3. *Note on a Paper by Satô.**

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1. Satô[1] has been led to consider the roots of the cubic

$$f(\tau)=\beta(\beta-1)\tau^3-\beta(2K+\beta-\alpha)\tau^2+K(K+2\beta)\tau-K^2=0 \qquad (1)$$

where α, β and $K=\alpha\beta-(\gamma-2)^2$ are positive real numbers. He needed the proof that (1) has only one real positive root which is smaller than K/β in this case. The proof for this fact as given by Satô was not a complete one, as he himself remarked in a footnote. As such a proof can be given simply, it is outlined below.

2. From $f(0)<0$ and $f(K/\beta)>0$ we can conclude at once that there exist either one or three roots with the properties mentioned. We shall make the tentative hypothesis that three roots exist between 0 and K/β and show that this leads to a contradiction thus allowing just one root in the interval. Under this hypothesis $\beta>1$. Furthermore each root is smaller than min $(1, \alpha)$. As the last coefficient $K^2/(\beta^2-\beta)$ of the normed equation (1) is equal to the product of the three roots, it follows that

$$K^2/(\beta^2-\beta)<1 \quad \text{and so} \quad K<\beta \qquad (2)$$

As $(2K+\beta-\alpha)/(\beta-1)$ is equal to the sum of the three roots, we have furthermore the inequality

$$(2K+\beta-\alpha)/(\beta-1)<3K/\beta \qquad (3)$$

3. The zeros of $f(\tau)$ are just the common points of $f_1(\tau)$ and $f_2(\tau)$ where

$$f_1(\tau)=\beta(\beta-1)\tau^3-\beta(2K+\beta-\alpha)\tau^2, \quad f_2(\tau)=K^2-K(K+2\beta)\tau\,.$$

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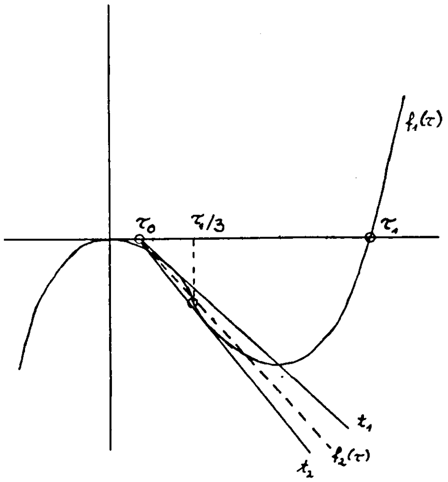
* Communicated by Y. Satô.

** Read by Y. Satô.

1) Y. SATÔ, "Rayleigh Waves Propagated along the Plane Surface of Horizontally Isotropic and Vertically Aeolotropic Elastic Media," *Bull. Earthq. Res. Inst.*, **28** (1950), 23.

$f_1(\tau)$ is a cubic parabola with a relative maximum at (0, 0) and its point of inflection is at the abscissa $\tau=(2K+\beta-\alpha)/(3\beta-3)$.



$f_2(\tau)$ is a straight line with negative slope which goes through $(K/(K+2\beta)>0,\ 0)$ (see the sketch).

4. If there is a straight line with the just mentioned properties of $f_2(\tau)$ which has several common points with $f_1(\tau)$, then besides the abscissa-axis, there can be drawn two tangents t_1 and t_2 to the cubic parabola through $(K/(K+2\beta), 0)$. We now see that this is impossible.

Because one tangent, the abscissa-axis, is known, one can easily derive the condition for the existence of other tangents under the conditions mentioned. Using the abbreviations

$$\tau_0=K/(K+2\beta), \quad \tau_1=(2K+\beta-\alpha)/(\beta-1)$$

the condition for existence of tangents is

$$\tau_1^2+9\tau_0^2 \geq 10\tau_0\tau_1$$

which is equivalent to $\tau_1 \geq 9\tau_0$ as the alternative, $\tau_1 \leq \tau_0$, is impossible. But from $\tau_1 \geq 9\tau_0$ it follows by using (3) that

$$9K/(K+2\beta) < 3K/\beta \quad \text{or} \quad K>\beta$$

which contradicts (2). Thus the proof is completed.

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3. 佐藤の論文に関する注意

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地震研究所彙報 第 28 号 (1950) に載せられた佐藤の論文 "水平等方弾性体を伝はるレーリー波" の中の証明についての注意をのべた. (佐藤泰夫抄)