Letters

Selective Retrieval of Memory and Concept Sequences through Neuro-Windows

Hideki Kakeya and Yoichi Okabe

***Abstract*—This letter presents a crosscorrelational associative memory model which realizes selective retrieval of pattern sequences. When hierarchically correlated sequences are memorized, sequences of the correlational centers can be defined as the concept sequences. The authors propose a modified neuro-window method which enables selective retrieval of memory sequences and concept sequences. It is also shown that the proposed model realizes capacity expansion of the memory which stores random sequences.**

I. Introduction

In the autocorrelational associative memory [2], [3], [8], [10], when hierarchically correlated patterns are memorized, the centers of correlation can be defined as the concept patterns in each hierarchical level. Kakeya and Kindo have shown that the memory patterns and the concept patterns can be retrieved selectively in the network composed of the neuro-window elements, the controllable window-type neural elements [4].

When correlated sequences are memorized in the crosscorrelational associative memory, the sequences of the correlational centers can be defined as the concept sequences. Though the existing neuro-window method realizes selective autoassociation of memory patterns and concept patterns, its simple adoption to the sequential associative memory does not work. In the present letter the authors take up the case where cyclic sequences of patterns with the same period are memorized, and present a modified neuro-window method which realizes selective association of memory and concept sequences. The authors also apply the proposed algorithm to capacity expansion of sequential memory.

In Section II, the neuro-window method for the autocorrelational associative memory is reviewed. In Section III, sequential associative memory based on the crosscorrelational learning is formulated, and the neuro-window method is modified and applied to the sequential memory to achieve selective retrieval of memory sequences and concept sequences. In Section IV, the proposed models are investigated through the eigenspace analysis of the memory matrix, and its application to capacity expansion is discussed.

II. Neuro-Windows in Auto-Associative Memory

In the autocorrelational associative memory, the nonmonotonic neural dynamics (the partial reverse method) given by

$$x_i(t+1) = \mathrm{sgn}\left(\sum_{j=1}^{N} w_{ij}\tilde{x}_j(t)\right) \tag{1}$$

$$\tilde{x}_i(t) = x_i(t) - \lambda\phi(u_i(t)) \tag{2}$$

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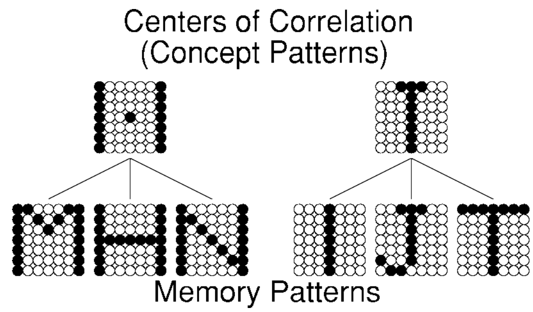


Fig. 1. Example of memory patterns and concept patterns in hierarchical autoassociative memory.

$$u_i(t) = \sum_{j=1}^{N} w_{ij}x_j(t) \tag{3}$$

$$\phi(u) = \begin{cases} -1, & \text{if } u < -h \\ 0, & \text{if } -h \le u < h \\ 1, & \text{if } h \le u \end{cases} \tag{4}$$

enlarge the capacity ($0 < \lambda \le 1, 0 < h$) [9], where $x_i(t)$ is the state of the ith neuron at time t, w_{ij} is the autocorrelational memory matrix, and sgn(u) is the sign function given by

$$\mathrm{sgn}(u) = \begin{cases} 1, & \text{if } u \ge 0 \\ -1, & \text{if } u < 0. \end{cases} \tag{5}$$

In these dynamics the output of neurons which have large absolute values of membrane potential $|u_i(t)| > h$ is weakened (or canceled when $\lambda = 1$).

When hierarchically correlated patterns are memorized, the centers of correlation can be defined as the concept patterns which represent the correlated groups of patterns (Fig. 1) [1]. Kakeya and Kindo have shown that selective association of memory patterns and concept patterns is enabled by controlling the parameter h in (4), for concept patterns have larger $|u_i|$ than memory patterns. Memory patterns can be stabilized exclusively when h is selected so that $|u_i| < h$ holds for memory and $|u_i| > h$ holds for concept [4]. If larger h is selected, memory patterns become unstable and concept patterns are retrieved as a result. This method for selective association is called the neuro-window method, because each neural element works as a window which transmits only the output of neurons with $|u_i| < h$. The size of the window h determines whether memory or concept is retrieved in the recalling process.

III. Neuro-Windows in Sequential Associative Memory

In this section we formulate crosscorrelational associative memory which stores sequences of correlated patterns, and propose a modified neuro-window method which realizes selective association of memory and concept sequences.

The neuro-windows can deal with the patterns which have the correlational structure with multiple layers of hierarchy generally. For simplicity, however, here we consider cyclic sequences of correlated patterns with two layers of hierarchy

$$\boldsymbol{s}^{(p_1,p_2)(0)} \to \boldsymbol{s}^{(p_1,p_2)(1)} \to \cdots \boldsymbol{s}^{(p_1,p_2)(Q-1)} \to \boldsymbol{s}^{(p_1,p_2)(0)}$$

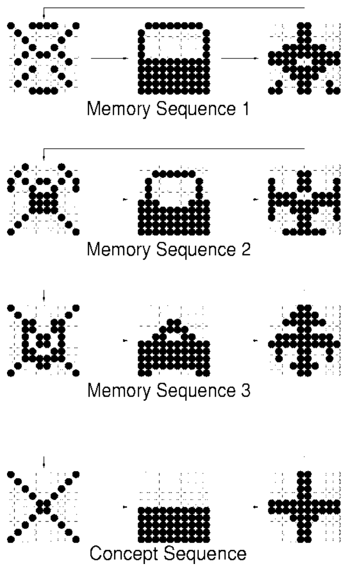


Fig. 2. Example of memory sequences and concept sequence. When correlated sequences of patterns are memorized, a concept sequence of patterns can be defined. The goal here is to retrieve memory sequences and concept sequences selectively.

where

$$\frac{1}{N} E[\boldsymbol{s}^{(p_1, p_2)(\tau)} \cdot \boldsymbol{s}^{(q_1, q_2)(\sigma)}] = \Delta_R(p_1, p_2, q_1, q_2)\delta_{\tau\sigma} \tag{6}$$

$$\Delta_R(p_1, p_2, q_1, q_2) = \begin{cases} 1, & \text{if } p_1 = q_1, \, p_2 = q_2 \\ R, & \text{if } p_1 = q_1, \, p_2 \neq q_2 \\ 0, & \text{if } p_1 \neq q_1. \end{cases} \tag{7}$$

Here $1 \leq p_1 \leq P_1$, $1 \leq p_2 \leq P_2$, and $\delta_{\tau\sigma}$ is the Kronecker's delta function. This means that the patterns which share the same suffix τ have the ultrametric correlation [11]. As in the autoassociative memory, the center of correlation whose components are given by

$$s_i^{(p_1)(\tau)} = \operatorname{sgn}\left(\sum_{p_2=1}^{P_2} s_i^{(p_1, p_2)(\tau)}\right) \tag{8}$$

can be defined as the concept sequence. An example of memory and concept sequences is illustrated in Fig. 2.

The crosscorrelational memory matrix which stores the above sequences is written in the form

$$w_{ij} = \frac{1}{N} \sum_{p_1=1}^{P_1} \sum_{p_2=1}^{P_2} \sum_{\tau=0}^{Q-1} s_i^{(p_1, p_2)((\tau+1) \bmod Q)} s_j^{(p_1, p_2)(\tau)}. \tag{9}$$

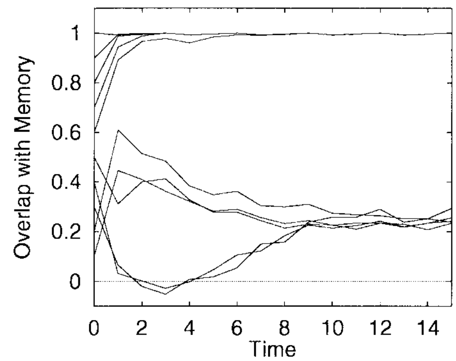
Unless the correlation R is small and negligible, this weight matrix enables only the retrieval of concept sequences

$$\boldsymbol{s}^{(p_1)(0)} \rightarrow \boldsymbol{s}^{(p_1)(1)} \rightarrow \cdots \boldsymbol{s}^{(p_1)(Q-1)} \rightarrow \boldsymbol{s}^{(p_1)(0)}$$

and cannot attain the retrieval of memory sequences under the standard dynamics given by

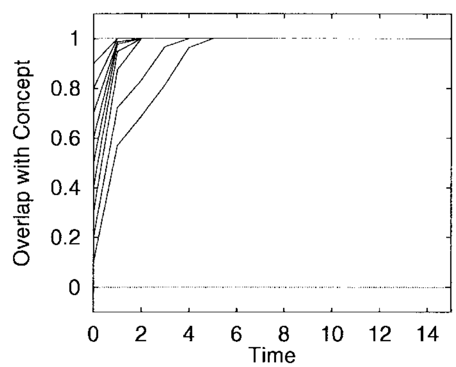
$$x_i(t+1) = \operatorname{sgn}\left(\sum_{j=1}^{N} w_{ij} x_j(t)\right). \tag{10}$$

As in the autocorrelational associative memory, it is expected that the neuro-window method realizes selective association of memory and concept in the sequential memory. The dynamics given by (1)–(4), however, cannot attain selective association of memory and concept when it is applied to crosscorrelational associative memory in the original form.



$h = 2.0$

(a)



$h = 10$

(b)

Fig. 3. Recalling process ($Q = 3$, $P_1 = 5$, $P_2 = 3$ $R = 0.49$, $N = 1000$): (a) time course of overlap with memory orbit from various initial conditions ($h = 2.0$, $\lambda = 0.5$) and (b) time course of overlap with concept orbit from various initial conditions ($h = 10$, $\lambda = 0.5$). Small h attains retrieval of memory sequences, while large h attains retrieval of concept sequences. Both memory orbit and concept orbit have basins of attraction. The basin of the concept orbit is wider than that of the memory orbit.

To realize selective association of memory and concept sequences which cycle with period Q, here we propose modified nonmonotonic dynamics written in the form

$$x_i(t+1) = \operatorname{sgn}\left(\sum_{j=1}^{N} w_{ij} \tilde{x}_j(t)\right) \tag{11}$$

$$\tilde{x}_i(t) = x_i(t) - \lambda\phi(\tilde{u}_i(t)) \tag{12}$$

$$\tilde{u}_i(t) = \sum_{j=1}^{N} v_{ij} x_j(t) \tag{13}$$

$$[v_{ij}] = V = W^Q \tag{14}$$

where the function $\phi(u)$ is the same as that shown in (4). In these dynamics it is possible to retrieve memory sequences and concept sequences by adjusting the parameter h to the proper value in the above dynamics.

Fig. 3 shows the results of the numerical experiments where the above dynamics are applied to selective association of sequential memory. Here the overlap with the orbit of memory (or concept) sequences $m(t)$ given by

$$m(t) = \frac{1}{N} \boldsymbol{s}^{(\tau)} \cdot \boldsymbol{x}(t) \tag{15}$$

is plotted ($\tau = t \bmod Q$). As shown in the figure, small h realizes retrieval of a memory sequence, while large h realizes retrieval of a concept sequence. It is also shown that both the memory orbit and the concept orbit have basins of attraction. Especially, the basin of the concept orbit is very large.

IV. Discussion

A. Geometry of Recalling Process

In this subsection we analyze the models proposed above from the geometrical viewpoint based on the eigenspace analysis of the weight matrix.

It is known that in the dynamics given by (10) linear transformation is the major driving force of state transitions, while nonlinear transformation terminates the state transitions where the linear flow is slow. Linear transformation drives the state in the direction of the eigenvectors of the weight matrix with large absolute eigenvalues. Therefore eigenspace analysis of the weight matrix is effective to grasp the dynamic property of the network [6], [7]. One of the important points given by the eigenspace analysis is that concept patterns in the hierarchical memory and spurious memory patterns in the random memory are composed mainly of the eigenvectors with large eigenvalues, which constitute stable space. Cancellation of the effect of neurons which have large membrane potential $|u_i|$ by the nonmonotonic dynamics makes the space with large absolute eigenvalues unstable, which leads to the retrieval of memory patterns [5].

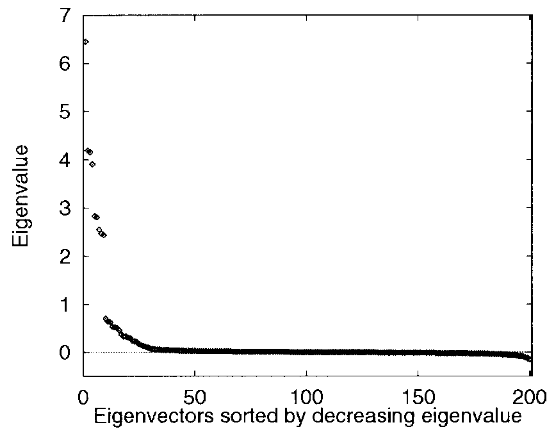
In the crosscorrelational associative memory, however, eigenspace analysis is not so simple as in the autocorrelational associative memory, because the memory matrix is not symmetric and includes complex eigenvalues. Complex eigenvalues cause rotation of state vectors in the recalling process, which enables emergence of limit cycles in the network.

The nonmonotonic dynamics given by (1)–(4) can cancel the effects of neurons with large membrane potential only when the signs of $u_i(t)$ and $x_i(t)$ are the same [notice (2) and (4)]. This relation holds in most cases in the autoassociative memory. Rotation of state vectors in the cross-associative memory, however, often leads to the situation where the signs of $u_i(t)$ and $x_i(t)$ are different, which results in enhancement of the space with large eigenvalues on the contrary. This is why the nonmonotonic dynamics given by (1)–(4) fail to give the same effect in the cross-associative memory.

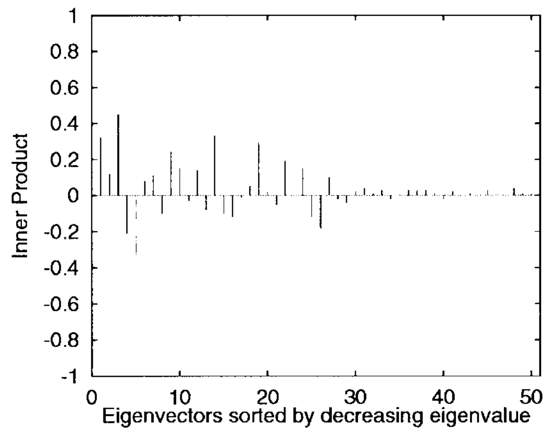
Though the crosscorrelational memory matrix W is asymmetric and has complex eigenvalues, $V = W^Q$ becomes almost symmetric, since the cross-associative memory with period Q works as an auto-associative memory system if it is observed after Q steps of dynamics, which include Q times linear transformation by W. Therefore the signs of $\tilde{u}_i(t) = \sum_j v_{ij} x_j(t)$ and $x_j(t)$ are the same in most cases. This is why the nonmonotonic dynamics (11)–(14) enable selective association of memory and concept sequences as in the auto-associative memory.

Since W^Q is almost symmetric, it is expected that eigenspace analysis of the symmetric part of the matrix W^Q, $[W^Q]_s$, helps understand the geometry of the recalling process. From the knowledge of the autocorrelational associative memory, it is expected that the state proceeds toward the eigenvectors of $[W^Q]_s$ with large eigenvalues.

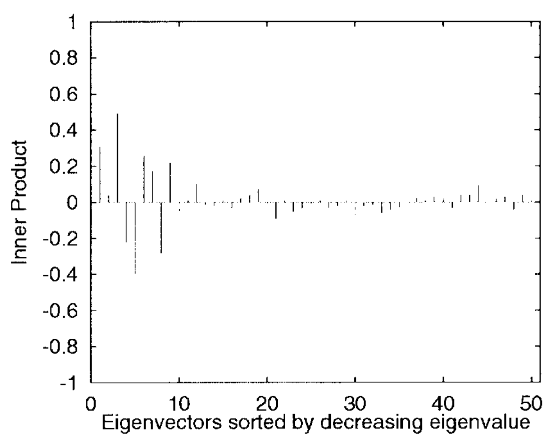
The eigenvalue distribution of $[W^Q]_s$ and the inner products of the eigenvectors and the pattern vectors in the memory and concept sequences are shown in Fig. 4. This figure shows that the patterns in the memory sequence are composed of all the eigenvectors in the memory space, which span the memory pattern vectors, sporadically, while the patterns in the concept sequence are composed mainly of the eigenvectors with large eigenvalues. Therefore concept sequences are stable in the standard neural dynamics. On the other hand, the modified neuro-window method proposed above can terminate the flow toward the eigenvectors with large eigenvalues in a favorable place by setting the parameter h to the proper value, which leads to



(a)

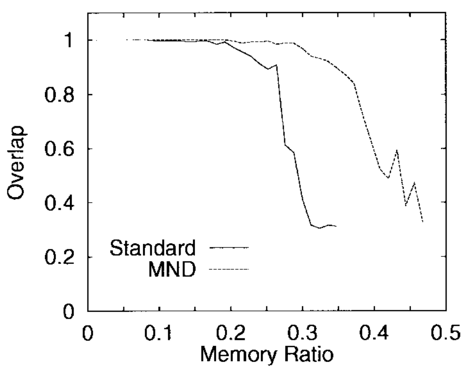


(b)



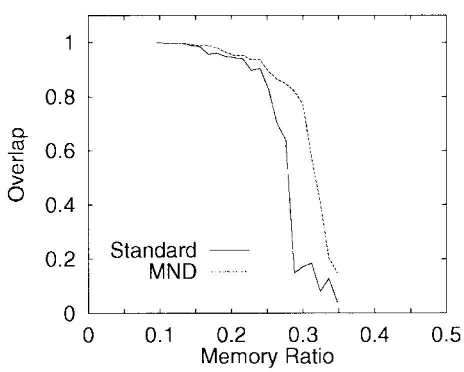
(c)

Fig. 4. Eigenspace structure of memory matrix ($N = 200$, $Q = 3$, $P_1 = 3$, $P_2 = 3$): (a) eigenvalues of the matrix $[W^Q]_s$; (b) inner products of the eigenvectors and a pattern vector in a memory sequence; and (c) inner products of the eigenvectors and a pattern vector in a concept sequence. As for the inner products, only the eigenvectors with the largest 50 eigenvalues are shown. Memory patterns are composed mainly of the eigenvectors with the largest $P_1 P_2 Q$ (27 in this case) eigenvalues, while concept patterns are composed mainly of the eigenvectors with the largest $P_1 Q$ (nine in this case) eigenvalues.



$Q = 3$

(a)



$Q = 6$

(b)

Fig. 5. Overlap with memory sequence after 30 steps of state transitions under different memory rations $a = (PQ/N)$. In the modified nonmonotonic dynamics (MND), the parameters $h = 2.0$, $\lambda = 0.5$ are used when $Q = 3$, and the parameters $h = 3.5$, $\lambda = 0.5$ are used when $Q = 6$. The memory capacity increases when the modified nonmonotonic dynamics are used. The increase of the capacity is greater when the length of the sequence Q is shorter.

the stabilization of memory sequences. Thus selective association of memory and concept sequences is attained.

B. Capacity Expansion

In the model which stores random patterns, the patterns in a spurious memory sequence are composed mainly of the eigenvectors of $[W^Q]_s$ with large eigenvalues. Therefore the flow toward spurious memory sequences is terminated and the memory sequences are stabilized and retrieved by the modified neuro-window method using the matrix W^Q even when the memory ratio is rather large.

Fig. 5 shows the memory capacity given by the standard dynamics and the proposed nonmonotonic dynamics. Here the stability of memory sequences is examined in the cases where $Q = 3$ and $Q = 6$. The results show that the proposed model realizes larger capacity in both cases, while the enhancement of capacity by the nonmonotonic model is weaker when $Q = 6$. As Q increases further, the capacity enhancement by the nonmonotonic model gradually disappears.

V. Conclusion

In the present letter the nonmonotonic neural dynamics which realize selective association of memory sequences and concept sequences have been presented by modifying the neuro-window method. It has been shown that capacity of crosscorrelational associative memory which stores sequences of random patterns can be expanded based on the same dynamics. The basic mechanism of the proposed model has been clarified by the eigenspace analysis of the memory matrix.

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