36. *Model Experiments of Electromagnetic Induction within the Earth.*

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Summary

This is a supplementary note to Rikitake-Yokoyama's paper "The Anomalous Behaviour of Geomagnetic Variations of Short Period in Japan and Its Relation to the Subterranean Structure". It is shown that model experiments are very useful for solving problems of electromagnetic induction within the earth, especially in the cases of complicated distribution of electric conductivity. By means of the model experiments, the problem of anomalous distribution of induced subterranean electric currents in the vicinity of Japan, which was pointed out by Rikitake and Yokoyama, was studied with the result that a possible subterranean distribution of electric conductivity responsible for the said anomalous currents may be something like that shown in Fig. 3.

1. Introduction

It has long been noticed that transient variations in the geomagnetic field on the earth's surface consist of the external part which originates in the outside of the earth, say the ionized upper atmosphere, and the internal part which is induced within the earth by the external part. The latter is due to electromagnetic induction within the electrically conductive earth. We may then be able to find the subterranean distribution of electric conductivity by analyzing the distribution of various geomagnetic variations over the earth's surface. Studies on the electric behaviour of the earth along this line have been developed with fruitful results by S. Chapman[1),2)], A. T. Price[3),4),5),6),7)] and their colleagues, and

1) S. CHAPMAN and A. T. PRICE, *Phil. Trans. Roy. Soc. London, A*, **229** (1930), 427.
2) S. CHAPMAN and T. T. WHITEHEAD, *Trans. Cambr. Phil. Soc.*, **22** (1922), 463.
3), 4) A. T. Price, *Proc. London Math. Soc., Ser.* 2, **31** (1930), 217; **33** (1931), 233.
5) B. N. LAHILI and A. T. PRICE, *Phil. Trans. Roy. Soc. London, A*, **237** (1939), 509.
6) A. T. PRICE, *Quart. Journ. Mech. Appl. Math.*, **2** (1948), 3.
7) A. A. ASHOUR and A. T. PRICE, *Proc. Roy. Soc. A*, **195** (1948), 198.

have recently been carefully revised and extended by T. Rikitake[8),9),10),11)]. The main purpose of their studies is to find the average distribution of electric conductivity along the earth's radius, though they sometimes took into account the effect of the oceans.

Thus, so far as the uppermost part of the earth about 2000 *km* thick is concerned, it seems likely that a fairly reliable knowledge of change in the average electric conductivity with depth has been obtained.

However a question still remains to be solved in connection with the problem of electromagnetic induction within the earth, namely the remarkable heterogeneity of lateral distribution of the internal parts of short period variations in the geomagnetic field. Since the internal part of the shorter period variations is subject to the conductivity of the shallower parts of the earth, the said heterogeneity ought to be mainly due to some heterogeneous distribution of electric properties of the earth's upper part. One of the most striking results of this study is that obtained recently by T. Rikitake and I. Yokoyama[12),13),14),15),16),17),18)]. In their studies, Rikitake and Yokoyama have concluded that the internal electric currents induced within the earth by eastward external electric currents of short duration are eastward in the south-east parts of Japan, though in the other parts of her neighbourhood the corresponding internal currents flow westwards as usually expected from the simple theory of electromagnetic induction.

This result may suggest the presence of a very abnormal distribution of subterranean electric conductivity in the above-mentioned area of Japan. If the reason why such an abnormal phenomenon takes place is made clear from the viewpoint of the electromagnetic induction theory, it would give us an important information as to the physical structures of the earth's upper part.

Mathematical procedures of calculating the induced electric currents in the earth having heterogeneous distribution of electric conductivity are so difficult and complicated, especially for transient phenomena, that only several problems of specially simple cases have been solved hitherto. There should be, however, an alternative method of attacking this problem with the aid of model experiments. This short note gives some

8), 9) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 45; **28** (1950), 219.

10), 11) T. RIKITAKE, *ibid.*, **29** (1951), 263; **29** (1951), 539.

12)-16) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *ibid.*, **30** (1952), 207; **31** (1953), 19; **31** (1953), 89; **31** (1953), 101; **31** (1953), 119.

17) T. RIKITAKE and I. YOKOYAMA, *Bull. Earthq. Res. Inst.*, **33** (1955), 297.

18) T. RIKITAKE and I. YOKOYAMA, *Journ. Geomag. Geoelectr.*, **5** (1953), 59.

results of such experimental researches, which seem to be promising in this field of research.

The Earthquake Research Institute has been interested in electric and magnetic phenomena in the solid earth since some twenty years ago. This is because these phenomena can show us additional, and sometimes very useful, information about the physical conditions of the earth's crust and interior, even in connection with earthquake and volcanic phenomena. One of the authors (T.N.) fortunately had the honour to join the research group of geoelectricity and geomagnetism in this Institute in the earliest stage of the research. This short note is dedicated to this world-wide known research institute of solid geophysics on its 30th anniversary.

2. Model Experiments of Electromagnetic Induction within the Earth

When a phenomenon of electromagnetic induction is sufficiently slowly changing, it is considered to be quasi-stationary, the displacement currents being neglected. In such a case the general equations of electromagnetic induction within a body of σ and μ in electric conductivity and magnetic permeability are given by

$$\mathrm{curl}\,\boldsymbol{E}=-\frac{\partial \boldsymbol{B}}{\partial t}\,, \qquad \boldsymbol{i}=\sigma\boldsymbol{E}\,, \qquad \boldsymbol{B}=\mu\boldsymbol{H}\,, \tag{1}$$

where generally σ is not uniform being expressed by

$$\sigma=\sigma_0 f\,, \tag{2}$$

f being a normalized function of coordinates. Now, the coordinates are taken as the spherical ones $(r,\ \theta,\ \varphi)$, with which the surface of the conductive sphere concerned is expressed by $r=a$. Let us further consider such a normalized spherical coordinate system as expressed by $(r/a,\ \theta,\ \varphi)$, and let the curl with respect to this normalized coordinate system be denoted by curl_0. Then

$$\mathrm{curl}=\frac{1}{a}\,\mathrm{curl}_0 \tag{3}$$

Putting (2) and (3) into (1), we get

$$\mathrm{curl}_0(\boldsymbol{i}/f)=-a\sigma_0\frac{\partial}{\partial t}\boldsymbol{B}\,. \tag{4}$$

It is thus shown that the relation between changes in magnetic induction $\boldsymbol{B}$ and distribution of electric currents induced into the spherical conductor of $\sigma_0 f$ in conductivity is uniquely determined, provided that $a\sigma_0 \frac{\partial}{\partial t}$ is kept constant. In other words, the law of similitude in the case of electromagnetic induction within a spherical conductor is given by

$$a\sigma_0 p = \text{invariant}\,, \qquad (5)$$

where p denotes the operator of time derivative $\frac{\partial}{\partial t}$ as usual.

Thus the electrically conductive earth can be represented by a metallic sphere or spherical shell of a suitable dimension. Changes in magnetic field owing to the presence of this conductive sphere are estimated by measuring the time derivative $\frac{\partial \boldsymbol{H}}{\partial t}$ of the radial component of the field over the spherical surface by means of a search coil 9 *mm* in effective radius and 3×10^{-5} *sec.* in time constant. The change of output electromotive force of the search coil is given as a curve on a cathode ray oscilloscope. Such measurements were carried out at a large number of points over the spherical surface, and their results were compared with those of similar measurements carried out at the same point without the conductive sphere. The difference of the former curves from the latter ones represents the time derivative of the secondary magnetic field caused by the electric currents induced within the sphere. Therefore, integration of the difference curve can give the change with time of the secondary field.

3. Effect of a Semi-Spherical Ocean on the Electromagnetic Induction within the Earth

In all the experiments mentioned below, the external magnetic field is almost uniform in the space occupied by the model earth, and the intensity of uniform magnetic field $\boldsymbol{H}$ is expressed by

$$\boldsymbol{H} = \boldsymbol{H}_0(1 - e^{-t/t_0})\,,$$

where t_0 is actually 10^{-3} *sec.* That is to say, the type of change of the external magnetic field is nearly similar to that of the sudden commencement of magnetic storms.

In order to check the reliability of the model experiment, the electromagnetic induction within a spherical shell of uniform conductivity was first measured and the result was compared with a theoretical estimation[19]. The degree of agreement of the observed changes with the theoretical ones is satisfactory so far as the charges during $t=0\sim4\ m\ sec.$ are concerned. It seems likely that the discrepancy between the experiment and the theory in the later stages ($t>4\ m\ sec.$) is chiefly due to accumulation of the errors in the process of numerical integration.

Now, one of the chief causes of the anomalous phenomenon of electromagnetic induction within the earth in the vicinity of Japan may be the effect of the electrically conductive ocean, since Japan Islands are situated at the north-western margin of the Pacific Ocean. For the purpose of demonstrating the said effect of the ocean, electromagnetic induction within a semi-spherical shell was experimentally studied.

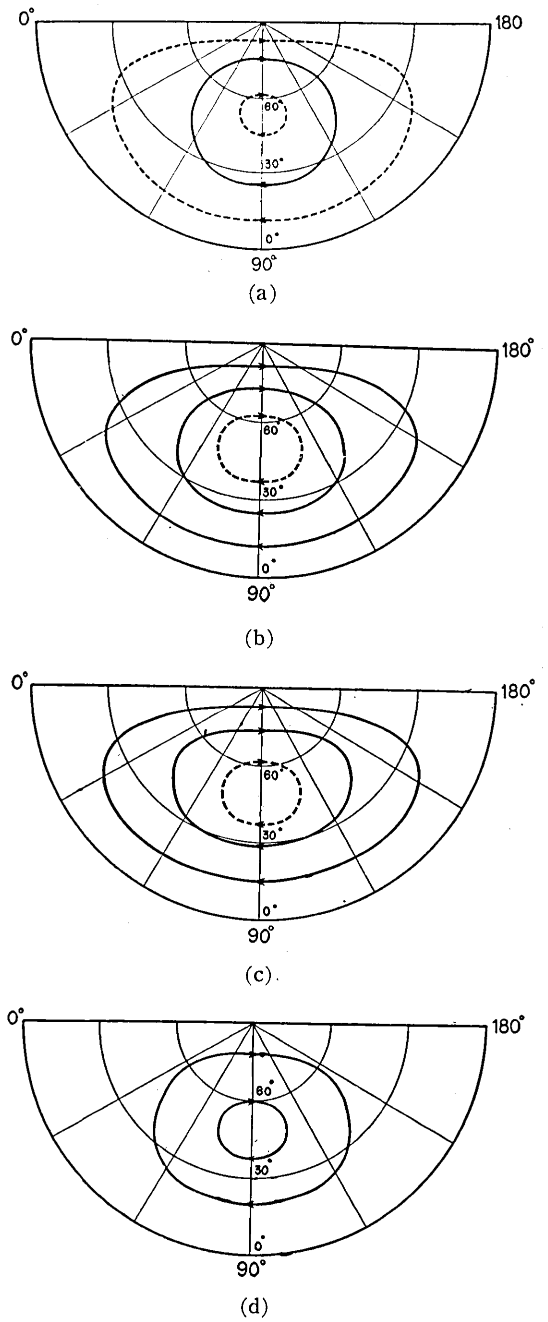

Fig. 1. The distribution of induced electric currents in the northern half part of semi-spherical shell for $t=0.3$, 1.0, 2.0 and 3.0 *m sec.*, viewed from North Pole. Electric currents of about 1 *amp.* flow between successive full stream lines and about 0.5 *amp.* between successive full and broken lines.

19) T. OGUTI and T. NAGATA, *Rep. Ionosphere Res. Japan*, **8** (1954), 171.

The semi-spherical copper shell is of 1.5 *mm* in thickness. With the aid of the law of similitude mentioned before, it was found that this model corresponds to a semi-spherical ocean of 170 *m* in depth for the case of sudden commencement of magnetic storms, and to that of about 4,000 *m* in depth for the case of a geomagnetic bay of 2 hours in duration.

The distribution of induced electric currents in the northern half part of such a shell for $t=0.3$, 1.0, 2.0 and 3.0 *m sec.* is illustrated in Fig. 1. These experimental results are in good agreement with those obtained theoretically by Rikitake and Yokoyama[17]. Our problem here is concerned with the mode of change in the magnetic field outside this spherical shell, because the continental part where we have magnetic stations may be represented by the non-conductive semi-spherical surface and the ocean by the conductive shell.

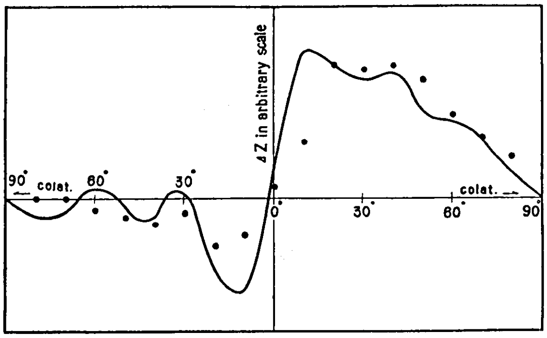

Fig. 2. The distribution of vertical component (ΔZ) of magnetic force for $t=2$ *m sec.* along the 90°–270° meridian line covering a half of the conductive semi-spherical surface and a half of the non-conductive one. The full line shows the result calculated by T. Rikitake and I. Yokoyama and full circles show the observed values.

Fig. 2 shows the distribution of vertical component (ΔZ) of magnetic force at $t=2$ *m sec.* along the 90°–270° meridian line (*cf.* Fig. 1) covering half of the conductive semi-spherical surface and half of the non-conductive one. In this figure, the full circles represent the experimental results while the full line shows the result of theoretical calculation made by Rikitake and Yokoyama[17] through a suitable method of approximation. It seems that the theoretical result is in agreement with the experimental one within the limit of probable errors in the calculation and in the experiment.

As discussed by Rikitake and Yokoyama[17] in their paper, these results may show that the observed anomaly of the subterranean electric currents in the vicinity of Japan can hardly be attributed to any effect of the presence of the Pacific Ocean.

4. Probable Subterranean Structure Responsible for Observed Geomagnetic Anomalous Variation

Under the circumstances mentioned in the preceding section, a possible explanation may be that some particular distribution of high electric conductivity in the earth's crust or the upper part of the earth's mantle is responsible for the observed anomalous distribution of geomagnetic variations. This interpretation may be justified, as pointed out by Rikitake and Yokoyama also, by considering that the region where the anomaly is observed is near the Fuji (Huzi) Volcanic Zone under which magma of high temperature and high conductivity may be distributed in a fairly complex form.

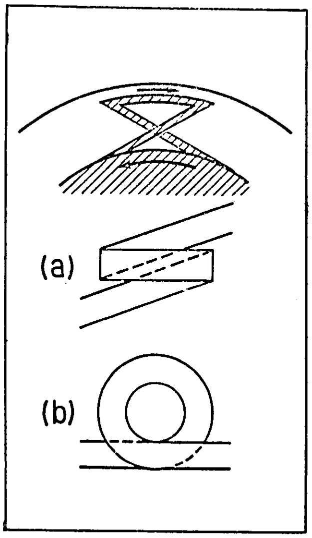

Fig. 3. Proposed subterranean structures responsible for observed geomagnetic anomalous variation.

After several trials along this line of study, we have been led to conclude so far that only a considerable distribution of high electric conductivity, from the viewpoint of electromagnetic induction, will be such as schematically illustrated in Fig. 3, namely, an upheaved zigzag circuit (a) or an upheaved loop (b). In these cases, a part of the general westward currents induced in the major parts of the earth's interior may be forced to pass through this branch circuit, and the electric currents passing through this branch in the opposite direction may have a predominant effect on the observed geomagnetic field because of its shorter distance from the earth's surface. Fig. 4 shows examples of experimental results for $t=0.3$, 1.0, 2.0, 3.0 and 4.0 *m sec.* in the case (a), where the zigzag belt of copper is 1.5 *mm* in thickness. Although in the later stages we can see slight eastward currents around the centre of locality of the upheaved zigzag belt, this tendency of local inversion of induced current is still much smaller than that actually observed in the vicinity of Japan.

It must be taken into consideration, however, that there may be some ambiguity about the absolute intensity of general currents in the case of the analysis of a regional distribution of geomagnetic variations. If so, it may be rather reasonable to compare the observed result with the experimental one in which a suitable amount of general zonal cur-

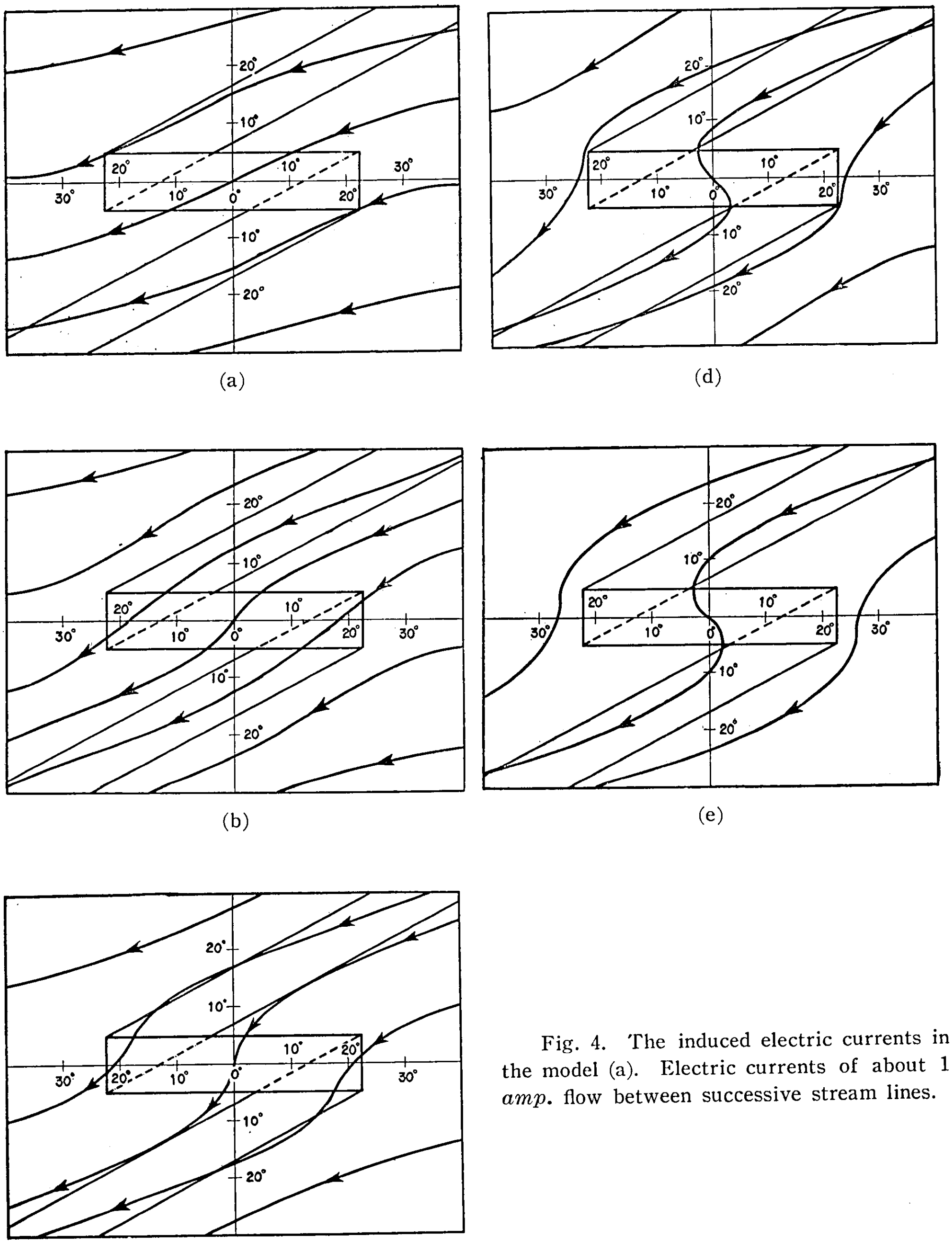

Fig. 4. The induced electric currents in the model (a). Electric currents of about 1 *amp.* flow between successive stream lines.

rent is subtracted or added. Fig. 5. illustrates an example where the intensity of zonal currents is so adjusted that the effect of the zigzag belt becomes remarkable.

Thus, the local inversion of subterranean induced currents can, at any rate, be demonstrated experimentally. But this is only the first step towards the real solution of the observed important problem.

Actual underground structures may probably be much more complicated, even from the viewpoint of electromagnetic induction, and moreover, a reliable information of the phenomenon must be obtained in relation to geological data.

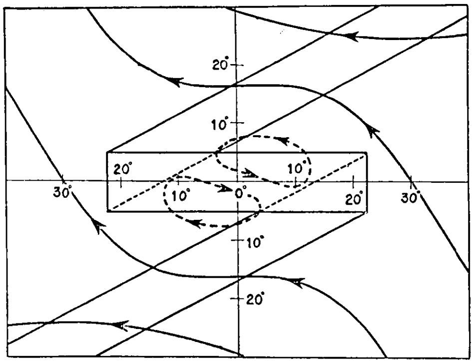

Fig. 5. An example of induced electric current where the intensity of zonal currents is so adjusted that the effect of the zigzag belt becomes remarkable. The representation of the electric current intensity is as same as that in Fig. 1.

Since the model experiments dealt with in this paper seem to be very useful in solving the problems of electromagnetic induction, especially in the cases of complicated distribution of electric conductivity, it is hoped that this kind of study will be extended in future towards the final solution of the preblem.

In conclusion, the authors wish to express their thanks to Dr. T. Rikitake and Mr. I. Yokoyama for their valuable discussion on this subject.

36. 地球内電磁誘導現象の模型実験

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小口 高
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この論文は力武，横山両氏の論文に対する一補遺であつて，複雑な電気伝導度分布を持つ地球内部への電磁誘導現象を考究するに当つて，模型実験が極めて有効である事を示した．この方法を用いて力武，横山が発見した“日本近傍における地磁気短周期変化の異常分布現象”の解釈を試みたが，現在のところ第 3 図に示した様な地下の電気的構造が最も都合がよい．