21. *An Explanation for the Ground Vibration caused by Periodically Exerted Force.*

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Figs. 1 and 2 illustrate the relation between the distance from the vibration origin and the amplitude of the actual ground vibration generated by a vibrator[1]. As seen in these figures, in Fig. 1 the amplitude decreases in accordance with the distance from the origin but in Fig. 2 it does not decrease in such a simple way.

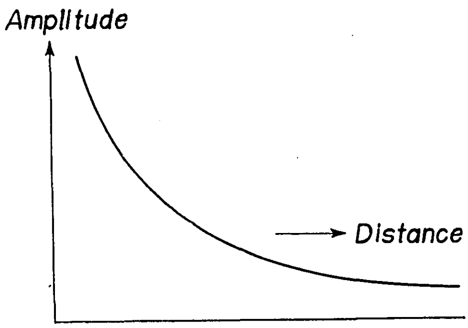

Fig. 1

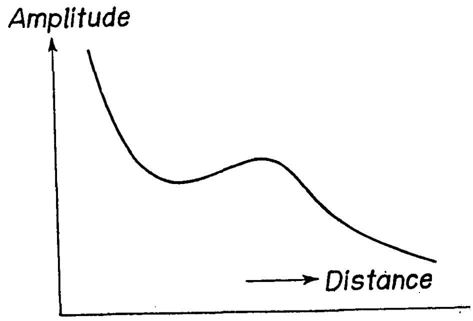

Fig. 2

The relation of the distance to the amplitude which is shown by a sinuate curve is considered to be a superposition of two waves of different velocity. For instance, it has been considered that if there exists a surface layer, the relation of the distance from the origin to the amplitude is an superposition of the direct waves and the refracted waves which run under the surface layer[2].

But when the relation curve scarcely waves, as in Fig. 1, or when it has waves of finite number, the explanation mentioned above does not clarify the phenomena.

1) K. KANAI and T. SUZUKI, *Monthly Meeting of the Earthq. Res. Inst.*, (June 1952), K. KANAI, T. TANAKA, T. SUZUKI and K. NAKAGAWA, *ditto*, (May 1953), K. KANAI and N. NASU, *ditto*, (Nov. 1954).

2) T. WATANABE, *Journ. Seism. Soc., Japan*, [ii], **3** (1951), 31, (in Japanese).

Now, let us assume the amplitude at the vibration origin to be a, the phase difference ϕ, the constant concerning the attenuation of amplitude in case of wave-propagation α, the constant of damping due to solid viscosity k, the propagation velocity v, $p=2\pi/T$, the period T, the thickness of the surface layer H, the time t, the distance from the origin x, and the index of reflection $R(x)$. Then the amplitude of direct wave A_D and the amplitude of reflected wave A_R can be expressed as follows:

$$\left.\begin{aligned} A_D &= ax^{-\alpha}e^{-kx}\sin\left\{\phi+p\left(t-\frac{x}{v}\right)\right\}, \\ A_R &= aR(x)(4H^2+x^2)^{-\alpha}\,e^{-k(4H^2+x^2)^{1/2}}\cdot\sin\{\phi+p(t-\sqrt{4H^2+x^2}/v)\}. \end{aligned}\right\} \quad (1)$$

And the condition necessary for the amplitude to become the maximum or the minimum at a certain point will be

$$\frac{d}{dx}(A_D+A_R)=0, \quad (2)$$

from which we get

$$\left\{\alpha\frac{d\alpha}{dx}+\beta\frac{d\beta}{dx}+\cos(X-Y)\frac{d(\alpha\beta)}{dx}\right\}+\alpha\beta\frac{d\cos(X-Y)}{dx}=0. \quad (3)$$

where

$$\left.\begin{aligned} \alpha &= \frac{e^{-kx}}{x}, & \beta &= R(x)\frac{e^{-k\sqrt{4H^2+x^2}}}{\sqrt{4H^2+x^2}}, \\ X &= \frac{px}{v}, & Y &= \frac{p\sqrt{4H^2+x^2}}{v}. \end{aligned}\right\} \quad (4)$$

Here, in order to find the outline of the problem, the term put in { } of Equation (3) is omitted by considering the derivatives of the terms of damping and the index of reflection to be negligible. Therefore, roughly speaking, the maximum and the minimum of the amplitude-distance curve will appear under the following condition:

$$\alpha\beta\frac{d\cos(X-Y)}{dx}=0. \quad (5)$$

From Equation (5) the condition finally becomes

$$\frac{p}{v}(\sqrt{4H^2+x^2}-x)=n\pi. \quad [n=1,\ 2,\ 3,\cdots] \quad (6)$$

Equation (6) can be rewritten as follows:

$$x=\frac{(4H)^2-(nvT)^2}{4nvT}. \qquad (7)$$

As a practical question, x must be greater than 0. Therefore,

$$4H>nvT. \qquad (8)$$

Equation (8) may be the necessary condition for the amplitude-distance curve to be wavy. In other words, if $4H$ is smaller than vt, the curve will not be sinuate. The larger the value of $4H$ is than vt, the greater the value of n can be, that is, the number of wave becomes greater.

From Equation (8) it can be said qualitatively that the number of waves of the amplitude-distance curve becomes larger as H (the thickness of surface layer) gets thicker and the wave length gets shorter (the velocity of propagation v is smaller or the period T is shorter).

Equation (8) explains qualitatively the following experimental results: At the up-town of Tokyo, where the ground is firm, non-wavy curves of amplitude-distance are observed, while at the down-town where the ground is rather soft, there are many spots where we obtained sinuate curves[3].

Next, we can rewrite Equation (6) as follows:

$$v=\frac{2\{\sqrt{x^2+4H^2}-x\}}{nT} \qquad (9)$$

from which we can obtain the propagation velocity v if the values of H, x and T are known. If, the thickness of the surface layer H are found by means of the ordinary seismic prospecting, and the distance x which corresponds to the maximum and the minimum of the amplitude-distance curve are also found from the ground vibration caused by the periodically exerted force, the propagation velocity v will be obtained, since the period of exerted force T is already known.

When an amplitude-distance curve has more than two waves, the unknown values of H and v can be obtained at the same time. As a practical question, there still exists the effect of the terms neglected in the course of the derivation of Equation (5) from Equation (3). Therefore there are some cases where v or H obtained from Equation (9) may include a considerable error. Since the error can be

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2) *loc. cit.*, 1).

estimated from the amplitude-distance curve, by taking Equation (3) into consideration, the error may be lessened.

Conducting one or two experiments on the actual ground and applying the method stated above, the plausible value of the velocity of S-waves was obtained. As to some examples of its application, we will report in the next paper.

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21. 周期的加力による地盤振動に対する1つの解釈

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地表面で周期的加力を与えたときの，地表面の振巾は，震動源からの距離とともに単純には減少しないで，波を打つて減少することは，古くから気付かれていた事柄である．

その理由としては，普通，速度のちがう2つの波の合成が考えられて来た．2つの波としては，地表面に沿つて伝わる直接波と表面層と下層との境界面を伝わる屈折波を考える場合，或は表面波と実体波を考える場合などがあつた．

ところが，ここ数年来，各種地盤上に建つ建物の振動実験を行うにあたつて，建物の振動にともなう地盤の振動を測定した結果，振巾と距離の間にはいろいろ変つた関係が見られ，その関係は地盤の性質に左右されることがわかつた．すなわち，一般に東京の山手のような地盤では Fig. 1 のようになり，下町のようなところでは Fig. 2 のような形になる傾向があることがわかつたわけである．

このような現象は，従来の考え方だけで解釈することは容易でない．

それで，地表面に沿つて伝わる直接波と表面層と下層との境界面で反射してくる波との合成を考えてみると，この現象が，定性的には一応説明できることになつた．

そして，もし，これらの現象が，直接波と反射波との合成で起るものならば，地震波地下探査法で表面層の厚さを求めた後，周期的加力を与える方法で表面層内の S 波の速度を求めることが可能なことになる．又，やり方によつては，周期的加力の方法だけで，表面層の厚さと S 波の速度を求めることも必ずしも不可能ではない．