37. *Growth of the Magnetic Field of the Self-exciting Dynamo in the Earth's Core.*

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Summary

An investigation is attempted in order to see whether or not the earth's dynamo can grow from a state of zero field. The dynamo considered by E. C. Bullard and H. Gellman seems likely to grow, the rate of the growth being also obtained. A period of about 10^4 *years* is required in order to have a field e times of the initial one.

1. Introduction

The idea that the earth's magnetic field is maintained by a self-exciting process in the earth's core has been developed by a number of investigators[1),2),3),4),5)]. Although the theory of the self-exciting dynamo is quite difficult to establish, many difficulties of the theory have been removed, one by one, through the elaborate works of the investigators cited above. The use of powerful calculating machines has also been of great help in promoting the studies. Now we may consider that the theory is approximately applicable to the explanation of the earth's magnetic field though there will be many more problems left for dispute. One of them is the stability of the dynamo which should be highly stable in order that the earth's magnetic field may be maintained over a long period. E. C. Bullard[6)] examined the oscillation of finite amplitude of a self-exciting disc dynamo. He found that there exist oscillations of finite amplitude even though small disturbances are unstable. Making use of

1) W. M. ELSASSER, *Phys. Rev.*, **69** (1946), 106; **70** (1946), 202; **72** (1947), 821. *Rev. Mod. Phys.*, **22** (1950), 1.

2) E. C. BULLARD, *Proc. Roy. Soc. London A*, **197** (1949), 433; **199** (1949), 413.

3) H. TAKEUCHI and Y. SHIMAZU, *Journ. Phys. Earth*, **1** (1952), 1; 56, **2** (1954), 5. *Journ. Geophys. Res.*, **58** (1953), 497.

4) E. C. BULLARD and H. GELLMAN, *Phil. Trans. Roy. Soc. London A*, **247** (1954), 213.

5) E. N. PARKER, *Tech. Rep. Dept. Phys., Univ. Utah* (1954).

6) E. C. BULLARD, *Proc. Cambr. Phil. Soc.* **51** (1955), 744.

the analogies between the disc dynamo and the homogeneous one, he discussed the magnetic oscillations of celestial bodies, the reversals of the earth's magnetic field and other points. T. Rikitake[7] investigated possible oscillations of small amplitude of the earth's dynamo with the result that the small disturbances of the dynamo considered by Bullard and Gellman might be unstable. As was suggested by Bullard, however, there is no reason to believe that the earth's dynamo is utterly unstable, because some finite amplitude oscillations might be possible. S. Lundquist[8] has also studied the stability of magnetic lines of force which are twisted by the magneto-hydrodynamic action. He pointed out that the existence of a strong toroidal field is a little doubtful because instabilities may be caused by the twisting. In spite of all the investigations mentioned above, no definite conclusion concerning the stability problem has been obtained yet.

Another thing to be done is the examination of the growth of the earth's dynamo starting from the zero field. Whether an accidental field given to the system with no field grows or not is not a self-evident problem. The problem would be easier to study than the stability problem of the steady field because we start from the zero field, while the growing field can be treated as a small quantity. The rate of the growth might give some clue to the interval of the occurrence of the normal and reverse magnetization of the earth as has been suggested from the studies of palaeomagnetism of rocks. The writer would here like to investigate the growth of the magnetic field both in homopolar and homogeneous dynamos. Some discussions on the limit of the growth will also be added in the final part of this paper.

2. Growth of the magnetic field of a disc dynamo

As was fully studied by Bullard, the simplest dynamo would be like the one shown in Fig. 1. A conducting disc can rotate about an axle, while a brush is attached on the periphery of the disc. The brush is connected with a ring-shape circuit whose other end touches the axle as shown in the figure. If there is a magnetic field in the direction of the axle, an electromotive force will be induced in the rotating disc, so that we have some potential difference between the axle and the periphery. The electric current driven by the potential difference in the circuit will

7) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **33** (1955), 1.

8) S. LUNDQUIST, *Phys. Rev.*, **83** (1951), 307.

produce a magnetic field at the disc. If the rotation velocity of the disc is sufficiently large, the initial magnetic field will grow indefinitely. At a critical velocity, the magnetic field will remain unchanged while it exponentially decreases in the case of a lower rotation velocity than that of the critical one.

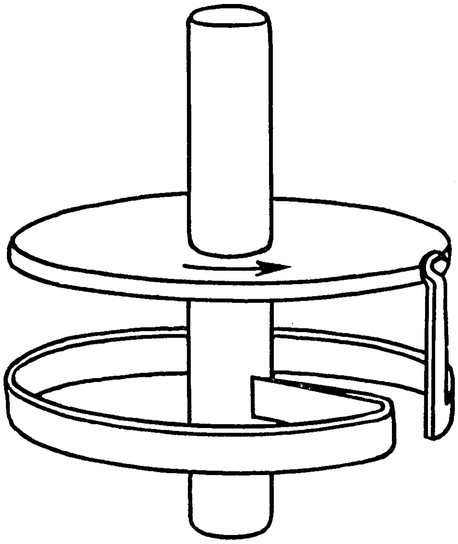

Fig. 1. Schematic view of the disc dynamo after Bullard.

Since this dynamo was investigated in detail by Bullard, the discussion in this section will have nothing new to add. But by going over his discussion, the later discussion is hoped to be better understood.

Bullard has given the differential equations which govern the electric current flowing in the circuit and the motion of the dynamo as follows:

$$\left.\begin{aligned} L\frac{dI}{dt}+RI&=M\omega I\ ,\\ C\frac{d\omega}{dt}&=G-MI^2\ ,\end{aligned}\right\} \qquad (1)$$

where L, R, $M/2\pi$, I, ω, C and G denote respectively the self inductance and the resistance of the circuit, the mutual inductance between the coil and the periphery of the disc, the electric current, the angular velocity, the monent of inertia of the disc, and the couple driving the disc. It is also assumed that there is no friction at the axle.

Suppose the disc is rotating with an angular velocity ω_0 for $t<0$, and a small magnetic field $\vec{h}_0$ perpendicular to the disc is given at $t=0$, the equations which hold for small values of the electric current or magnetic field will be simplified as

$$\left.\begin{aligned} L\frac{di}{dt}+Ri&=M\omega_0 i\ ,\\ C\frac{d\omega}{dt}&=G\ ,\end{aligned}\right\} \qquad (2)$$

in which we ignore the second order quantities writing i in place of I. The first equation of (2) can easily be integrated as

$$i = i_0 e^{\frac{M\omega_0 - R}{L} t} , \tag{3}$$

whence we see that the initial field will exponentially grow or decay respectively for $\omega_0 > R/M$ or $\omega_0 < R/M$. As was shown by Bullard, however, the field does not grow indefinitely but reaches a maximum, the dynamo performing a finite amplitude oscillation as can be seen from the solution of (1) which should be taken for large electric current. Although it is not known whether the supposed dynamo in the earth's core will behave like the disc dynamo or not, the condition of the growth or decay of the magnetic field of the earth's dynamo should be seriously affected by the velocity of the fluid motion in the earth's core.

3. Growth of the magnetic field of the earth's dynamo

In a homogeneous dynamo, whose magnetic field or electric current are regarded as the first-order small quantities, the fundamental equations are given as

$$\left.\begin{aligned} &\vec{i} = \sigma(\vec{e} + \vec{V}_0 \wedge \vec{h}) , \\ &\operatorname{curl} \vec{e} = -\partial \vec{h}/\partial t , \\ &\operatorname{curl} \vec{h} = 4\pi \vec{i} , \\ &\rho d\vec{v}/dt = -\operatorname{grad} p , \end{aligned}\right\} \tag{4}$$

where $\vec{i}$, σ, $\vec{e}$, $\vec{V}_0$, $\vec{h}$, ρ, $\vec{v}$, and p denote respectively the electric current density, the electrical conductivity, the electric field, the steady velocity of the fluid motion, the magnetic field, the density, the disturbing velocity and pressure. It is also assumed that the system has steady motions with the velocity $\vec{V}_0$ for $t < 0$ during which the non-electromagnetic force is balanced by the steady pressure gradient. In this case, the differential equations for the magnetic and electric quantities are independent of the one for the fluid motion.

On eliminating $\vec{i}$ and $\vec{e}$ from (4) and measuring times in units of $4\pi\sigma a^2$ and velocities in units of $1/4\pi\sigma a$, where a denotes the radius of the earth's core, we obtain

$$D\vec{h} - \nabla^2 \vec{h} = V \operatorname{curl} (\vec{v}_0 \wedge \vec{h}) , \tag{5}$$

in which

$$\vec{V}_0 = \frac{V\vec{v}_0}{4\pi\sigma a} \tag{6}$$

and D is written in place of $\partial/\partial t$.

Now we are in a position to solve (5) under certain conditions at the boundary. As has been done in the theory of the earth's dynamo and its stability problem, the only tractable way for solving (5) would be to consider suitable combinations of spherical harmonic constituents of the magnetic field. Since the magnetic field is non-divergent, $\vec{h}$ can be expressed as

$$\vec{h} = \sum_{n,m} \vec{h}_{s,n}^m + \sum_{n,m} \vec{h}_{t,n}^m \tag{7}$$

where $\vec{h}_{s,n}^m$ is of the poloidal type, the r, θ and ϕ components of it being written as

$$\vec{h}_{s,n}^m = \begin{cases} \dfrac{n(n+1)}{r} s_n^m(r) Y_n^m , \\ \dfrac{1}{r} \dfrac{ds_n^m}{dr} \dfrac{\partial Y_n^m}{\partial \theta} , \\ \dfrac{1}{r \sin\theta} \dfrac{ds_n^m}{dr} \dfrac{\partial Y_n^m}{\partial \phi} , \end{cases} \tag{8}$$

and $\vec{h}_{t,n}^m$ is of the toroidal type which is written as

$$\vec{h}_{t,n}^m = \begin{cases} 0 , \\ \dfrac{t_n^m(r)}{r \sin\theta} \dfrac{\partial Y_n^m}{\partial \phi} , \\ -\dfrac{t_n^m(r)}{r} \dfrac{\partial Y_n^m}{\partial \theta} , \end{cases} \tag{9}$$

in which

$$Y_n^m = P_n^m(\cos\theta) \begin{matrix}\cos \\ \sin\end{matrix} m\phi . \tag{10}$$

As has been shown by the present writer[9] in the theory of the magneto-hydrodynamic oscillations of a spherical fluid body, it is almost impossible to take into account all the constituents, so that we shall take a combination of the constituents which seem to be important as has been done by Takeuchi, Shimazu, Bullard and Gellman in the eigen-value problem of the self-exciting dynamo. The simplest one is the so-called A-approximation in which only the S_1^0, T_2^0, T_2^{2c} and T_2^{2s} type fields are

9) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **33** (1955), 175.

taken into account, all other constituents being ignored. As for the fluid motion, only the S_2^{2c} and T_1^0 velocity fields are considered. They are to be expressed also in a form of (8) or (9), their radial functions being respectively denoted by Q_S and Q_T following Bullard and Gellmans' expression. In that case, it is easy to obtain the following four equations[10] from (5):

$$\left.\begin{aligned}
&-Dr^2s_1^0+r^2\frac{d^2s_1^0}{dr^2}-2s_1^0=-\frac{216}{5}VQ_St_2^{2s}\ ,\\
&-Dr^2t_2^0+r^2\frac{d^2t_2^0}{dr^2}-6t_2^0\\
&\qquad=-\frac{2}{3}V\left(\frac{dQ_T}{dr}-\frac{2Q_T}{r}\right)s_1^0-\frac{72}{7}V\left\{Q_S\frac{dt_2^{2c}}{dr}+2\left(\frac{dQ_S}{dr}-\frac{Q_S}{r}\right)t_2^{2c}\right\}\ ,\\
&-Dr^2t_2^{2c}+r^2\frac{d^2t_2^{2c}}{dr^2}-6t_2^{2c}\\
&\qquad=-\frac{6}{7}V\left\{Q_S\frac{dt_2^0}{dr}+2\left(\frac{dQ_S}{dr}-\frac{Q_S}{r}\right)t_2^0\right\}+2VQ_Tt_2^{2s}\ ,\\
&-Dr^2t_2^{2s}+r^2\frac{d^2t_2^{2s}}{dr^2}-6t_2^{2s}\\
&\qquad=-2VQ_Tt_2^{2c}-\frac{2}{3}V\left\{3Q_S\frac{d^2s_1^0}{dr^2}+\left(\frac{dQ_S}{dr}-\frac{6Q_S}{r}\right)\frac{ds_1^0}{dr}\right.\\
&\qquad\qquad\left.+\left(\frac{d^2Q_S}{dr^2}-\frac{2}{r}\frac{dQ_S}{dr}\right)s_1^0\right\}\ .
\end{aligned}\right\}\qquad(11)$$

In order to solve these equations, we have to specify the radial functions Q_S and Q_T, of which considerable varieties may be expected. One of their usual forms which have been considered in the theory of the self-exciting dynamo is

$$Q_S=r^3(1-r)^2\ ,\qquad Q_T=\epsilon r^3\ ,\qquad(12)$$

where ϵ is a constant which determines the ratio of the T_1^0 and S_2^{2c} velocities. If (12) is adopted, (11) becomes

$$\begin{aligned}
&-Dr^2s_1^0+r^2\frac{d^2s_1^0}{dr^2}-2s_1^0=-\frac{216}{5}V(r^5-2r^4+r^3)t_2^{2s}\ ,\\
&-Dr^2t_2^0+r^2\frac{d^2t_2^0}{dr^2}-6t_2^0\\
&\qquad=-\frac{2}{3}V\epsilon r^2s_1^0-\frac{72}{7}V\left\{(r^5-2r^4+r^3)\frac{dt_2^{2c}}{dr}+(8r^4-12r^3+4r^2)t_2^{2c}\right\},
\end{aligned}$$

10) See Bullard and Gellmans' paper. Expressions (25) of p. 225 and (29) of p. 236.

$$
\left.
\begin{aligned}
&-Dr^2t_2^{2c}+r^2\frac{d^2t_2^{2c}}{dr^2}-6t_2^{2c}\\
&\qquad=-\frac{6}{7}V\left\{(r^5-2r^4+r^3)\frac{dt_2^0}{dr}+(8r^4-12r^3+4r^2)t_2^0\right\}+2V\epsilon\, r^3t_2^{2s},\\
&-Dr^2t_2^{2s}+r^2\frac{d^2t_2^{2s}}{dr^2}-6t_2^{2s}\\
&\qquad=-2V\epsilon\, r^3t_2^{2c}-\frac{2}{3}V\left\{(3r^5-6r^4+3r^3)\frac{d^2s_1^0}{dr^2}\right.\\
&\qquad\qquad\left.+(-r^4+4r^3-3r^2)\frac{ds_1^0}{dr^2}+(10r^3-20r^2+3r)s_1^0\right\}.
\end{aligned}
\right\}\quad(13)
$$

A considerable variety of methods will be available for the solution of the simultaneous equations. In order to examine whether an accidental field given to the system at $t=0$ grows or not, we may choose a crude way of examination making use of power series of r for s_1^0, t_2^0, t_2^{2c} and t_2^{2s}. Let us assume the following expressions:

$$
\left.
\begin{aligned}
s_1^0&=\sum_n a_n r^{n+2},\\
t_2^0&=\sum_n b_n r^{n+1},\\
t_2^{2c}&=\sum_n c_n r^{n+1},\\
t_2^{2s}&=\sum_n d_n r^{n+1},
\end{aligned}
\right\}\quad(14)
$$

all of which remain finite at $r=0$. Introducing (14) into (13), we obtain

$$
\left.
\begin{aligned}
&-D\sum a_nr^{n+4}+\sum n(n+3)a_nr^{n+2}\\
&\qquad=-\frac{216}{5}V\sum d_nr^{n+6}+\frac{432}{5}V\sum d_nr^{n+5}-\frac{216}{5}V\sum d_nr^{n+4},\\
&-D\sum b_nr^{n+3}+\sum(n^2+n-6)b_nr^{n+1}\\
&\qquad=-\frac{2}{3}V\epsilon\sum a_nr^{n+4}-\frac{72}{7}V\sum(n+9)c_nr^{n+5}\\
&\qquad\quad+\frac{144}{7}V\sum(n+7)c_nr^{n+4}-\frac{72}{7}V\sum(n+5)c_nr^{n+3},\\
&-D\sum c_nr^{n+3}+\sum(n^2+n-6)c_nr^{n+1}\\
&\qquad=-\frac{6}{7}V\sum(n+9)b_nr^{n+5}+\frac{12}{7}V\sum(n+7)b_nr^{n+4}\\
&\qquad\quad-\frac{6}{7}V\sum(n+5)b_nr^{n+3}+2V\epsilon\sum d_nr^{n+4},
\end{aligned}
\right\}\quad(15)
$$

$$
\left.\begin{aligned}
&-D\sum d_n r^{n+3}+\sum(n^2+n-6)d_n r^{n+1}\\
&\quad=-2V\epsilon\sum c_n r^{n+4}-\frac{2}{3}V\sum(3n^2+8n+14)a_n r^{n+5}\\
&\qquad+\frac{4}{3}V\sum(3n^2+7n+12)a_n r^{n+4}-2V\sum(n+1)^2 a_n r^{n+3}\ .
\end{aligned}\right\}
$$

If we ignore a_n, b_n, c_n and d_n for $n>5$, the following relations can be obtained by equating the coefficients of the corresponding terms of the left- and right-hand sides of the above equations:

$$
\left.\begin{aligned}
&a_0=a_1=a_2=a_3=b_0=b_1=b_2=b_3=c_0=c_1=c_3=d_0=d_1=d_3=0\ ,\\
&\quad 40a_5=\frac{432}{5}Vd_2\ ,\\
&\quad -Da_4=-\frac{216}{5}Vd_2-\frac{216}{5}Vd_4\ ,\\
&\quad -Db_2+14b_4=-\frac{504}{7}Vc_2\ ,\\
&\quad 24b_5=\frac{1296}{7}Vc_2\\
&\quad -Dc_2+14c_4=-6Vb_2\ ,\\
&\quad 14c_5=\frac{108}{7}Vb_2+2V\epsilon d_2\ ,\\
&\quad -Dd_2+14d_4=0\ ,\\
&\quad 14d_5=-2V\epsilon c_2\ .
\end{aligned}\right\}\qquad(16)
$$

The boundary conditions that the magnetic field is continuous at the surface of the core give the relations such as

$$
\sum(n+3)a_n=\sum b_n=\sum c_n=\sum d_n=0\ ,\qquad(17)
$$

from which we have

$$
\left.\begin{aligned}
&7a_4+8a_5=0\ ,\\
&b_2+b_4+b_5=0\ ,\\
&c_2+c_4+c_5=0\ ,\\
&d_2+d_4+d_5=0\ ,
\end{aligned}\right\}\qquad(18)
$$

in which a_n, b_n, c_n and d_n for $n>5$ are neglected as before.

In order to have non-vanishing solutions for (16) and (18), the following relation should hold good:

$$\begin{vmatrix}
0 & 0 & 0 & -\frac{432}{5}V & -40 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & D & -\frac{504}{7}V & 0 & 0 & -14 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1296}{7}V & 0 & 0 & 0 & 0 & 0 & -24 & 0 & 0 \\
0 & -6V & D & 0 & 0 & 0 & -14 & 0 & 0 & 0 & 0 \\
0 & \frac{108}{7}V & 0 & 2V\epsilon & 0 & 0 & 0 & 0 & 0 & -14 & 0 \\
0 & 0 & 0 & D & 0 & 0 & 0 & -14 & 0 & 0 & 0 \\
0 & 0 & -2V\epsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 \\
7 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{vmatrix} = 0 . \tag{19}$$

(19) can be written as

$$D^3+14D^2+(588-1728V^2+4V^2\epsilon^2)D-2744-7056V^2-54V^2\epsilon^2=0 . \tag{20}$$

If V and ϵ are larger than 10, the equation has a root which is approximately given by

$$D=\frac{2744-7056V^2-54V^2\epsilon^2}{588-1728V^2+4V^2\epsilon^2} .$$

Bullard and Gellman have shown that a steady state of the self-exciting dynamo is possible, for example, for the combination of $\epsilon=100$ and $V=22.06$. Corresponding to these values of ϵ and V, the above root becomes 14.3, the other roots of (20) being complex with negative real parts. It is obvious that the complex roots whose real parts are negative specify simple harmonic oscillations superposed by exponential decays of the magnetic field. Hence the magnetic field does not grow in these cases. On the other hand, we see that the magnetic field grows exponentially when the root is positive. In that case, we obtain

$$\vec{h}=\vec{h}_0 e^{\frac{14.3t}{4\pi\sigma a^2}} \tag{21}$$

If we take $\sigma=3\times10^{-6}$ *emu* and $a=3.5\times10^8$ *cm*, the period during which the magnetic field increases e times of its initial value is calculated to be 10^4 *years* from (21). It is of importance that the magnetic field of the dynamo, the fluid motion in which is studied by Bullard and Gellman, can grow starting from the zero field provided an accidental field is given. Therefore it might be assumed that the earth's magnetic field was generated either from the interstellar magnetic field which is supposed to amount to $10^{-6} \sim 10^{-5}$ *gauss* or from an unknown accidental field given to the earth when the fluid motion in the core was arranged favourably. If the initial field were of opposite direction to the present one, the field grown from it would be the reversed one. Suppose the earth's dynamo is destroyed by some causes and the state of no magnetic field is apparently attained, the new growth of the field might take place in the above-described way, provided there are suitable fluid motions in the core. Since the probability of a large magnetic field being applied to the earth would be quite small, the period for getting a new field of appreciable strength would exceed 10^4 *years*. The above theory can not be applicable for cases in which electromagnetic forces are comparable with the mechanical forces because we assumed that the electric and magnetic quantities are the first-order small ones. A theory of finite magneto-hydrodynamic oscillations in the earth's core will be advanced later.

4. The limit of the growing field

Although it is obvious that we can have no exact understanding about the growth and other time-dependent behaviours of the earth's dynamo until we solve the fundamental equations of electromagnetism and fluid motion, which are intrinsically non-linear, a number of attempts have been made to determine the limit of the amplification of the magnetic field as well as the period needed to attain to that state from analogical and physical points of view.

G. K. Batchelor[11] studied the behaviour of the magnetic field associated with turbulent motions of a conducting liquid. Making use of the results concerning the statistical behaviour of homogeneous turbulence, he suggested that a magnetic field can grow from an initial field arriving finally at a certain steady state. The rate of the growth, the time needed for the complete growth and the magnetic energy at the steady state were approximately obtained. According to Bullard, however, the equality between the kinetic and magnetic energies at the steady state as is supposed by Batchelor for turbulent motions does not always hold

11) G. K. BATCHELOR, *Proc. Roy. Soc. London A*, **201** (1950), 405.

good for large-scale motions such as considered here, so that Batchelor's way of discussion is hardly applicable for the present case.

Elsasser[1)] has pointed out that the mechanical forces produced by the magnetic field in the earth's core will become significant when they become comparable in magnitude with the purely mechanical forces which control the fluid motion. If the magneto-mechanical forces are supposed to be much larger, the motion that would be caused by the forces would modify the fluid motion in a way so as to decrease the magnetic field. He supposed that the magnetic forces can be amplified up to the same order of magnitude with the Coriolis force that seems most important for relatively slow motions in the core. If this equating is taken for granted, the limit of the amplification is obtained to be about 12 *gauss* by Elsasser, while Bullard suggested some larger values.

The writer is of opinion, however, that the steady value of the earth's magnetic field would be determined from the energetics in the earth's core. The energetics has been discussed by Elsasser, Bullard and others from time to time. The mean magnetic energy in the earth's core is estimated at 2.3×10^3 *ergs/cm*^{3} according to Bullard's recent study. The figure might become smaller if we take the view that the toroidal magnetic field in the core is not so large as is estimated by Bullard. The rate of dissipation of magnetic energy is therefore roughly 5×10^{-9} *ergs/cm*^{3} *sec.* when we take into consideration the fact that the free decay period of the core's field amounts to 1.4×10^4 *years*. To maintain the magnetic field, the energy of this order of magnitude should be always supplied from the fluid motion or ultimately from the thermal source. It is known that the dissipation due to fluidal friction is much smaller than the figure cited above. On the other hand, investigations of meteorites suggested that thermal energy of 0.3×10^{20} *ergs/sec.* would be generated in the whole core, whence we have 1.7×10^{-7} *ergs/cm*^{3} *sec.* as the value of generation of thermal energy. The efficiency of this heat engine then becomes about 3%. The limit of the efficiency of the thermodynamical process in the core is given by the adiabatic difference between the top and bottom of the convection currents after the second law of thermodynamics, the ideal efficiency being supposed to be of the order of 10%. The convert of the thermal energy into the magnetic energy is therefore possible.

However, the above discussions have nothing to do with the determination of the magnitude of the magnetic field in the core. On the contrary, the magnitude of the toroidal magnetic field obtained by making the Coriolis force equal to the electromagnetic force has been adopted in estimating the magnetic energy in the earth's core. The matter would become clearer if the logical order is inverted, that is if we point out that the magnitude of the magnetic field produced by the earth's dynamo would be determined from the amount of thermal energy transferable to magnetic energy. This is equivalent to determining the efficiency of the thermodynamical engine. In order to do this, however, the detail of thermal behaviour of the dynamo should be known. The introduction of thermal energy into the system of the dynamo, which has been described by the fundamental equations of fluid motion and electromagnetism, would be quite a task, and no satisfactory results can be expected within a short time.

5. Concluding remarks

As a result of a rough examination, it is shown that the dynamo in the earth's core considered by Bullard and Gellman can grow from the state of zero field. The rate of the growth is also obtained. The time needed for getting at a field e times of the initial one is of the order of 10^4 *years*. No satisfactory result concerning the limit of the growing field is obtained from a survey of related problems.

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37. 地磁気ダイナモに於ける磁場の生長

地震研究所　力　武　常　次

Bullard-Gellman および竹内一島津等によつて求められて来た地球核内のダイナモの零磁場状態に微小磁場を与えた時，その微小磁場が生長するか否かを検討した．ある定常ダイナモを形成し得ると考えられる流体運動に対しては，微小磁場の生長が確められ，初期値の e 倍に生長するのに10^4年かかることがわかつた．つまりこのような流体運動は磁場を増幅する作用を有することになる．
