;r)}{c(p;r)}\right\}$	
		Honolulu	Kodaikanal	Honolulu	Kodaikanal
$2\frac{1}{2}$	216		70		
3	180	81	75	1.38	−0.75
4	135	95	80	1.57	−0.18
$4\frac{1}{2}$	120	100	81	1.65	−3.04
5	108	104	79	1.71	0.35
$5\frac{1}{3}$	101.25	106		1.75	
6	90	106	71	1.80	0.89
7	77.1	102	63	1.83	1.43
$7\frac{1}{2}$	72	96	57	1.82	−1.41
8	67.5	89	51	1.81	2.02
9	60	72	40	1.77	2.57
10	54	53	32	1.56	3.12
11	49.1	35	22	1.27	−2.68
$11\frac{1}{4}$	48	32	20	−1.97	2.15
12	45	23	15	0.79	2.28
13	41.5	17	9	0.07	−2.04
14	38.6	15	6	−0.50	−2.05
15	36	12	4	−1.10	−2.30
16	33.75	11	4	−1.84	−2.43
17	31.8	11	4	−2.50	−2.60
18	30	11	3	−3.05	−2.70
19	28.4	11	4	2.73	−2.80
20	27	8	4	2.30	−2.69
21	25.7	6	4	1.62	−2.37
22	24.5	5	4	0.82	−1.92
$22\frac{1}{2}$	24	5	4	0.63	−1.48
23	23.4	6	3	0.15	−1.39
24	22.5	6	3	−0.41	−1.46
25	21.6	5	2	−0.94	−0.11
26	20.8	5	2	−1.61	0.40
27	20	4	1	−2.15	0.71
28	19.3	4	1	−2.60	0.12
29	18.6	3	1	2.95	0.37
30	18	3	1	2.26	−0.12
31	17.4	2	2	1.70	−0.42
32	16.9	2	1	0.96	−0.48
33	16.4	2	1	0.61	−0.10
34	15.9	1	1	0.66	−0.01
35	15.4	0	1	0.06	0.13
36	15	0	1	2.01	0.72

we again[7] encounter the difficulty of determining which of the curves given by the dots in the figure is our true dispersion curve.

In Fig. 2a, A has too large a phase velocity while D has too small

7) *loc. cit.*, 1). p. 44.

a one, so we may omit these two at once. Of the remaining two, B branch seems preferable to the author, because C has a decreasing phase velocity at the period range of less than 50 seconds, which fact does not seem natural.

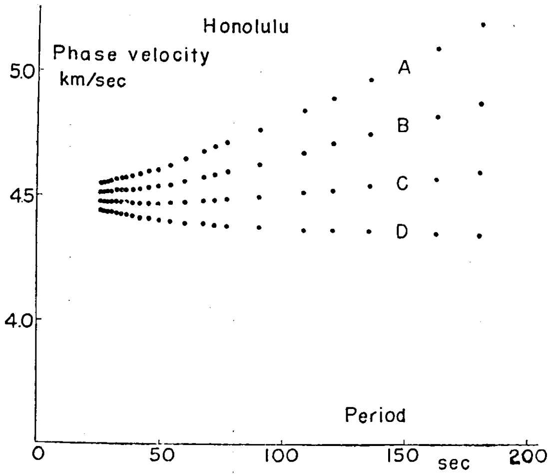

Fig. 3 a.

Thus the only remaining branch is B. From Fig. 3 a we can make a graph phase velocity versus wave-length, and by graphical differentiation we can obtain the group velocity. The result is given in Fig. 4, and the formula for this calculation is

$$\text{Group velocity} = V - L\frac{dV}{dL}, \quad (4.1)$$

where L is the wave-length.

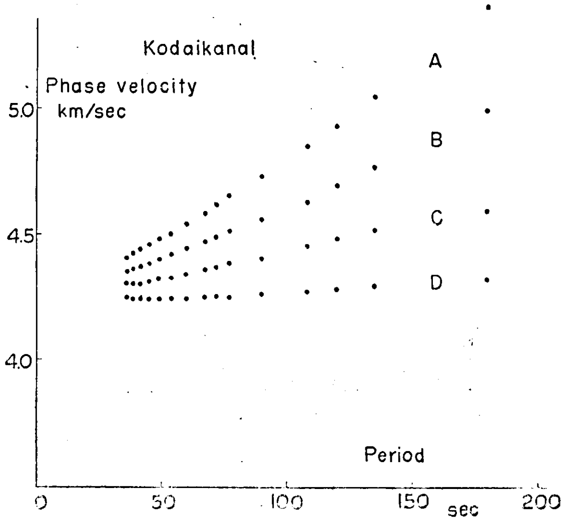

Fig. 3 b.

By a similar consideration of Fig. 3b we must choose one of the branches shown in this figure. B or C will be adequate, but neither of these curves coincides with the branch B in Fig. 3a. This implies that the dispersion curves belonging to the two paths from the epicenter to Honolulu and Kodaikanal are not the same. This circumstance will be recognized more fully, if we apply the formula (2.3) to the values in Table II. If the dispersion formula is common to these two paths, we shall get a better result than by the approximate formula (2.2)′. But the curves obtained by this process do not meet our expectation, and present very unreasonable features. (Fig. 5.)

5. Concluding remarks

In this paper we have applied our formula for the analysis of the dispersed surface waves to the seismograms prepared by J. T. Wilson. Although the result is seemingly true, the data are not abundant and we cannot apply the exact formula (2.3). Hence the dispersion curves in Figs. 3 and 4 are not very trustworthy in their details. The author eagerly wishes to get good seismic data, analyse them and determine an exact dispersion formula. Then we shall be able to obtain the underground condition by means of the method of T. Takahashi[8]. The determination of the velocity distribution under the Pacific Ocean is the author's great hope. Also we may get some clue to the mechanism of the focus by the synthesis of the movement near the origin.

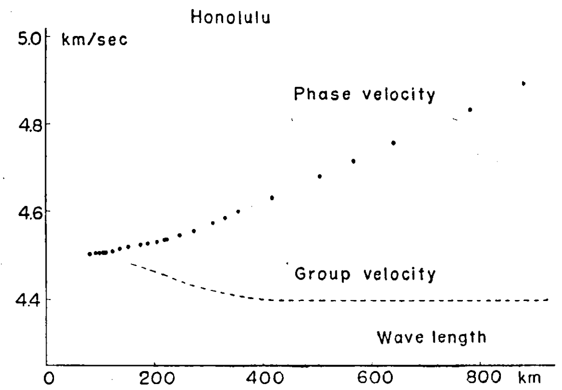

Fig. 4

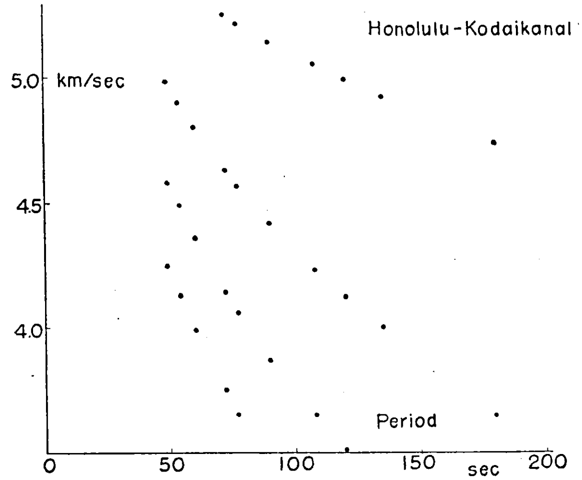

Fig. 5

Finally the author wishes to express his heartiest thanks to James T. Wilson. The calculations in this paper were performed only by use of his excellent study.

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8) T. TAKAHASHI, "Analysis of the Dispersion Curves of Love-Waves," *Bull. Earthq. Res. Inst.*, 33 (1955), 287.

10. 分散した地震波の解析 III.
南大西洋の地震の記録解析

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1. 第1報においてのべた[1]，Fourier 変換を用ゐて分散した表面波を解析する方法を，実際の地震の記録に応用した．用ゐたのは，1933 年 8 月，南大西洋に起つた地震を，J. T. Wilson が整理して発表したものである．

2. 解析のための公式は筆者の前の論文と全く同じである．$f(t; r)$ を以て震央距離 r の所の動きとすれば，

$$F(p;\, r)^2 = c(p;\, r)^2 + s(p;\, r)^2 = \frac{r_0}{r} F(p;\, r_0)^2$$

$$c(p;\, r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\tau;\, r) \cos(p\tau) d\tau$$

$$s(p;\, r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\tau;\, r) \sin(p\tau) d\tau \qquad \cdots\cdots(2.1)$$

で与へられる F によつて波のスペクトルが，

$$V(p) = p(r^{(1)} - r^{(2)}) / [\mathrm{ARCTAN}\{s(p;\, r^{(1)})/c(p;\, r^{(1)})\} - \mathrm{ARCTAN}\{s(p;\, r^{(2)})/c(p;\, r^{(2)})\}] \qquad \cdots\cdots(2.3)$$

によつて位相速度が与へられる．

3. 用ゐた記録は前述の如く，J. T. Wilson の論文中のものであるが，その中で特に殆ど分散を示してゐないと見られる Honolulu 及び Kodaikanal のもの (Fig. 1a 及び 1b) を此処では使用した．それは，太平洋底を伝はつた表面波が殆ど分散を呈しない事が従来から知られて居り[6]，筆者が特にこの現象に興味をいだいてゐたからである．

4. 計算の結果得られた波のスペクトルは Fig. 2a 及び 2b に示す．位相速度は前にも経験した事であるが[7]，逆三角函数の多価性の為に一義的に定まらない．他の事情を考慮すれば，Honolulu に対しては Fig. 3a の B の曲線をとるべきと考へられる．今この線を採用するならば，公式 (4.1) を応用して図上で微分を行い，群速度を求めることができる. (Fig. 4). Kodaikanal に対しては Fig. 3b が得られるが，このうち B もしくは C の曲線をとるべきであろう．観測が 1 ヶ所しかない為に位相角を与える $\beta(p)$ を省略した簡単な公式 (2.2)′ を用ゐてゐるので，速度に関しては，いか程の精度があるか疑はしい．

5. もし Honolulu 及び Kodaikanal に到る道筋の分散公式が同じであるならば，両者を (2.2) に代入すればよりよい結果が得られる筈であるが，計算の結果は Fig. 5 の如くなつて到底真実のものとは考へられない．これは二つの分散公式が異ることを暗示するものであろう．

6. よりよい資料をえて研究を進め，一方では分散公式から媒質の構造を定めると共に，他方発震機構に関する推論迄も進み得る日の近い事を望む．