2. *Analysis of Dispersed Surface Waves by means of Fourier Transform II. Synthesis of the Movement near the Origin.*

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1. Introduction

In our previous paper[1] we have discussed the result of an experiment performed upon the ice sheet covering the surface of a lake. By means of some simple principle based on Fourier transform, the spectrum of the movement observed at the distant places as well as the dispersive property of the medium through which the surface waves are propagated are obtained numerically. We show the data in Figs. 1 and 2. The former is the calculated phase velocity, while the latter is the spectrum at a distance of 141.75 m. Both were obtained using the seismogram in Fig. 3 of the previous paper (Fig. 5a in this paper).

We will try in the following sections to synthesize numerically the movement near the origin in conformity with the above two data.

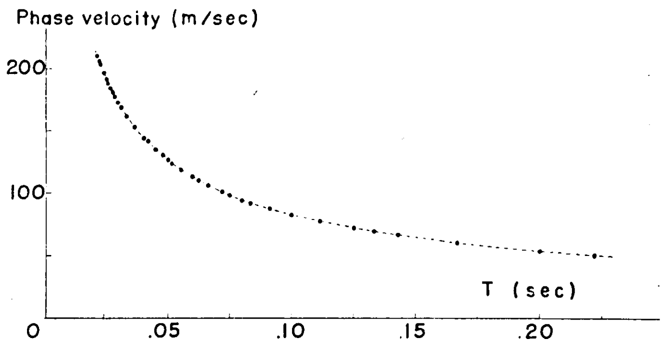

Fig. 1.

1) Y. SATÔ, *Bull. Earthq. Res. Inst.*, **33** (1955), 33.

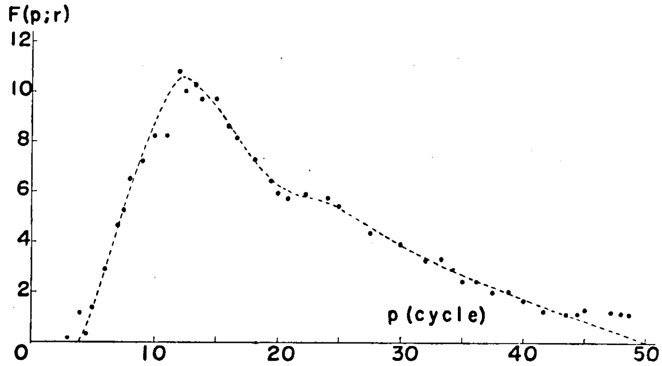

Fig. 2. Spectrum curve $F(p;\ r)$. ($r=141.75$ m). Smooth broken line was used for the calculation.

2. Fundamental formula

In this section we will try to deduce the formula for obtaining the movement at any point $r=r$ using the data of the spectrum at some point $r=r_1$ and the dispersive formula $V=V(p)$.

Assume the movement at a point $r=r_0$, which is very near the origin to be

$$f(t\ ;\ r_0)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f^*(p\ ;\ r_0)\exp{(ipt)}dp\ . \tag{2.1}$$

If the disturbance is propagated to the point $r=r$, the amplitude decreases by the law of the inverse square root and besides the phase

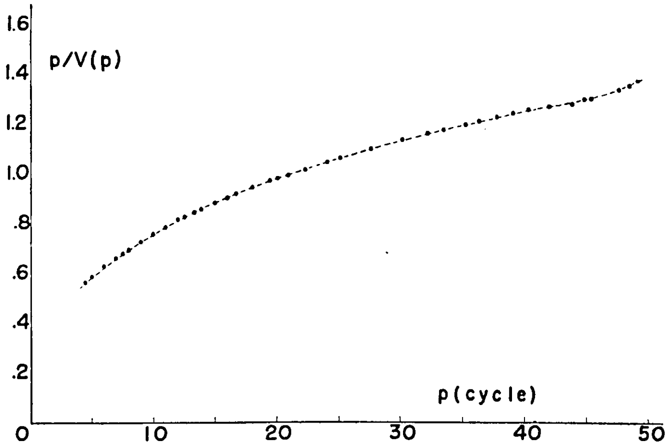

Fig. 3. $p/V(p)$. Smooth broken line was used for the calculation.

shift occurs, therefore

$$f(t\,;\,r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sqrt{\frac{r_0}{r}}f^*(p\,;\,r_0)\exp\left\{ip\left(t-\frac{r-r_0}{V(p)}\right)\right\}dp\,. \qquad (2.2)$$

On the other hand we have the record obtained at the point $r=r_A$. Therefore

$$f(t\,;\,r_A)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sqrt{\frac{r_0}{r_A}}f^*(p\,;\,r_0)\exp\left\{ip\left(t-\frac{r_A-r_0}{V(p)}\right)\right\}dp$$
$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f^*(p\,;\,r_A)\exp(ipt)dp\,, \qquad (2.3)$$

which is a known function with respect to t. We calculated the Fourier transform of this function numerically and obtained

$$f^*(p\,;\,r_A)=\sqrt{\frac{r_0}{r_A}}f^*(p\,;\,r_0)\exp\left(-ip\frac{r_A-r_0}{V(p)}\right)\,. \qquad (2.4)$$

Introducing this expression into the integrand of (2.2), we have

$$f(t\,;\,r)=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{r_A}{r}}\int_{-\infty}^{\infty}f^*(p\,;\,r_A)\exp\left\{ip\left(t-\frac{r-r_A}{V(p)}\right)\right\}dp\,. \qquad (2.5)$$

This is the fundamental formula for the study in this paper. Since $f^*(p\,;\,r_A)$ and $V(p)$ are known by the operation in the previous paper we can calculate $f(t\,;\,r)$, the motion at any point and at any time.

3. Modification for the practical calculation

Although the last formula is theoretically complete, it is desirable for us to have a formula with a more convenient form.

1) $$f^*(p\,;\,r_A)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;\,r_A)\exp(-ip\tau)d\tau$$
$$\equiv F(p\,;\,r_A)\exp\{-i\beta(p)\}\cdot\exp\left\{-i\frac{p(r_A-r_0)}{V(p)}\right\}\,, \qquad (3.1)$$

where $F(p\,;\,r_A)$ is a real function.

While

$$f^*(-p\,;\,r_A)=F(-p\,;\,r_A)\exp\{-i\beta(-p)\}\cdot\exp\left\{i\frac{p(r_A-r_0)}{V(-p)}\right\}$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(\tau\,;\,r_A)\exp(ip\tau)d\tau$$

$$=\text{conjugate complex of } f^*(p\,;\,r_A)$$

$$=F(p\,;\,r_A)\exp\{i\beta(p)\}\cdot\exp\left\{i\frac{p(r_A-r_0)}{V(p)}\right\}\,. \qquad (3.2)$$

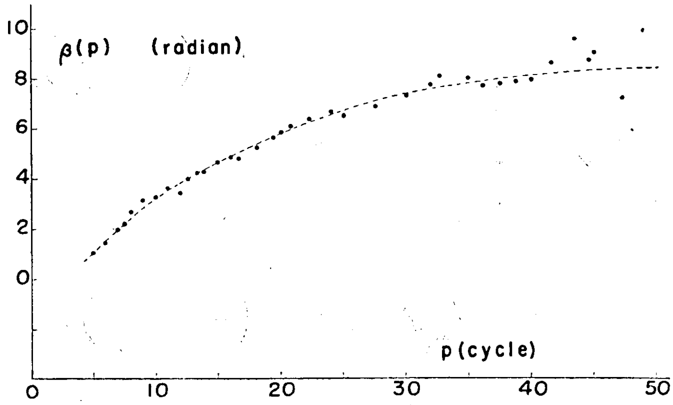

Fig. 4. $\beta(p)$. Smooth broken line was used for the calculation.

Comparing the first and forth line, we have

$$\begin{cases} F(-p\,;\,r_A)=F(p\,;\,r_A)\,, \\ \beta(-p)=-\beta(p)\,, \\ V(-p)=V(p)\,. \end{cases} \qquad (3.3)$$

2) $$f(t\,;\,r)=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{r_A}{r}}\left[\int_{-\infty}^{0}+\int_{0}^{\infty}\right]f^*(p\,;\,r_A)\exp\left\{ip\left(t-\frac{r-r_A}{V(p)}\right)\right\}dp$$

$$=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{r_A}{r}}\int_0^{\infty}\left[f^*(-p\,;\,r_A)\exp\left\{-ip\left(t-\frac{r-r_A}{V(-p)}\right)\right\}\right.$$

$$\left.+f^*(p\,;\,r_A)\exp\left\{ip\left(t-\frac{r-r_A}{V(p)}\right)\right\}\right]dp$$

$$=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{r_A}{r}}\int_0^{\infty}\left[F(p\,;\,r_A)\exp\left\{i\beta(p)+i\frac{p(r-r_0)}{V(p)}-ipt\right\}\right.$$

$$\left.+F(p\,;\,r_A)\exp\left\{-i\beta(p)-i\frac{p(r-r_0)}{V(p)}+ipt\right\}\right]dp$$

$$=\sqrt{\frac{2}{\pi}}\sqrt{\frac{r_A}{r}}\int_0^{\infty}F(p\,;\,r_A)\cos\left\{\beta(p)+\frac{p(r-r_0)}{V(p)}-pt\right\}dp\,. \qquad (3.4)$$

3) If we put $r=r_0$,

$$f(t\ ;\ r_0)\equiv f(t)=\sqrt{\frac{2}{\pi}}\sqrt{\frac{r_A}{r_0}}\int_0^\infty F(p\ ;r_A)\cos\{\beta(p)-pt\}\,dp\,. \qquad (3.5)$$

This shows the movement near the origin.

4) If r_0 may be assumed to be extremely small, then we put $r_0\rightarrow 0$ in the expression (3.4).

$$f(t\ ;\ r)=\sqrt{\frac{2}{\pi}}\sqrt{\frac{r_A}{r}}\int_0^\infty F(p\ ;\ r_A)\cos\left\{\beta(p)+\frac{pr}{V(p)}-pt\right\}dp$$

$$=\sqrt{\frac{2}{\pi}}\sqrt{\frac{r_A}{r}}\left[\int_0^\infty F(p\ ;\ r_A)\cos\left\{\beta(p)+\frac{pr}{V(p)}\right\}\cos pt\,dp\right.$$

$$\left.+\int_0^\infty F(p\ ;\ r_A)\sin\left\{\beta(p)+\frac{pr}{V(p)}\right\}\sin pt\,dp\right]. \qquad (3.6)$$

By this expression we can caluclate the movement at any point $r=r$.

4. Numerical calculation

At first $f(t)$ was calculated by means of the formula (3.5). The result is given in Table II and Fig. 5c. Auxiliary graphs are given in Figs. 2, 3 and 4. Necessary values $F(p;\ r)$, $V(p)$ and $\beta(p)$ are given by these figures and tabulated in Table IV. The data used for this calculation are given in Table I, which is essentially the same as Table II given in the previous paper, with the data increased and the misprints corrected.

Next we calculated the expression (3.6) introducing $r=40$ m. The result is shown in Fig. 5b. and Table III. Table IV also gives important numerical values that appear in (3.6).

5. Discussion

Since the calculation is based on so simple a principle, there are a number of doubts that the obtained results do not show the true movement at that distance.

For example, the record is not integrated to obtain a true motion of the particle of the medium. This is because the characteristic of the amplifier is not clear enough to be used for the integration of the curve.

Next, the mechanism of the generation of the surface waves is not considered at all, only the law of propagation is considered and calculated.

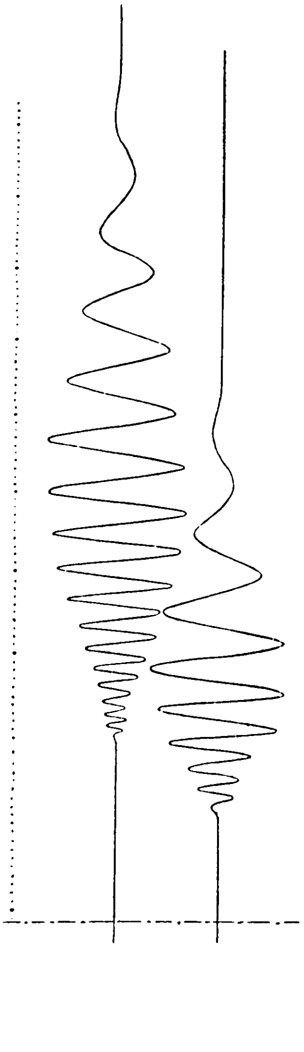

Fig. 5a. Observed curves at $r=141.75$ m (above) and $r=76.2$ m (below). One interval is 0.01 sec.

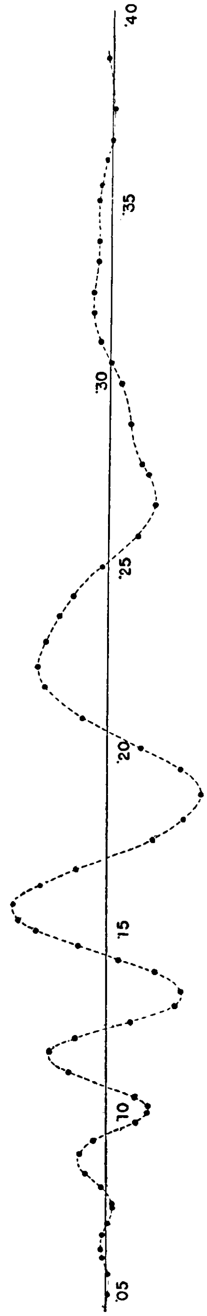

Fig. 5b. Calculated curve. ($r=40$ m)

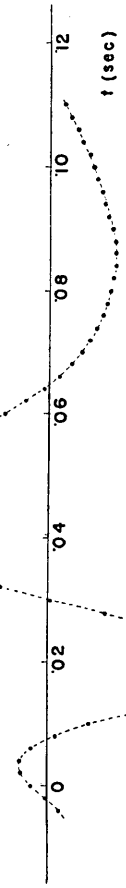

Fig. 5c. Calculated curve. ($r \doteqdot 0$)

Table I.

		A (r=141.75m)				B (r=76.2m)				*
p(cycle)	T(sec)	$-p\cdot c(p;r)$	$p\cdot s(p;r)$	$\|f^*(p;r)\|$	$-\arg f^*(p;r)$	$-p\cdot c(p;r)$	$p\cdot s(p;r)$	$\|f^*(p;r)\|$	$-\arg f^*(p;r)$	
2.5	0.40	−4.80	−1.60	2.00		0.27	−0.82	0.32	−1.89	
3	0.333	0.45	0	0.17	3.14	−1.45	−0.19	0.49	−0.13	3.27
4	0.25	−2.05	4.20	1.18	1.12	2.13	−0.37	0.52	−2.97	4.09
4.5	0.222	0.1	−1.6	0.36	−1.69	−7.0	−9.1	2.5	−0.92	−0.77
5	0.2	6.7	2.4	1.42	2.79	0.64	1.86	0.40	1.90	0.89
6	0.167	15.5	8.2	2.91	2.65	−6.3	−7.8	1.67	−0.89	3.54
7	0.143	2.6	32.6	4.67	1.65	10.9	15.1	2.66	2.20	−0.55
7.5	0.133	12.2	−37.8	5.29	−1.88	19.8	−15.5	3.35	−2.48	0.60
8	0.125	−39.2	34.5	6.52	0.72	−20.6	−23.3	4.14	−0.86	1.58
9	0.111	−55.5	−33.7	7.21	−0.55	24.2	49.0	6.08	2.03	−2.58
10	0.1	48.2	−67.4	8.28	−2.19	9.0	−71.9	7.25	−1.70	−0.49
11	0.0909	41.8	80.4	8.24	2.05	−60.1	53.9	7.34	0.73	1.32
12	0.0833	−124.7	38.3	10.87	0.30	84.4	6.4	7.06	3.07	−2.77
12.5	0.08	62.4	109.1	10.05	2.09	41.4	−75.4	6.88	−1.97	4.06
13.333	0.075	−60.0	−123.8	10.31	−1.12	−86.4	−20.0	6.58	−0.23	−0.89
13.882	0.072	−61.4	120.1	9.71	1.10	−52.2	78.8	6.82	0.99	0.11
15	0.0667	−43.4	−139.6	9.75	−1.27	98.6	−14.0	6.64	−3.00	1.73
16	0.0625	77.1	114.8	8.64	−0.98	−50.4	−89.4	6.42	−1.06	0.08
16.666	0.06	45.3	−128.1	8.17	−1.91	−109.2	10.3	6.59	0.09	−2.00
18.055	0.0554	103.3	82.2	7.31	2.47	93.5	51.2	5.91	2.64	−0.17
19.444	0.0514	−116.6	46.3	6.45	0.38	−43.7	−102.2	5.73	−1.17	1.55
20	0.05	52.6	107.0	5.96	2.03	−108.3	−19.5	5.50	−0.18	2.21
20.833	0.048	14.6	−118.2	5.72	−1.69	−30.1	103.0	5.15	1.29	−2.98
22.222	0.045	65.2	114.2	5.92	2.09	101.0	−30.6	4.75	−2.85	4.94
24	0.0417	−117.4	73.5	5.77	0.56	−97.0	−13.9	4.08	−0.14	0.70
25	0.04	134.0	−23.1	5.44	−2.97	−30.9	97.8	4.10	1.26	−4.24
27.5	0.0364	121.8	−27.4	4.40	−2.92	−10.0	−91.6	3.35	−1.46	−1.46
30	0.0333	118.5	−0.4	3.95	−3.14	35.4	71.3	2.65	2.03	−5.17
32	0.03125	−27.5	101.9	3.30	1.31	7.4	−68.1	2.14	−1.68	2.99
33.333	0.03	34.0	−92.6	3.34	−1.92	−62.7	7.0	1.89	0.11	−2.03
35	0.0286	0.6	86.1	2.46	1.58	24.5	59.8	1.85	1.96	−0.38
36.111	0.02777	68.1	−57.2	2.47	−2.44	64.0	5.7	1.78	3.05	−5.49
37.5	0.0267	−65.1	38.4	2.02	0.53	4.0	−66.0	1.76	−1.63	2.16
38.888	0.0257	78.9	−17.3	2.08	−2.92	−69.4	−6.1	1.79	−0.09	−2.83
40	0.025	−54.1	−41.0	1.70	−0.65	−25.6	64.2	1.73	1.19	−1.84
41.666	0.024	43.6	31.0	1.28	2.52	52.5	−5.9	1.27	−3.03	5.55
42	0.0238					56.0	−15.1	1.38	−2.88	
43.555	0.0230	−23.3	−43.7	1.14	−1.08	−19.9	−53.0	1.30	−1.21	0.13
44.444	0.0225	−0.1	52.5	1.18	1.57	−37.8	−6.5	0.86	−0.17	1.74
45	0.0222	39.9	44.8	1.33	2.30	−28.2	12.5	0.68	0.42	1.88
47.222	0.0212	−56.2	18.5	1.25	0.32	2.6	10.6	0.23	1.81	−1.49
48	0.0208	11.9	54.0	1.15	−1.35	6.5	13.8	0.32	2.01	−3.36
48.611	0.0206	53.4	12.0	1.12	2.92	1.5	12.8	0.27	1.69	1.23

* ARCTAN $\{s(p;r_A)/c(p;r_A)\}$ − ARCTAN $\{s(p;r_B)/c(p;r_B)\}$

Table II. $f(t;\ r_0) \equiv f(t)$

t (sec)	$f(t)$	t (sec)	$f(t)$	t (sec)	$f(t)$	t (sec)	$f(t)$
-.004	-97						
-.002	-17						
0	132	.030	-26	.060	344	.090	-505
.002	210	.032	377	.062	182	.092	-474
.004	216	.034	753	.064	38	.094	-444
.006	130	.036	1046	.066	-85	.096	-420
.008	-65	.038	1253	.068	-187	.098	-382
.010	-335	.040	1381	.070	-266	.100	-344
.012	-705	.042	1453	.072	-333	.102	-319
.014	-996	.044	1431	.074	-385	.104	-263
.016	-1269	.046	1383	.076	-435	.106	-224
.018	-1468	.048	1264	.078	-460	.108	-168
.020	-1516	.050	1172	.080	-491	.110	-120
.022	-1427	.052	1036	.082	-516		
.024	-1200	.054	866	.084	-529		
.026	-863	.056	694	.086	-531		
.028	-466	.058	517	.088	-522		

Table III. $f(t;\ r)$ $(r=40\ m)$

t (sec)	$360t$ (°sec)	$f(t;r)$	t (sec)	$360t$ (°sec)	$f(t;r)$
0.05	18	-24	0.180555	65	-842
0.0555555	20	-45	0.1875	67.5	-1039
0.06	21.6	42	0.194444	70	-812
0.0625	22.5	54			
0.0666666	24	45	0.2	72	-372
0.0694444	25	-14	0.208333	75	272
0.0740740	26.4	-72	0.216666	78	686
0.075	27	-67	0.222222	80	763
0.0796296	28.4	50	0.229166	82.5	679
0.0833333	30	227	0.236111	85	532
0.0888888	32	289	0.241666	87	380
0.0925925	33.333	67			
0.0972222	35	-326	0.25	90	63
			0.258333	93	-328
0.1	36	-452	0.266666	96	-518
0.1018518	36.666	-460	0.275	99	-427
0.1041666	37.5	-309	0.277777	100	-370
0.1111111	40	412	0.288888	104	-235
0.1166666	42	619			
0.1203703	43.333	341	0.3	108	-125
0.125	45	-263	0.305555	110	-15
0.1296296	46.666	-750	0.311111	112	108
0.1333333	48	-820	0.319444	115	185
0.1388888	50	-532	0.325	117	184
0.1416666	51	-130	0.333333	120	138
0.1458333	52.5	311	0.338888	123	131
			0.35	126	139
0.15	54	767	0.354166	127.5	111
0.1527777	55	968	0.361111	130	51
0.1574074	56.666	1034	0.366666	132	-2
0.1625	58.5	727	0.375	135	-32
0.1666666	60	333	0.388888	140	43
0.175	63	-503			

Table IV.

p (cycle)	$F(p;\ r)$ ($r=40$ m)	$p/V(p)$ (1/m)	$\beta(p)$ (°)	$F(p;r_A)\cdot\cos\left\{\beta(p)+\frac{pr_A}{V(p)}\right\}$	$F(p;r_A)\cdot\sin\left\{\beta(p)+\frac{pr_A}{V(p)}\right\}$
5	13	.588	60.2	1.1	− 0.7
6	29	.630	87.1	− 0.5	29.0
7	46	.665	114.0	−43.7	−14.3
8	61	.699	138.7	31.1	−52.5
9	75	.730	162.2	61.1	43.2
10	88	.760	183.9	−51.4	71.5
11	99	.788	202.2	−86.0	−48.9
12	106	.816	219.4	35.5	−99.7
13	104	.841	235.5	103.9	5.3
14	100	.865	251.0	28.6	95.8
15	94	.888	265.8	−73.0	59.1
16	87	.910	279.6	−78.7	−37.0
17	80	.932	293.4	− 1.0	−80.0
18	73	.952	306.5	62.1	−38.3
19	67	.972	319.1	59.8	30.2
20	63	.991	331.6	8.4	62.4
21	60	1.009	342.6	−42.5	42.4
22	58	1.028	353.5	−57.2	− 9.6
23	57	1.047	364.4	−25.0	−51.2
24	56	1.065	374.7	24.1	−51.8
25	54	1.084	384.4	53.0	−10.5
26	51	1.103	393.6	38.2	33.8
27	48	1.120	402.2	0.8	48.0
28	45	1.138	409.9	− 1.5	45.0
29	42	1.153	416.5	−31.1	28.2
30	39	1.168	422.8	−30.0	1.1
31	36	1.183	428.6	− 6.4	−35.4
32	34	1.197	433.7	15.5	−30.3
33	31	1.212	438.3	28.3	−12.6
34	29	1.227	442.3	28.1	7.2
35	27	1.240	446.3	18.0	20.1
36	25	1.253	449.8	3.7	24.7
37	23	1.268	452.6	−11.0	20.2
38	21	1.282	455.5	−18.8	9.3
39	19	1.295	457.8	−18.9	− 1.9
40	18	1.808	460.1	−14.2	−11.0
41	16	1.319	463.0	− 6.5	−14.6
42	14	1.330	465.2	0.8	−14.0
43	12	1.342	467.5	6.6	−10.0
44	10	1.356	469.8	9.2	− 3.8
45	9	1.370	471.5	8.8	1.8
46	7	1.389	472.2	3.9	5.8
47	5	1.410	473.8	− 1.3	4.8
48	4	1.433	474.4	− 3.7	1.5
49	2	1.456	474.4	− 1.7	− 1.0

The obtained curve corresponding to the origin, or $r=r_0\to 0$ is a fairly slow oscillation and is not similar to the motion of the P- and S-phases. The reason is not clear and must be examined in future.

In spite of the above defects, the author believes that the result is an interesting one, and that similar investigation are desired in many fields of the propagation of dispersive waves.

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2. 分散した地震波の解析 II.
震源附近の動きの合成

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1．さきには分散した地震波を解析して spectrum 及び分散公式を求める事を試みた．本論文に於いては，之を逆に合成して，任意の点，特に震源附近での動きを求めようとするものである．

2．既に各 component wave の振幅—即ち spectrum—とその速度がわかつているのであるから，此を適当に加え合せれば任意の地点での動きを表わす式を求められる事は予想にかたくない．この思想を数式をかりて表現したのが §2 である．計算は簡単であり，その結果のみを示せば

$$f(t;\,r)=\frac{1}{\sqrt{2\pi}}\sqrt{\frac{r_A}{r}}\int_{-\infty}^{\infty}f^*(p;\,r_A)\exp\left\{ip\left(t-\frac{r-r_A}{V(p)}\right)\right\}dp \quad \cdots(2.5)$$

ここに r_A における spectrum と $V(p)$ とは既知とする．

3．上式は数値計算に便利な形に変形する事ができる．即ち，震源に極めて近い $r=r_0\to 0$ では，

$$f(t;\,r_0)\equiv f(t)=\sqrt{\frac{2}{\pi}}\sqrt{\frac{r_A}{r_0}}\int_0^{\infty}F(p;\,r_A)\cos\{\beta(p)-pt\}dp \quad \cdots(3.5)$$

任意の r に対しては

$$f(t;\,r)=\sqrt{\frac{2}{\pi}}\sqrt{\frac{r_A}{r}}\left[\int_0^{\infty}F(p;\,r_A)\cos\left\{\beta(p)+\frac{pr}{V(p)}\right\}\cos pt\,dp\right.$$
$$\left.+\int_0^{\infty}F(p;\,r_A)\sin\left\{\beta(p)+\frac{pr}{V(p)}\right\}\sin pt\,dp\right] \quad \cdots(3.6)$$

4．この式によつて求めた曲線を Fig. 5b 及び Fig. 5c に示す．数値は Table II 及び III にある．

5．この結果は極めて簡単な Fourier 変換の原理にもとづくものであり，初めての試みでもあるから，なお議論の余地も多いことと思う．例えば，記象が積分してない為，何を求めているか明らかでない点，波の伝播のみを考えて発震機構に関する考慮のなされていない点等である．

えられた震源附近の動きは実体波と異り，かなりゆつくりしたものである．いかなる理由によるものか，将来の研究にまつ所少なくない．
