23. *The Anomalous Behaviour of Geomagnetic Variations of Short Period in Japan and Its Relation to the Subterranean Structure. The 6th report. (The results of further observations and some considerations concerning the influences of the sea on geomagnetic variations.)*

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Summary

New results of the recent observations are described in regard to the anomalous distribution of short-period geomagnetic variations in Japan. The possible influence of the sea on the variations is also studied from the standpoint of the electromagnetic induction theory. It is made clear that the observed results can not be explained by the effect of the electric currents induced in the sea. An interpretation concerning a possible cause of the phenomenon is also attempted.

Introduction

The anomalous behaviour of geomagnetic variations of short period in Japan has been investigated by the writers. As has been published in a series of papers in this bulletin[1),2),3),4),5)] and elsewhere[6),7)], the characteristic features of the anomaly were examined in detail by all available means. As a result of these investigations, it becomes clear that the anomalously large amplitudes of the vertical component of

1) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **30** (1952), 207.
2) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **31** (1953), 19.
3) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **31** (1953), 89.
4) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **31** (1953), 101.
5) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **31** (1953), 119.
6) T. RIKITAKE and I. YOKOYAMA, *Journ. Geomagn. Geoelectr.* **5** (1953), 59.
7) T. RIKITAKE and I. YOKOYAMA, *Naturwissenschaften*, **41** (1954), 420.

geomagnetic variations as observed in central Japan on occasions of bays and storms have nothing to do with things outside the earth but must be explained by some agencies inside the earth. So there must be a complicated distribution of the induced electric currents at the time of geomagnetic variations. On the basis of this fact, we are now inclined to imagine a peculiar distribution of the electrical conductivity underneath Japan.

It is also of interest that an anomalous phenomenon of the same character concerning short-period geomagnetic variations has been recently discovered in Germany[8),9)]. It seems likely that the anomaly is caused by a concentrated distribution of the induced electric currents underneath the northern part of Germany, though the distribution of the electric currents seems somewhat simpler than that in Japan.

Although we still know very little about the cause of the phenomenon, there was no knowledge of this kind in the past. Consequently, an intensive study in this line may throw some light on the study of the earth's crust or deeper interior. It might be possible to obtain some knowledge on the physical property of the substances at a depth of about 100 *km* and its relation to the geological structure, seismic and volcanic activities, provided we can carry out sufficient observations for detailed analyses.

In the light of the importance of the phenomenon, it is quite desirable to make further observations for the three geomagnetic components at as many places as possible, especially observations on the islands in the southern sea of Japan being of special interest in order to determine the southern boundary of the area in which we observe the large amplitude of the vertical component. The writers set up a temporary station in Ooshima Island, about 100 *km* south of Tokyo, as the first step. As will be stated in this paper, the behaviours of short-period variations there do not differ much from those observed at Aburatsubo and Kakioka, the observations at the latter two observatories being already reported in the previous papers.

In the cases of bay disturbances in Japan, which are mostly used in the previous analyses, the magnetic vector in the horizontal plane is directed towards the north. This is due to the fact that bays occur during nights when the magnetic vectors are necessarily directed towards approximately the north as has been made clear by the statistical

8) U. FLEISCHER, *Naturwissenschaften*, **41** (1954), 114.
9) H. WIESE, *Zeits. f. Meteorologie*, **8** (1954), 77.

studies of geomagnetic bays[10),11)]. Accordingly, the results discussed in the writers' previous papers are based on geomagnetic variations which change nearly in one direction. If there is a special distribution of the electrical conductivity in the earth, the magnetic field caused by the induced electric currents might show an apparent anisotropy, the analyses of which would be useful for the present study. Some examinations are carried out with regard to the observations at a few observatories from this point of view though there are not many favourable cases because of the above-mentioned characters of bays. The writers have got an empirical law between the vertical force and the north and east forces, the relation thus made clear being sometimes useful in inferring the distribution of the induced currents.

One might suspect that the electric currents induced in sea water play an important role in the interpretation of the anomaly because of the high conductivity of sea water. The writers have been of the opinion that the anomalous distribution of geomagnetic variations in Japan are not to be explained by the influence of the sea judging from the duration-time or period of the variations and the mode of distribution. However, it will be useful to study the possible influence of the sea so as to keep at the same question from cropping up all the time. So the writers would here like to describe the possible influence of the sea on the basis of the theory of the electromagnetic induction in a sheet. In the study, the effect of the self-induction, which is important for rapid variations, is fully taken into account.

In the final part of this paper, a speculation about the cause of this anomalous phenomenon is added. Although we have to accomplish many more observations and analyses in order to establish a physical picture of the origin which is situated underneath the earth's crust, it would be of some use to construct a possible model of the electrical state for future studies. But it seems somewhat puzzling to accept the model thus constructed from both the geological and geophysical stand-points because the model is not simple.

Chapter I. Results of the new observation

1. The observation on Ooshima Island

In order to bring out more clearly the anomalous characteristics

10) H. HATAKEYAMA, *Geophys. Mag.*, **12** (1938), 16.

11) N. FUKUSHIMA, *Journ. Faculty Sci. Tokyo Univ.* Sec. II, **8** (1953), 291.

of geomagnetic variations in the central part of Japan, a temporary station for a continuous observation of three geomagnetic components has been set up in Ooshima Island (34°44′ N, 139°22′ E) in July and August, 1954, while the routine observations at Aburatsubo (35°09′N, 139°37′ E) and Maze (37°44′ N, 138°48′ E) were going on as usual. It is not easy to keep the observation of this kind in good condition over a long period on Ooshima Island, because we have no commercial electric supply there except for a few hours in the evening.

The magnetic variometers were installed in a cave where the changes in geomagnetic declination in connection with the activities of Volcano Mihara have been continuously recorded since 1951. The sensibility of the D (declination)-variometer is $0.58'/mm$ with a $3\,m$ optical lever, a magnet being suspended with a quartz fibre of $10\,\mu$. The H (horizontal intensity)-variometer has a suspending quartz fibre of $60\,\mu$ with a sensibility of usually $1.5\gamma/mm$. The Z (vertical intensity)-variometer is the one of the Watson type with a sensibility of $0.7\gamma/mm$ as a rule. At this station, absolute measurements of the three components have often been carried out with a G.S.I. type magnetometer. The values at the epoch of 1954.0 are given in the foot-note[12]. In the magnetograms obtained here, we can see that the strong parallelism between the short period changes in the horizontal (ΔH) and vertical (ΔZ) components is very conspicuous.

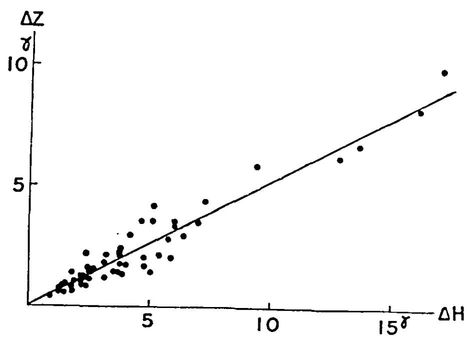

Fig. 1. The relation between ΔZ and ΔH for short period variations obtained by the observation on Ooshima Island.

The relation between ΔH and ΔZ for short period variations are shown in Fig. 1. The mean value of $\Delta Z/\Delta H$ is estimated at 0.52. Judging from the value of $\Delta Z/\Delta H$ and the above-mentioned parallelism between ΔZ and ΔH, we may presume that Ooshima Island belongs to the same district as Kakioka and Aburatsubo where the anomalous behaviours of geomagnetic variations have been observed.

With the aid of the value of $\Delta Z/\Delta H$ newly obtained by the con-

12)

D	H	I
6°11′.9W	30045γ	47°18′·7

tinuous observations on Ooshima Island and Shimosato (33°34′ N, 135°55′ E) of Hydrographic Office, Maritime Safety Agency, the preliminary chart showing the statistical distribution of $\Delta Z/\Delta H$ for short-period variations in Japan is modified as shown in Fig. 2.

In Fig. 3 the relation between the ratio $\Delta Z/\Delta H$ and the duration-time of geomagnetic variations is shown. The shorter the period is, the more dispersive the value of $\Delta Z/\Delta H$ becomes (the variations whose duration-time is shorter than 5 *min.* are omitted in the figure). This phenomenon is caused by small and irregular fluctuations, especially in Z-component, which seem to be always observed except for a few hours during midnight. It is probable that the disturbances are caused by the stray electric currents from electric railways in Tokyo district.

2. Apparent anisotropy of geomagnetic variations in the central part of Japan

The accumulation of the observations enables

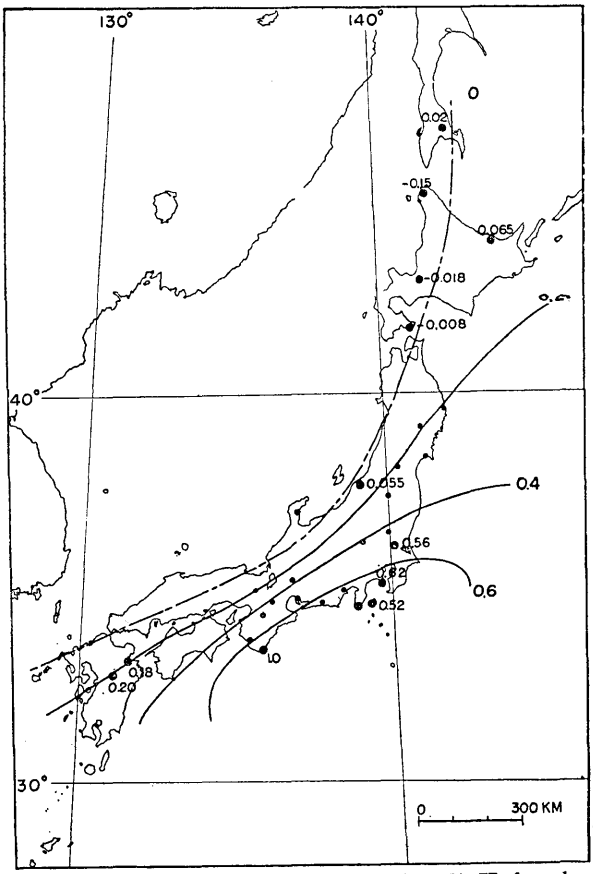

Fig. 2. The revised distribution of $\Delta Z/\Delta H$ for short period variations in Japan.

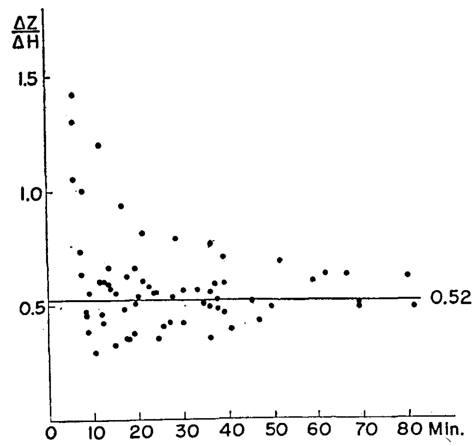

Fig. 3. The relation between $\Delta Z/\Delta H$ and the period of geomagnetic variations at Ooshima Island.

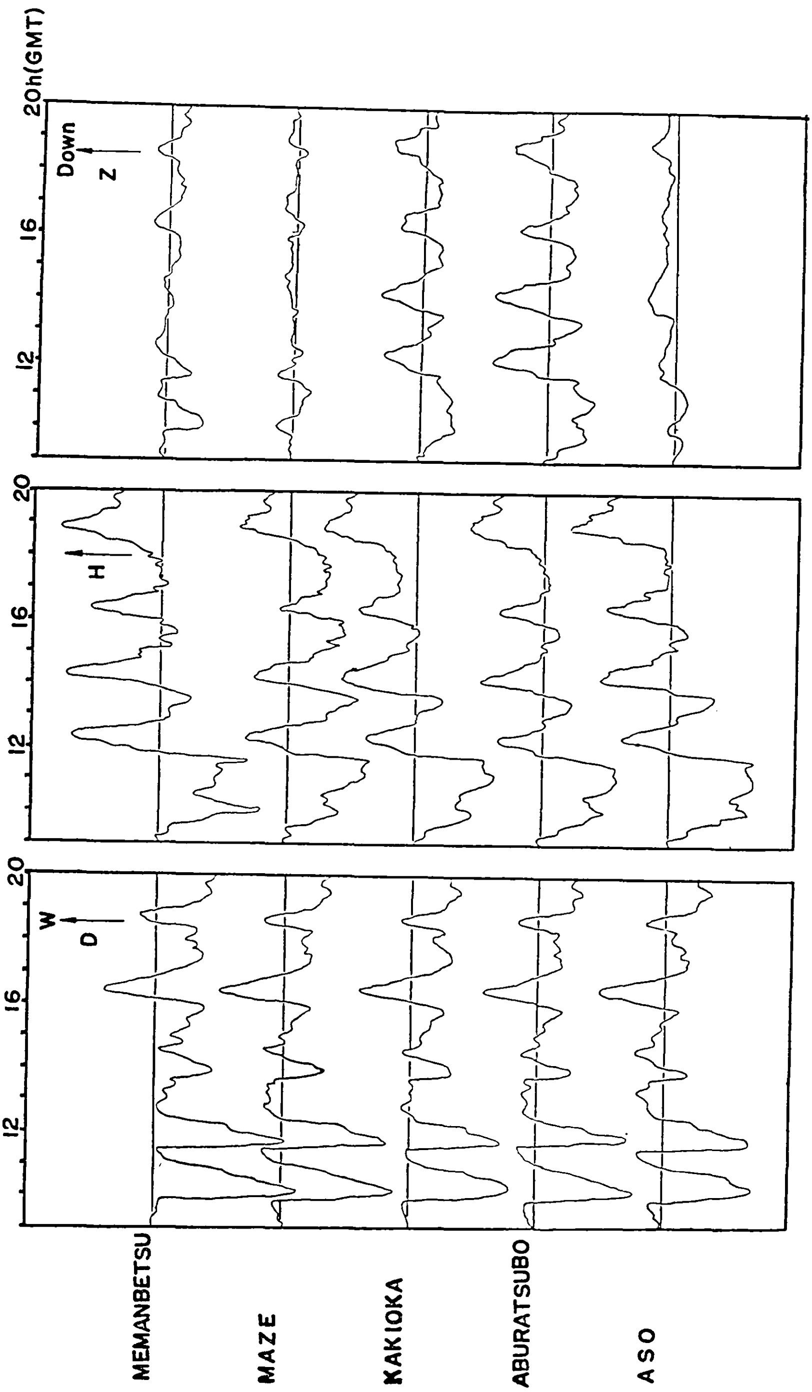

Fig. 4. The changes in the three components observed at the five stations in Japan at the time of the repeated occurrences of bays on Sept. 23 and 24, 1953. The distance between two datum lines corresponds to 40 *gammas*.

us to examine some further characteristics of geomagnetic variations of short period in the central part of Japan. At Maze, it is found out that ΔZ is fairly large whenever the magnetic vector is directed towards approximately east or west, though ΔZ is quite small for usual variations whose magnetic vectors lie nearly in the north-south direction. This tendency has been only recently discovered in spite of three years of observation, because bay disturbances, which were mostly

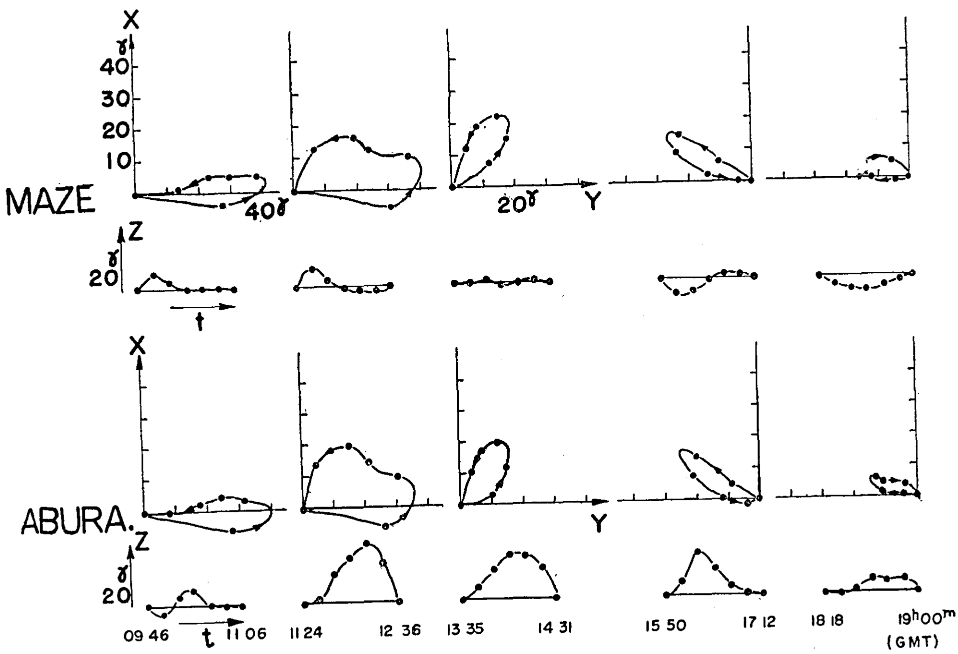

Fig. 5. The vector diagrams in the horizontal plane and changes in the vertical component respectively for Maze and Aburatsubo on occasions of the repeated occurrences of bays on Sept. 23 and 24, 1953.

analysed before, often occur during nights when the magnetic vector of the disturbance mainly points to the north[10),11)]. By a similar examination in regard to the observation at Aburatsubo, a quite different characteristics has been found there. ΔZ is quite small for the east-west changes of the magnetic vector though we observe an anomalously large ΔZ when the vector changes in the north-south direction. So we may say that there is a sort of anisotropy of geomagnetic variations. In order to make clear the underground structure, it will be of importance to study the anisotropy or the dependency of ΔZ on the direction of the inducing magnetic field.

The characteristics stated above is well demonstrated by the following examples. We found a successive occurrence of several bay disturbances during one evening on Sept. 23 and 24, 1953. Copies of the magnetograms at that time from five stations in Japan, Memanbetsu (43°54′ N, 144°12′ E), Kakioka (36°14′ N, 140°13′ E), Aso (32°55′ N, 131°04′ E) and our two stations, are shown in Fig. 4. Fig. 5 shows the vector diagrams in the horizontal plane and the modes of ΔZ-variation for each bay at Maze and Aburatsubo. It is noteworthy that an apparent anisotropy of geomagnetic variation exists at both stations.

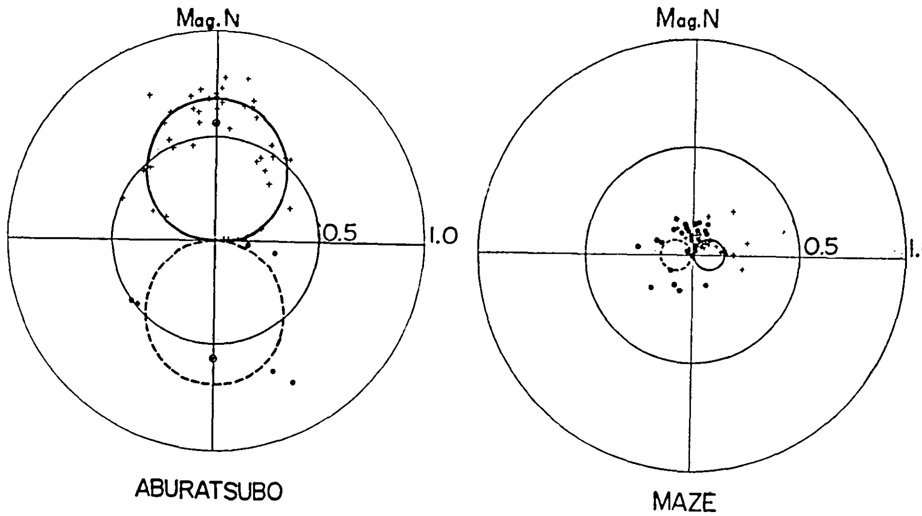

Fig. 6a. The relation between $\Delta Z/\Delta R$ and the direction of the magnetic vector in the horizontal plane at Aburatsubo.

Fig. 6b. The relation between $\Delta Z/\Delta R$ and the direction of the magnetic vector in the horizontal plane at Maze.

+ : positive value of ΔZ,
● : negative value of ΔZ.

In the following discussion, we will examine ΔZ in relation to ΔR (changes of the resultant component in the horizontal plane) in place of ΔH. At Aburatsubo, large amplitude of ΔZ becomes smaller in the cases of east- or westward variations, while slight parallelism between ΔZ and ΔR can be seen at Maze during the same variations.

The values of $\Delta Z/\Delta R$ at both stations are plotted in the diagrams as shown in Fig. 6. The radial distance gives the value of $\Delta Z/\Delta R$ while the direction of the magnetic vector is given by the azimuth. In the figures the small circle and the cross denote respectively the positive and negative values of ΔZ. If we express $\Delta Z/\Delta R$ by empirical formulae, we get the following expressions by means of the least square method:

$$\frac{\Delta Z}{\Delta R}=0.68\frac{\Delta X}{\Delta R}+0.01\frac{\Delta Y}{\Delta R} \qquad \text{for Aburatsubo,} \tag{1}$$

$$\frac{\Delta Z}{\Delta R}=-0.01\frac{\Delta X}{\Delta R}+0.15\frac{\Delta Y}{\Delta R} \qquad \text{for Maze,} \tag{2}$$

where ΔX and ΔY denote respectively the north and east components of variations. These formulae are shown in the figures with full and broken lines respectively for positive and negative values of ΔZ.

Here, we shall consider normal distribution of $\Delta Z/\Delta R$ in the cases of bay type variations assuming the uniform core model of the earth. As has been studied by one[13] of the writers, the distribution of the magnetic potential of a bay type disturbing field in low and middle latitude may be expressed approximately by the expression

$$W=\left(er+i\frac{R^3}{r^2}\right)\sin\phi Q_1^1(\cos\theta), \tag{3}$$

where ϕ, θ, R, e and i denote respectively the geomagnetic longitude, co-latitude, the radius of the earth, external and internal parts of the coefficient of the magnetic potential. ϕ is equivalent to the local time. Then the magnetic components at $r=R$ become as follows:

$$\left.\begin{aligned} X&=(e+i)\sin\phi\frac{dQ_1^1}{d\theta}, \\ Y&=-(e+i)\cos\phi\frac{Q_1^1}{\sin\theta}, \\ Z&=(e-2i)\sin\phi Q_1^1. \end{aligned}\right\} \tag{4}$$

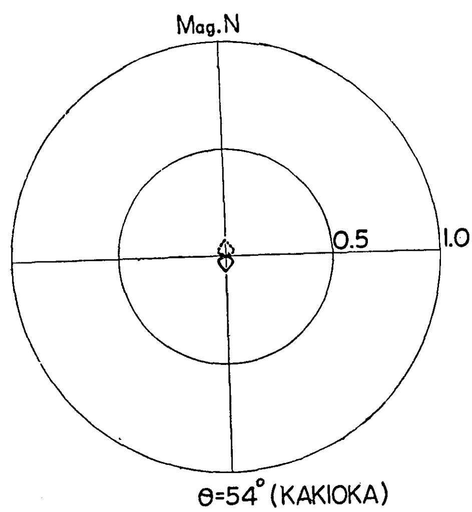

Fig. 7. The calculated relation between $\Delta Z/\Delta R$ and the direction of the magnetic vector in the horizontal plane at a point for $\theta=54°$, the relation being obtained with respect to an idealized uniform earth.

On taking the uniform core model which has been accepted from various reasons, the relation between $i(t)$ and $e(t)$ is given by

13) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 45.

$$i(t)=\frac{q^3}{2}e(t)=0.415e(t) \quad (q=0.94)\ . \tag{5}$$

Calculating each magnetic component by (4) and (5) for various values of ϕ, the distribution of $\Delta Z/\Delta R$ is shown in Fig. 7 where θ is taken as 54° (Kakioka).

Comparing Fig. 7 with Fig. 6, we find that the distributions of $\Delta Z/\Delta R$ at both Aburatsubo and Maze are quite different from those theoretically derived from the uniform core model of the earth. So we may say that the variations at the two stations are of the type which can not be explained by assuming a uniform conducting earth. The subterranean structure by which we can explain the anomalous behaviour of geomagnetic variations in the central part of Japan should be such as can account for these observed results.

Chapter II. The influences of the sea on geomagnetic variations

1. Review of the problem

Although the characteristic features of geomagnetic variations in Japan as well as in Germany have been considered to be caused by the induced electric currents flowing in the earth's interior, the question that the induced electric currents in the sea might be responsible for the particular distribution of geomagnetic variations has constantly been discussed by some geomagneticians. J. Coulomb[14] suggested that the sea surrounding Japan might have an appreciable influence. K. Burkhart[15] is of the opinion that all the characteristic distribution of geomagnetic variations can be explained by a particular distribution of the electrical conductivity near the surface of the earth. In view of these opinions, it is desirable to clarify the possible effect of the electric currents flowing in the sea on short-period geomagnetic variations.

From the beginning of the quantitative studies of geomagnetic variations, the influence of the seas or oceans has often been considered in relation to the interpretation of the relation between the external and internal parts of geomagnetic variations. However, no definite results were obtained because of the irregularities of the distribution of land and sea.

14) J. COULOMB, Discussion to the writers' paper at the Rome Assembly of IUGG ATME (1954).

15) K. BURKHART, Personal communication.

S. Chapman and T. T. Whitehead[16] made a mathematical study on the possible effect of a hypothetical ocean covering the whole surface of the earth with a uniform depth. They found that the internal origin part of a geomagnetic variation would be considerably affected even by a shallow ocean. It is not likely, however, that the influence of the real ocean will be as great as studied by them because the land areas will greatly reduce the said effect. The same kind of investigation was also made by B. N. Lahiri and A. T. Price[17] with special application to the electromagnetic induction within a non-uniform earth.

Since an appreciable effect of an ocean which is supposed to spread all over the earth can be expected as stated above, it becomes important to see to what extent geomagnetic variations will be influenced by the real distribution of the seas though a rigorous estimate would be of great difficulty to make. J. M. de Wet[18] calculated the induced electric current by solar diurnal variation in the oceans. On the basis of the distribution of ocean depth, he constructed a world-map of the distribution of the electrical conductivity over the earth's surface, the boundaries between the lands and seas being approximated with suitable meridians and parallel circles of every 15°. Ignoring the effect of self-induction, he calculated the induced electric currents by means of the relaxation method. So he got the induced currents whose distributions change according to the universal time. As may be expected, current vortices were obtained in large oceans such as the Pacific, Atlantic and Indian Oceans, so that we may suppose that a comparatively large magnetic field due to the induced currents exists there. T. Rikitake[13] also investigated the possible influence of a sea which is bounded by two meridians $\pi/2$ apart. According to his study it was concluded that the induced currents by S_q variation would produce a magnetic field of several *gammas* as its maximum. Near the boundaries between lands and seas, the field would become smaller. Since, however, the self-induction is ignored in the two studies mentioned above, the results can not be applied to rapid variations. A. A. Ashour[19] studied the induction of electric currents in a uniform circular disk. He applied the result to the problem of the induced current in the ocean which is regarded as a large disk.

In spite of all the studies reviewed here, the question of whether or

16) S. Chapman and T.T. Whitehead, *Trans. Cambr. Phil. Soc.*, **22** (1922), 463.
17) B.N. Lahiri and A.T. Price, *Phil. Trans. Roy. Soc. London A*, **237** (1939), 509.
18) J.M. de Wet, *Thesis, London University*, (1949).
19) A.A. Ashour, *Quart. Journ. Mech. and Applied Math.*, **3** (1950), 119.

not the anomalous behaviour of short-period geomagnetic variations as observed in Japan can be explained as being the effect of the electric currents induced in the sea still remains unsolved. Although it seems unlikely that the anomalous behaviour is caused by the electric currents in the sea judging from the duration-time of variations and also from the mode of the distribution of the anomaly, we have to prove that the sea is not responsible for the phenomenon, otherwise no rigorous conclusion can be obtained. The writers would here like to investigate the behaviour of time-dependent magnetic field in the neighbourhood of the boundary between land and sea.

The present investigations will be divided into two parts. In the first part, the electromagnetic induction in a hemi-spherical shell of uniform conductivity will be investigated for the purpose of examining the effect of ocean of a world-wide scale. A similar study will be made in regard to a plane sheet in the second part, the sheet having different conductivities respectively for idealized land and sea. The plane sheet model may be applied to a smaller scale distribution of land and sea.

2. Electromagnetic induction in a hemi-spherical shell of uniform conductivity[20]

We will examine here the electromagnetic induction in a hemi-spherical shell of uniform conductivity, the shell being placed in a time-dependent uniform magnetic field. Although the model differs from the real distribution of oceans over the earth, the hemi-spherical shell can, as a rough approximation, be regarded as an ocean of world-wide extent. Since no exact examination has hitherto been undertaken, the following study might be of some use for studying rapid changes in geomagnetic field because the effect of self-induction is fully taken into account.

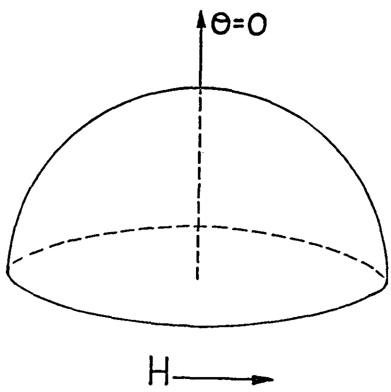

Fig. 8.

Let us take a hemi-spherical shell as shown in Fig. 8 where the

20) A part of this study was carried out by one of the writers (T.R.) under the direction of Professor A.T. Price in relation to the screening effect of the ionosphere which had been suggested by Professor S. Chapman. He is very grateful to Professor Price for the guidance given by him.

external magnetic field is applied in a direction as shown by the arrow. The centre of the sphere is taken as the origin of a spherical polar coordinate whose $\theta=0$ axis is also shown in Fig. 8.

The magnetic potential of the external field is given as

$$W_e=-rHP_1^1(\cos\theta)\cos\phi\ . \tag{6}$$

It is readily seen that we can express the current function of the induced electric currents in a form as

$$\Psi=\sum_n a_n P_n^1(\cos\theta)\cos\phi\ , \tag{7}$$

the expression being effective not only on the shell but on the whole surface of the sphere whose radius is denoted by a. The current function is to be determined by the following differential equation which is true only on the conductor, the equation being written as

$$\frac{1}{a^2}\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right\}\Psi=K\frac{\partial H_r}{\partial t} \tag{8}$$

in which K and H_r denote respectively the integrated conductivity which is uniform on the hemi-sphere and the normal component of the magnetic field. The magnetic permeability is assumed to be unity in electromagnetic unit.

Let us now suppose that the external magnetic field changes as

$$\begin{aligned} H&=0 && t<0 \\ &=H_0 && t\geqq 0\ . \end{aligned} \tag{9}$$

In that case, a particular distribution of the induced currents takes place in the shell at $t=0$. Since there is no change in the external field after that, the currents thus excited will decay freely. So the first task is to obtain the initial current distribution.

In the case of a sudden change in the external field, the self-induction is so large that we can neglect the lefthand-side of (8). Hence we have

$$H_r=0 \qquad \text{for} \quad t=0 \tag{10}$$

from which we see that the magnetic field produced by the initial currents cancels the normal component of the applied magnetic field. Hence the problem becomes equivalent to that of a hemi-spherical shell moving in an ideal fluid, the stream lines corresponding to the magnetic lines of force.

If we take into account (6) and (7), the relation (10) may be written as

$$\frac{aH_0}{4\pi}P_1^1+\sum_n\frac{n(n+1)}{2n+1}a_nP_n^1=0 \tag{11}$$

which is true only for $0<\theta<\pi/2$. (11) can be also written as

$$\sum_n a_nP_n^1=-\sum_n\left\{\frac{n(n+1)}{2n+1}-1\right\}a_nP_n^1-\frac{aH_0}{4\pi}P_1^1 \qquad 0<\theta<\pi/2. \tag{12}$$

On the other hand, the current function should be zero on another half of the sphere, so that we have

$$\sum_n a_nP_n^1=0 \qquad \pi/2<\theta<\pi . \tag{13}$$

Table I. R_{nN} $(=R_{Nn})$.

n \ N	1	2	3	4	5	6
1	0.666667	0.433013	0	−0.131762	0	0.071603
2		0.400000	0.176778	0	−0.034939	0
3			0.2857143	0.181546	0	−0.058463
4				0.222222	0.114820	0
5					0.181818	0.115549
6						0.153846
7						
8						
9						
10						
11						
12						

n \ N	7	8	9	10	11	12
1	0	−0.046876	0	0.033798	0	−0.025875
2	0.014321	0	−0.0075646	0	0.0045805	0
3	0	0.033490	0	−0.022809	0	0.016946
4	−0.027235	0	0.012430	0	−0.0070562	0
5	0	−0.037823	0	0.022090	0	−0.015311
6	0.084573	0	−0.021890	0	0.010604	0
7	0.133333	0.067344	0	−0.027944	0	0.016473
8		0.117647	0.066878	0	−0.018223	0
9			0.105263	0.066957	0	−0.022150
10				0.095238	0.055287	0
11					0.086957	0.055325
12						0.076923

From (12) and (13), we can determine the coefficients in the following way. After multiplying by $P_N^1(\cos\theta)\sin\theta$, we shall integrate (12) and (13) with respect to θ from $\theta=0$ to $\theta=\pi$. In that case (12) is true between 0 to $\pi/2$ while we have to take (13) between $\pi/2$ to π. So we obtain

$$2a_nR_{nN}+\sum_n\left\{\frac{n(n+1)}{2n+1}-1\right\}a_nR_{nN}=-\frac{aH_0}{4\pi}R_{1N}\,, \tag{14}$$

where

Table II. The coefficients and the right-hand members of the simultaneous equations.

a_1	a_2	a_3	a_4	a_5	a_6	a_7
1	0.077942	0	−0.144606	0	0.143757	0
−0.164020	1	0.143488	0	−0.068578	0	0.044480
0	0.045590	1	0.285462	0	−0.168171	0
0.061391	0	0.181258	1	0.277213	0	−0.104053
0	−0.010311	0	0.206604	1	0.380357	0
−0.036669	0	−0.064158	0	0.306633	1	0.355055
0	0.004538	0	−0.052623	0	0.298937	1
0.025369	0	0.038839	0	−0.106070	0	0.298861
0	−0.002505	0	0.025100	0	−0.080865	0
−0.018963	0	−0.027423	0	0.064222	0	−0.128565
0	0.001563	0	−0.014863	0	0.040367	0
0.015487	0	0.021734	0	−0.047485	0	0.080848

a_8	a_9	a_{10}	a_{11}	a_{12}	right-hand member*)
−0.136493	0	0.128917	0	−0.122140	0.600000
0	−0.032122	0	0.024668	0	0.492060
0.139714	0	−0.124650	0	0.114499	0
0	0.064925	0	−0.046742	0	−0.184173
−0.180569	0	0.138111	0	−0.118384	0
0	−0.125676	0	0.077210	0	0.110008
0.345230	0	−0.187656	0	0.136773	0
1	0.405758	0	−0.140218	0	−0.076107
0.358301	1	0.469912	0	−0.192640	0
0	0.421150	1	0.441024	0	0.056890
−0.100609	0	0.399844	1	0.494705	0
0	−0.148620	0	0.470790	1	−0.046461

*) The right-hand members should be multiplied by $-\frac{aH}{4\pi}$.

$$R_{nN}=\int_0^1 P_n^1(x)P_N^1(x)dx\ . \tag{15}$$

R_{nN}'s are constants which can be evaluated by (15) as given in Table I up to the 12th degree of the spherical surface harmonics. With the aid of these values we can obtain a set of simultaneous equations for a_n's. On taking a_n up to $n=12$, the coefficients of a_n's and the righthand members are calculated as given in Table II. Since

Table III. The solutions of the simultaneous equations. $\left(\text{Unit}: -\frac{aH}{4\pi}\right)$

a_1	a_2	a_3	a_4	a_5	a_6
0.4737	0.5549	0.0922	−0.2210	−0.0403	0.1498
a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
−0.0025	−0.1025	0.0074	0.0813	−0.0312	−0.0417

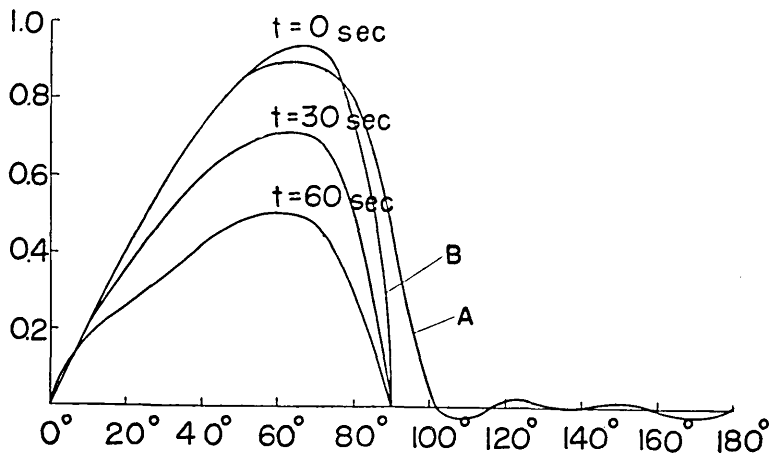

Fig. 9. The distribution of the current function on $\phi=0$. $\left(\text{Unit}: -\frac{aH}{4\pi}\right)$ Curve A shows the distribution for $t=0$ as obtained by (7). Curve B corresponds to that obtained by (16). The decay of the initial distribution is also shown for $t=30$ *sec.* and $t=60$ *sec.*

the coefficients of the diagonal terms are relatively large, we may solve the equations though the rigorous proof of convergency of the solution is disregarded here. The solutions which are solved by the relaxation method are given in Table III.

With these a_n's the current function is obtained from (7), the distribution on $\phi=0$ being shown in Fig. 9. It is clearly seen that the current function is nearly zero between $\theta=\pi/2$ and $\theta=\pi$. The fact that the current function is not exactly zero is due to the procedure of calculation in which we take harmonics of $n=1, 2, \cdots, 12$ only. This is also the cause of small fluctuations between $\theta=\pi/2$ and $\theta=\pi$.

If we take into consideration the fact that the normal component of the electric currents should vanish at $\theta=\pi/2$, we may have an alternative expression for the current function

$$\Psi=\sum_m c_{2m}P_{2m}^1(\cos\theta)\cos\phi \tag{16}$$

which is true only for $0<\theta<\pi/2$. So we have

$$\left.\begin{aligned}&\sum_n a_nP_n^1=\sum_m c_{2m}P_{2m}^1 && 0<\theta<\pi/2\,,\\ &\sum_n a_nP_n^1=0 && \pi/2<\theta<\pi\,,\end{aligned}\right\} \tag{17}$$

from which we deduce the following relation between a_n and c_{2m} by means of a procedure similar to the determination of a_n, the relation being given as

$$2R_{NN}a_n=\sum_m c_{2m}R_{2m,N}\,. \tag{18}$$

Since a_n's are given in Table III, we can solve c_{2m}'s from (18). The current function thus obtained is also shown in Fig. 9 while the current system viewed from a point on $\theta=0$ axis is shown in Fig. 10. Although we see some difference between the two curves, it would be meaningless to discuss the difference unless we take into account many more harmonics.

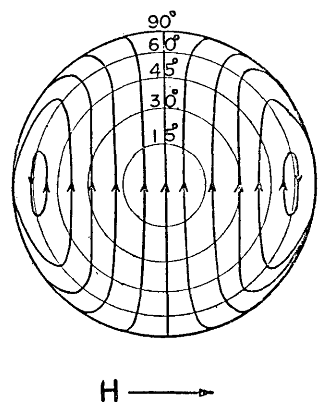

Fig. 10. The distribution of the initial currents, the same amount of the electric current flowing between two adjacent lines.

Now we are in a position to investigate the decay of the induced electric currents which are excited at $t=0$. We shall start again from (8) which can be written as

$$-\sum_n n(n+1)a_nP_n^1=a^2KpH_0P_1^1+4\pi aKp\sum_n\frac{n(n+1)}{2n+1}a_nP_n^1 \quad \text{for} \quad 0<\theta<\pi/2 \tag{19}$$

where p denotes $\partial/\partial t$.

It is also obvious that

$$\sum_n n(n+1)a_nP_n^1=0 \qquad \text{for} \quad \pi/2<\theta<\pi\,. \tag{20}$$

After multiplying by $P^1_N(\cos\theta)\sin\theta$ and integrating (19) and (20) with respect to θ from $\theta=0$ to $\theta=\pi$, we obtain

$$-2a_N R_{NN} N(N+1) = a^2 K p H_0 R_{1N} + 4\pi a K p \sum_n \frac{n(n+1)}{2n+1} a_n R_{nN}\ . \tag{21}$$

If it is assumed that a_n can be expressed as

$$a_n = \sum_s a_n^s p^{-s}\ , \tag{22}$$

(21) becomes

$$-\frac{R_{NN} N(N+1)}{2\pi a K} \sum_s a_N^s p^{-s-1} = \frac{aH_0}{4\pi} R_{1N} + \sum_s \sum_n \frac{n(n+1)}{2n+1} a_n^s R_{nN} p^{-s}\ . \tag{23}$$

On equating the coefficients of the corresponding powers of p, we obtain

$$\left.\begin{aligned}
&\sum_n \frac{n(n+1)}{2n+1} a_n^1 R_{nN} = -\frac{R_{NN} N(N+1)}{2\pi a K} a_N^0\ ,\\
&\sum_n \frac{n(n+1)}{2n+1} a_n^2 R_{nN} = -\frac{R_{NN} N(N+1)}{2\pi a K} a_N^1\ ,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&\sum_n \frac{n(n+1)}{2n+1} a_n^s R_{nN} = -\frac{R_{NN} N(N+1)}{2\pi a K} a_N^{s-1}\ .
\end{aligned}\right\} \tag{24}$$

By solving the first set of the equations in (24), therefore, we can obtain a_N^1 starting from a_N^0 obtained in the preceding paragraph. On repeating the successive operations, the coefficients a_N^2, a_N^3, $\cdots$ can be obtained from (24). If we interpret the operational equation (22) term by term, we obtain

$$a_n = \sum_s \frac{a_n^s}{s!} t^s\ , \tag{25}$$

so that it is possible to estimate the change in the current function with lapse of time provided the series (25) are convergent. Actually, the coefficients of the first three harmonics can be written as

$$\left.\begin{aligned}
c_2 &= (1.16 - 7.67\omega t + 32.0\omega^2 t^2 - 124\omega^3 t^3 + 468\omega^4 t^4 \cdots)\left(-\frac{aH_0}{4\pi}\right),\\
c_4 &= (-0.458 + 6.07\omega t - 41.7\omega^2 t^2 + 206\omega^3 t^3 - 840\omega^4 t^4 \cdots)\left(-\frac{aH_0}{4\pi}\right),\\
c_6 &= (0.260 - 6.49\omega t + 68.5\omega^2 t^2 - 456\omega^3 t^3 + 2240\omega^4 t^4 \cdots)\left(-\frac{aH_0}{4\pi}\right),
\end{aligned}\right\} \tag{26}$$

where

$$\omega = \frac{1}{4\pi a K}. \tag{27}$$

In this case, the second way of expression for the current function which is true only for $0 < \theta < \pi/2$ as given in (16) is adopted. c_2^s, c_4^s, $\cdots$ are readily obtained from a_1^s, a_2^s, $\cdots$.

If we assume $K = 10^{-7} emu$[21], the distribution of the current function becomes like the curves which are also shown in Fig. 9 respectively for $t = 30$ *sec.* and $t = 1$ *min.* It is seen that the centres of the current

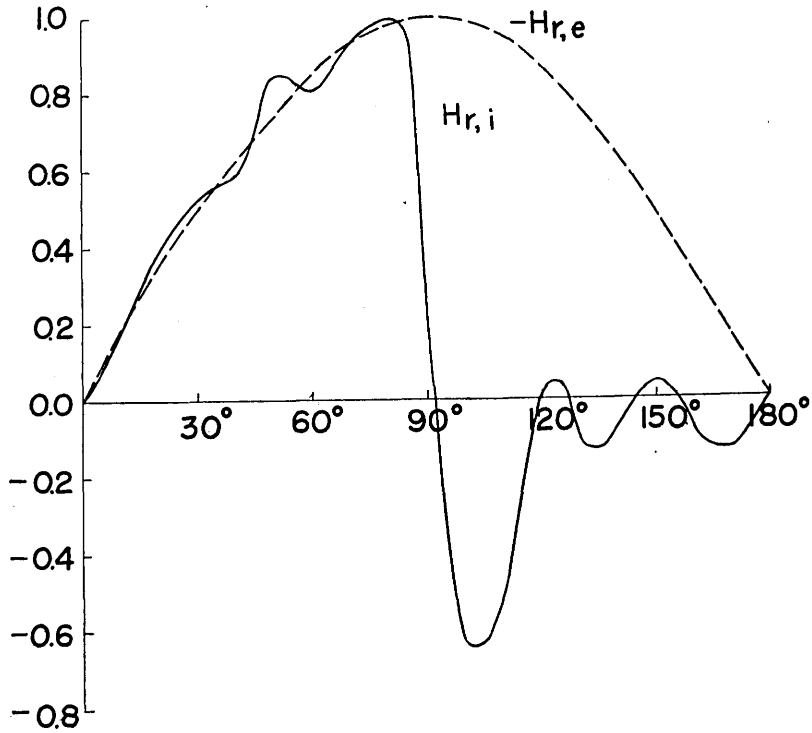

Fig. 11. The initial distribution of the normal component of the inducing and induced magnetic field on $\phi = 0$. (Unit: H_0)

vortices move towards $\theta = 0$ point while the intensity of the currents is decreasing. However, it seems difficult to obtain the current function for large t because the series as given by (25) does not converge for that case.

The normal component of the magnetic field produced by the induced

21) The electrical conductivity of sea water is about $10^{-11} emu$. So this value corresponds to an ocean whose depth is about $100 m$.

electric currents at $t=0$ are easily calculated with the coefficients given in Table I, its distribution on $\phi=0$ being shown in Fig. 11 in which the reversed external field is also shown. It is clearly seen that the induced field nearly cancels out the inducing one on the conducting shell though there are some fluctuations due to the method of approximation using spherical harmonics of finite number. On the other half of the sphere, we obtain a remarkable intensity of the normal component of the magnetic field which is of the same sign as that of the inducing one. We can therefore expect a large vertical field which intensifies the external one on the land near the boundary, the magnitude of which amounting to as much as 60 percent of that of the inducing field. This phenomenon may be called the "coast effect" in the case of rapidly varying geomagnetic field.

Since, however, the normal component of the external field is intensified by the presence of the sea on the land in the neighbourhood of the sea, the anomalous geomagnetic variation in Japan can not be ascribed to the "coast effect". If so, on occasions of sudden commencements of magnetic storms, the external vertical field should be upward in Japan, but we usually observe downward vertical field there as the writers examined in their previous paper. Even if we take into account not only sudden changes but general ones as will be mentioned in the following, it would still be impossible to explain the anomalous chage in Japan by the coast effect examined here.

If the external field changes in an arbitrary way starting from $t=0$, we have to regard H in (1) as an arbitrary function $H(t)$. In that case a_n can be determined by an operational equation such as

$$a_n(t)=\Phi_n(p)H(t) . \tag{28}$$

On solving (28), we obtain

$$a_n(t)=h_n(0)H(t)+\int_0^t H(t-u)h'_n(u)du , \tag{29}$$

where h_n denotes a_n when $H(t)$ changes as given by (9), so that we have

$$h_n(0)=a_n^0 \tag{30}$$

and also

$$h_n(0)h'_n(t)<0 . \tag{31}$$

When the sea is to be regarded as a perfect conductor, there is no decay of the induced currents whence we have always $h'_n(t)=0$. In that case, the $a_n(t)$'s are always proportional to $H(t)$. Even in the case of a sea of finite conductivity, the second term in the righthand-side of (29) is fairly small provided the depth is around hundreds or thousands of meters. So $a_n(t)$ is nearly equal to $h_n(0)H(t)$ even for this case. Furthermore, we see that the second term is of opposite sign to the first one except in cases for very small $H(t)$ because of (31). So we see that

$$|a_n(t)| \leq |a_n^0 H(t)| \tag{32}$$

together with

$$a_n(t) a_n^0 H(t) > 0 \tag{33}$$

for general cases provided $H(t)$ is of proper magnitude. For very small $H(t)$, however, the relations do not hold good, but such cases are not important here.

Thus far we see that the coefficients of the spherical harmonic constituents will not usually exceed their initial values multiplied by $H(t)$. Roughly speaking, therefore, the distribution of the current function as well as the magnetic field which are expressed by a sum of these spherical harmonic constituents will change proportionally to $H(t)$ provided the conductivity of the sea is assumed to be high. If it is so, the behaviour of the magnetic field is almost the same as that for sudden changes, its anomalous distribution such as has been observed in Japan being impossible. Even if we take into account the effect of finite conductivity, as is discussed in (32) and (33), we obtain current systems and magnetic fields whose distributions differ very little from the initial ones because the difference between $a_n^0 H(t)$ and $a_n(t)$ is not great. However, the discussions can not be applied to cases in which $H(t)$ is very small. They are also not applicable for parts of the sphere where we have very small amount of electric current and magnetic field because the zero point of the current function or magnetic field is likely to be displaced appreciably when we have slight changes in the amplitudes of the various spherical harmonic constituents. But those unfavourable cases are not important because of the small magnitude of electric currents and magnetic fields. Summarizing all the discussions stated here, it is unlikely that the tendency of the distribution which is observed for a sudden change would be seriously altered for variations of arbitrary shape.

So long as we consider the effect of an ocean as examined here, there is no possibility to explain the fact that the vertical component of the induced magnetic field overcomes the inducing one as has been observed in Japan.

3. Electromagnetic induction in a plane sheet having a special distribution of the electrical conductivity

The magnetic effect of the electric currents induced in an ocean of world-wide scale becomes clear in the last section. It is of interest to see how the effect will change if we take into account a distribution of land and sea of a smaller scale. For instance, we may regard the coast of the main island of Japan as a straight line. If we ignore the curvature of the earth's surface, we may imagine an idealized distribution of the electrical conductivity on a plane sheet as shown in Fig. 12 where the conductivity on the land ($-a<x<0$) is much smaller than that on the sea ($0<x<a$). For the purpose of mathematical convenience, it is assumed that the same distribution is repeated periodically along x-axis. In so far as the magnetic effects in the neighbourhood of the boundary between the land and sea are concerned, the present model would be useful. Furthermore, the distribution is assumed to be constant in the y-direction, while z-axis is taken to be upward.

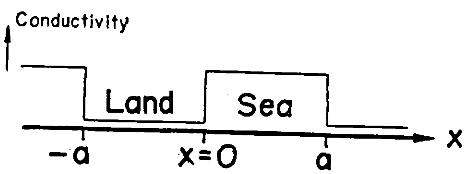

Fig. 12. The supposed distribution of the electrical conductivity on the land and sea.

We are now going to investigate the electromagnetic induction in a thin sheet having a distribution of the conductivity as shown in Fig. 12, the medium underneath the sheet being regarded as non-conducting. The theory of the electromagnetic induction in sheets or shells was fully studied by A.T. Price[22]. The following study will be made in a way similar to his study.

i) In the first place, we shall ignore the conductivity on the land while the integrated conductivity on the sea is denoted by K_s. In that case we have the boundary condition on the positive face of $z=0$ as follows;

$$\left.\begin{aligned} &W_i=0 && \text{for the land or} \quad -a<x<0\,, \\ &\frac{1}{K_s}\left(\frac{\partial^2 W_i}{\partial z^2}\right)=2\pi\frac{\partial}{\partial t}\left(\frac{\partial W_i}{\partial t}+\frac{\partial W_e}{\partial t}\right) && \text{for the sea or} \quad 0<x<a\,. \end{aligned}\right\} \tag{34}$$

22) A.T. PRICE, *Quart. Journ. Mech. and Applied Math.*, **2** (1949), 283.

In (34), W_e and W_i denote respectively the magnetic potential of the inducing and induced field, so that W_e and W_i must satisfy the differential equations

$$\nabla^2 W_e=0\ , \qquad \nabla^2 W_i=0 \tag{35}$$

in the space. Hence we may assume that W_e and W_i are given by the following expressions for $z>0$;

$$W_e=E(t)e^{qz}\cos qx\ , \tag{36}$$

$$W_i=\sum_m e^{-mqz}\{I_m(t)\cos mqx+J_m(t)\sin mqx\}\ . \tag{37}$$

If we assume that W_e is caused by inducing electric currents on a sheet $z=h$, the currents should flow in a direction parallel to the coast line.

If we denote the electric current function by Ψ, Ψ can be expressed as

$$\Psi=\frac{1}{2\pi}W_i \tag{38}$$

on the positive face of the sheet.

Before we investigate the decay-mode of the induced field, we seek the initial state for $t=0$ by solving the following equations in place of (34).

$$\left.\begin{array}{lll} W_i=0 & \text{for the land or} & -a<x<0\ , \\ \dfrac{\partial W_i}{\partial z}+\dfrac{\partial W_e}{\partial z}=0 & \text{for the sea or} & 0<x<a\ , \end{array}\right\} \tag{39}$$

in which we take into account only the self-induction effect ignoring the diffusion effect because of the sudden change considered. On introducing (36) and (37) into (39), we obtain

$$\sum_m \{I_m\cos mqx+J_m\sin mqx\}=0 \qquad \text{for } -a<x<0\ , \tag{40}$$

$$\sum_m m\{I_m\cos mqx+J_m\sin mqx\}+E\cos qx=0 \qquad \text{for } 0<x<a\ . \tag{41}$$

Similarly as in the last section, (41) can be transformed as

$$\begin{aligned}\sum_m m\{I_m\cos mqx+J_m\sin mqx\} \\ =-\sum_m (m-1)\{I_m\cos mqx+J_m\sin mqx\}-E\cos qx\end{aligned} \tag{42}$$

From (40) and (42), we can determine the coefficients I_m and J_m as follows. After multiplying by $\cos Mqx$ and $\sin Mqx$ where M is an integer, we shall integrate (40) and (42) with respect to x over the whole range and obtain the following two relations respectively.

$$\left.\begin{aligned} 2\sum(m-1)g(M,m)J_m &= \pi(M+1)I_M-\pi E \qquad \text{for} \quad M=1 \\ &= \pi(M+1)I_M \qquad\qquad \text{for} \quad M>1\,, \end{aligned}\right\} \tag{43}$$

$$2\sum(m-1)h(M,m)I_m=\pi(M+1)J_M+2k(M)\,, \tag{44}$$

where q is taken to be π/a for simplifying the calculations and $g(M, m)$, $h(M, m)$ and $k(M)$ are given by the following integrals.

$$\left.\begin{aligned} g(M,m) &= \frac{1}{q}\int_{-a}^{0}\cos Mqx\cdot\sin mqx dx\,, \\ h(M,m) &= \frac{1}{q}\int_{-a}^{0}\sin Mqx\cdot\cos mqx dx\,, \\ \text{and} \qquad k(M) &= \frac{1}{q}\int_{-a}^{0}\sin Mqx\cdot\cos qx dx\,. \end{aligned}\right\} \tag{45}$$

Table IV.

m	1	2	3	4	5	6
I_m	0.2995	0	−0.0339	0	−0.0090	0
J_m	0	0.2693	0	0.0767	0	0.0406

m	7	8	9	10	11	12
I_m	0.0008	0	0.0064	0	0.0115	0
J_m	0	0.0265	0	0.0275	0	0.0089

(Unit: E)

Solving the simultaneous equations (43) and (44), the coefficients I_m and J_m are evaluated as given in Table IV up to the 12th degree of the trigonometric harmonics. Using these values, we obtain the distribution of magnetic potential from (36) and (37) as shown in Fig. 13 (a) in which we see that the internal part is approximately zero on the land.

The vertical (downward) component of the magnetic field produced by the induced electric currents at $t=0$ are easily obtained with the coefficients given in Table IV. Fig. 13 (b) shows the distribution of the downward component. We see that the induced field nearly cancels out the inducing one for the sea region and the "coast effect" is existing as well as in the last section.

To investigate the decay of the induced electric currents which are excited at $t=0$, we go back to the equations (34). On introducing (36) and (37) into (34), we obtain the following two relations by the same procedure as before. We have

$$\left.\begin{aligned}&2\left(\sum\frac{m^2-1}{\pi}+\frac{2\pi K_s p}{q}\sum\frac{m}{\pi}\right)g(M,\,m)J_m-\left(M^2+1+\frac{2\pi K_s p}{q}M\right)I_M\\&\qquad=-\frac{2\pi K_s p}{q}E\,,\\&2\left(\sum\frac{m^2-1}{\pi}+\frac{2\pi K_s p}{q}\sum\frac{m}{\pi}\right)h(M,\,m)I_m-\left(M^2+1+\frac{2\pi K_s p}{q}M\right)J_M\\&\qquad=\frac{2\pi K_s p}{q}\,\frac{2k(M)}{\pi}E\,,\end{aligned}\right\}\tag{46}$$

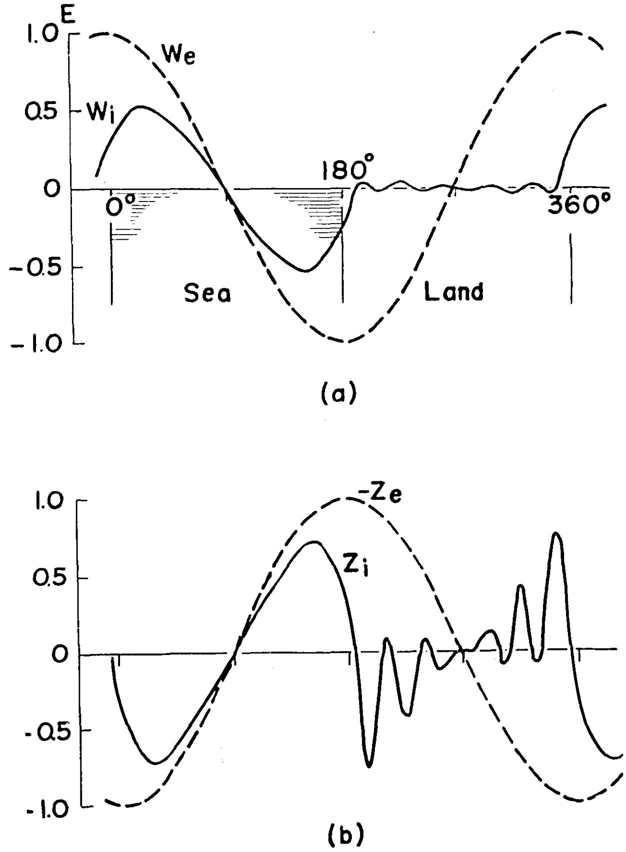

Fig. 13. (a) The inducing and induced magnetic potential or current function. (b) The vertical component of the inducing and induced field. $\left(\text{Unit: } \frac{\pi}{a}E\right)$

where p denotes $\partial/\partial t$.

On assuming the following expressions

$$I_M=\sum_{s=0} I^s_M p^{-s} \quad \text{and} \quad J_M=\sum_{s=0} J^s_M p^{-s}\,, \tag{47}$$

we obtain the following sets of simultaneous equations from (46),

$$\left.\begin{array}{l}
\begin{cases}
2\sum\frac{m^2-1}{\pi}g(M,m)\frac{q}{2\pi K_s}J^0_m+2\sum\frac{m}{\pi}g(M,m)J^1_m-\frac{q}{2\pi K_s}(M^2+1)I^0_M-MI^1_M=0\,, \\
2\sum\frac{m^2-1}{\pi}h(M,m)\frac{q}{2\pi K_s}I^0_m+2\sum\frac{m}{\pi}h(M,m)I^1_m-\frac{q}{2\pi K_s}(M^2+1)J^0_M-MJ^1_M=0\,,
\end{cases} \\
\cdots\cdots\cdots\cdots \\
\begin{cases}
2\sum\frac{m^2-1}{\pi}g(M,m)\frac{q}{2\pi K_s}J^{s-1}_m+2\sum\frac{m}{\pi}g(M,m)J^s_m-\frac{q}{2\pi K_s}(M^2+1)I^{s-1}_M-MI^s_M=0, \\
2\sum\frac{m^2-1}{\pi}h(M,m)\frac{q}{2\pi K_s}I^{s-1}_m+2\sum\frac{m}{\pi}h(M,m)I^s_m-\frac{q}{2\pi K_s}(M^2+1)J^{s-1}_M-MI^s_M=0\,.
\end{cases}
\end{array}\right\} \tag{48}$$

Here, we can obtain the decay-mode starting from I^0_m and J^0_m which are given in Table IV.

Next, we assume that the inducing electric currents on $z=h$ flow perpendicularly to the coast line. In this case W_e and W_i take the following forms for $z>0$;

$$W_e=E(t)e^{rz}\cos ry\,, \tag{49}$$

$$W_i=\cos ry\sum_m e^{-z\sqrt{r^2+m^2q^2}}\{I_m(t)\cos mqx+J_m(t)\sin mqx\}\,. \tag{50}$$

On introducing (49) and (50) into (39), we seek the initial state by the same procedure as in the former case. The relations between the coefficients I_m and J_m corresponding to (43) and (44) are given as follows;

$$\left.\begin{aligned}
\sum(\sqrt{1+m^2}-1)g(M,m)J_m&=\frac{\pi}{2}(\sqrt{1+M^2}+1)I_M-2aE \quad \text{for } M=0 \\
&=\frac{\pi}{2}(\sqrt{1+M^2}+1)I_M \qquad\qquad \text{for } M>0
\end{aligned}\right\}, \tag{51}$$

$$\sum(\sqrt{1+m^2}-1)h(M,m)I_m=\frac{\pi}{2}(\sqrt{1+M^2}+1)J_M-E\frac{2}{M}\,, \tag{52}$$

where q and r are both taken to be π/a for mathematical simplicity. Solving the simultaneous equations (51) and (52), we can evaluate the coefficients I_m and J_m as given in Table V. With the aid of these values, the distributions of magnetic potential and vertical (downward) component of the magnetic field are obtained as shown in Fig. 14. In the figure we also notice the " coast effect " which intensifies the vertical component of the external field.

Table V.

m	0	1	2	3	4	5	6
I_m	0.3770	0	−0.0619	0	−0.0101	0	−0.0053
J_m	—	0.5099	0	0.1095	0	0.0443	0

m	7	8	9	10	11	12
I_m	0	−0.0049	0	−0.0055	0	−0.0088
J_m	0.0230	0	0.0139	0	0.0095	0

(Unit: E)

ii) In the second case, we deem the sea to be perfectly conductive while the conductivity on the land is finite. The differential equations become

$$\left.\begin{aligned} \frac{1}{K_l}\left(\frac{\partial^2 W_i}{\partial z^2}\right)&=2\pi\frac{\partial}{\partial t}\left(\frac{\partial W_i}{\partial z}+\frac{\partial W_e}{\partial z}\right) \quad \text{for the land or} \quad -a<x<0\,, \\ 0&=2\pi\frac{\partial}{\partial t}\left(\frac{\partial W_i}{\partial z}+\frac{\partial W_e}{\partial z}\right) \quad \text{for the sea or} \quad 0<x<a\,, \end{aligned}\right\} \tag{53}$$

where K_l denote the conductivity on the land.

For brevity, we confine the later discussion to cases in which the inducing electric currents flow in a direction parallel to the coast line. In that case we have

$$\left.\begin{aligned} W_e&=E(t)e^{qz}\cos qx\,, \\ W_i&=\sum_m e^{-mqz}\{I_m(t)\cos mpx+J_m(t)\sin mpx\}\,. \end{aligned}\right\} \tag{54}$$

By the procedure similar to the previous one, we obtain the following relations.

$$\left.\begin{aligned} \left(\frac{M^2}{2}+\frac{2\pi K_l}{q}pM\right)I_M+\sum\frac{m^2}{\pi}g(M,m)J_m&=\frac{2\pi K_l}{q}pE \quad \text{for} \quad M=1\,, \\ &=0 \quad \text{for} \quad M>1\,, \\ \left(\frac{M^2}{2}+\frac{2\pi K_l}{q}pM\right)J_M+\sum\frac{m^2}{\pi}h(M,m)I_m&=0\,. \end{aligned}\right\} \tag{55}$$

On assuming that the coefficients I_m and J_m are expressed by a power series of the operator p, the internal part of magnetic potential W_i is

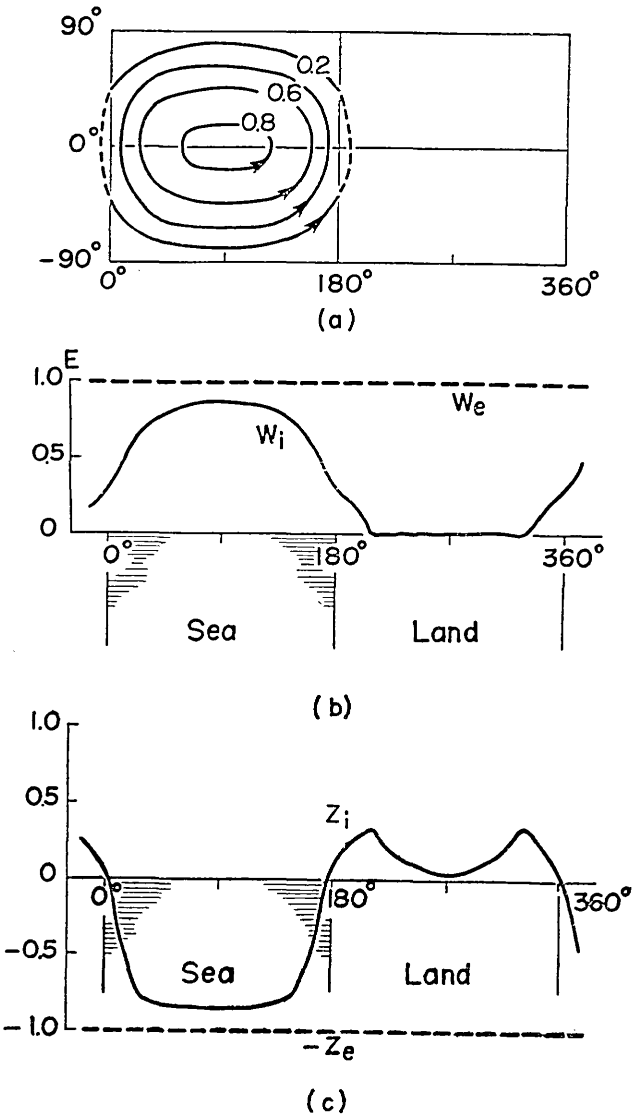

Fig 14. (a) The induced electric currents at $t=0$. (b) The distribution of the inducing and induced magnetic potential or current function on $y=0$. (c) The same distribution of the vertical component of the inducing and induced fields. $\left(\text{Unit: } \frac{\pi}{a}E\right)$

obtained as follows.

$$\begin{aligned}W_i=&\left\{1-0.2501\frac{t}{aK_l}+0.0849\left(\frac{t}{aK_l}\right)^2+\cdots\cdots\right\}E\cos px\\
&+\left\{\quad 0.1062\frac{t}{aK_l}-0.0400\left(\frac{t}{aK_l}\right)^2+\cdots\cdots\right\}E\sin 2px\\
&+\left\{\qquad\qquad 0.0054\left(\frac{t}{aK_l}\right)^2+\cdots\cdots\right\}E\cos 3px\\
&+\left\{\quad 0.0214\frac{t}{aK_l}-0.0133\left(\frac{t}{aK_l}\right)^2+\cdots\cdots\right\}E\sin 4px\\
&+\left\{\qquad\qquad -0.0005\left(\frac{t}{aK_l}\right)^2+\cdots\cdots\right\}E\cos 5px\\
&+\left\{\quad 0.0091\frac{t}{aK_l}-0.0079\left(\frac{t}{aK_l}\right)^2+\cdots\cdots\right\}E\sin 6px\\
&+\cdots\cdots\cdots\cdots\cdots\end{aligned}\quad . \qquad (56)$$

Fig. 15 shows one example of the decay-mode.

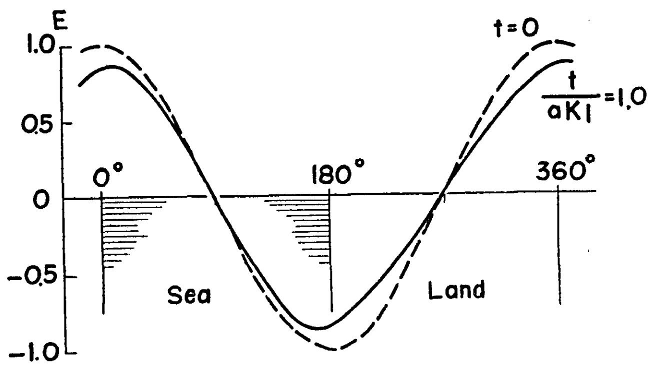

Fig. 15. The induced (broken line) magnetic potential or current function at $t=0$, while that at $t=0.1$ *sec.* is shown by the full line. The conductivity on the sea is taken to be infinity, while that on the land is assumed to be 10^{-9} *emu* and a to be 10^8 *cm.*

iii) In the third case, we take the conductivities both on the sea and on the land to be finite. Then the fundamental equations become as follows.

$$\left.\begin{aligned}&\frac{1}{K_l}\left(\frac{\partial^2 W_i}{\partial z^2}\right)=2\pi\frac{\partial}{\partial t}\left(\frac{\partial W_i}{\partial z}+\frac{\partial W_e}{\partial z}\right)\quad \text{for the land or}\quad -a<x<0\ ,\\
&\frac{1}{K_s}\left(\frac{\partial^2 W_i}{\partial z^2}\right)=2\pi\frac{\partial}{\partial t}\left(\frac{\partial W_i}{\partial z}+\frac{\partial W_e}{\partial z}\right)\quad \text{for the sea or}\quad 0<x<a\ .\end{aligned}\right\}\qquad (57)$$

We assume the following form for magnetic potential following the examples of the former cases.

$$\left.\begin{aligned} W_e &= E(t)e^{qz}\cos qx\ , \\ W_i &= \sum_m e^{-mqz}\{I_m(t)\cos mqx + J_m(t)\sin mqx\}\ . \end{aligned}\right\} \tag{58}$$

By the method similar to the previous cases, we obtain

$$\left.\begin{aligned} &\{M^2+aM(K_l+K_s)p\}I_M+2p\sum mJ_m g(M,m)\frac{a}{\pi}(K_l-K_s) \\ &\qquad\qquad = a(K_l+K_s)pE \qquad \text{for} \quad M=1\ , \\ &\qquad\qquad = 0 \qquad\qquad\qquad \text{for} \quad M>1\ , \\ &\{M^2+aM(K_l+K_s)p\}J_M+2p\sum mI_m h(M,m)\frac{a}{\pi}(K_e-K_s) \\ &\qquad\qquad = 2\frac{a}{\pi}(K_l-K_s)k(M)pE\ . \end{aligned}\right\} \tag{59}$$

Introducing the expressions (47) to (59), we get

$$\left.\begin{aligned} &I_1^0=E\ , \qquad I_2^0=I_3^0=\ldots\ldots\ldots=0\ , \\ &J_1^0=J_2^0=J_3^0=\ldots\ldots\ldots\ldots\ldots=0\ , \end{aligned}\right\} \tag{60}$$

and

$$\left.\begin{aligned} 2(K_l-K_s)\sum mJ_m^s g(M,m)\frac{a}{\pi} &= -M^2I_M^{s-1}-aM(K_l+K_s)I_M^s\ , \\ 2(K_l-K_s)\sum mI_m^s h(M,m)\frac{a}{\pi} &= -M^2J_M^{s-1}-aM(K_l+K_s)J_M^s\ , \end{aligned}\right\} \tag{61}$$

where $$s>0\ .$$

Although we are to obtain the decay-mode of the induced field by solving the simultaneous equations (61), the power series (47) become divergent when K_s much exceeds K_l. Assuming that the series (47) are convergent, the magnetic potential of the induced field is given as

$$\begin{aligned} W_i = &\left[1-3.7602\frac{t}{a(K_s-K_l)}+0.7062\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\cos px \\ &+\left[\qquad\qquad\qquad 0.0049\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\cos 3px \\ &+\left[\qquad\qquad\qquad 0.0089\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\cos 5px \\ &+\ldots\ldots\ldots \end{aligned}$$

$$+\left[\quad 0.0531\frac{t}{a(K_s-K_l)}-0.1194\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\sin 2px$$
$$+\left[\quad 0.0107\frac{t}{a(K_s-K_l)}-0.0281\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\sin 4px$$
$$+\left[\quad 0.0066\frac{t}{a(K_s-K_l)}-0.0247\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\sin 6px$$
$$+\left[\qquad\qquad -0.0128\left\{\frac{t}{a(K_s-K_l)}\right\}^2+\ldots\ldots\right]E\sin 8px$$
$$+\ldots\ldots\ldots \qquad . \quad (62)$$

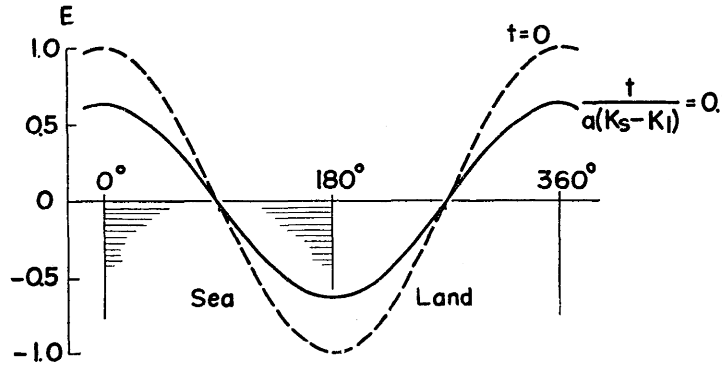

Fig. 16. The distribution of the induced (broken line) magnetic potential or current function at $t=0$, while that at $t=1$ *sec.* is shown by the full line. The conductivities are taken to be 2×10^{-7} and 10^{-7} *emu* respectively and a is assumed to be 10^8 *cm.*

One example of the decay-mode is shown in Fig. 16 in which we see that the effect of the difference of conductivity is very small.

Summarizing all the results for various cases, we see that the initial distribution of the induced field is exactly the same as that for a uniformly conducting sheet, provided there is a finite conductivity on the land. As far as a finite conductivity is concerned, however small it is, we have the same distribution. But if we put the conductivity on the land absolutely zero, the situation becomes greatly different. Physically speaking, the initial state of the induced field should soon decay changing into the pattern of the distribution which is described in the case of the absolutely non-conducting land, though it is difficult to trace the process of such a rapid change because of mathematical difficulties. It is also found that the general feature of the effect of the sea is almost the same in the neighbourhood of the boundary between sea and land both for the semi-spherical and plane models.

Discussions and conclusions

The results of the observations newly added do not materially change the statistical distribution of $\Delta Z/\Delta H$ which was already reported in the third paper. In spite of the observation on Oosoima Island, we could not determine the southern extent of the anomalous area in which we observe anomalously large ΔZ. We have to wait for the coming observations that will be carried out on other islands farther in the south. After those observations we might be able to say something definite about the southern boundary of the area.

Almost all the variations used in the statistics, however, are bay disturbances whose magnetic vector is directed approximately towards the north. In this paper, the writers are specially interested in cases in which the changing vectors are in a direction perpendicular to the above-examined direction. Although those cases are not many because bays usually occur during nights when the vectors lie in the north and south direction, we sometimes observed a different distribution in these cases. It is surprising that we should have observed fairly large amplitude for ΔZ at Maze (a station at the coast of Japan Sea) while ΔZ at Aburatsubo (at the coast of the Pacific Ocean) is practically zero, the direction of the magnetic vector being nearly east- or westwards in these cases. For the intermediate cases when we have the directions of the magnetic vectors that deviate from the above two directions, we observed ΔZ of either the same or the opposite sign respectively at the two stations. Thus we found that the distribution of the vertical component depends on the magnetic field applied from the outside.

In Germany, a similar anisotropy has been found. For instance, we have to suppose an induced electric current flowing in a direction from the west to the east underneath North Germany whenever the ionospheric current which is regarded as the origin of the inducing field flows from the south to the north. In the case of uniformly conducting earth, the induced currents should flow from the north to the south in this case. So it is obvious that there is an apparent anisotropy. However, the mode of distribution is not clear when the inducing field is perpendicular to the above-mentioned direction.

For the purpose of discussing the cause of the phenomenon, it becomes of importance to know all the behaviours of the magnetic field for the inducing field of any direction. Through the investigations carried out in Chapter I of this paper, the characteristics can be expressed by empirical formulae which are obtained for respective stations. The possible cause of the phenomenon should be presumed on the basis of these considerations.

Since the observations are not enough for presuming the cause, we can only speculate a possible model from the observed facts which have been hitherto accumulated. First of all, we have to imagine a roughly circular circuit under central Japan in order to understand the large ΔZ observed there. The electric current flowing along the circuit should be clockwise when it is viewed from above while the inducing magnetic field is directed to the north. In order to give rise to a vertical magnetic force of 20 *gammas*, the intensity of the current must be about 5,000 *amperes*, the diameter of the circuit being roughly a few hundred *kilometers*. It is not possible to suppose the induced currents in the specified direction from the electromagnetic induction theory even if we have good conductors along the circuit. Suppose the circuit is a ring-shaped conductor in the horizontal plane and the inducing field has some upward component such as is the case for usual bays, there will be some electric currents induced in the circuit. But we can not expect such an intense electric current required here. The only plausible interpretation is to suppose that the electric currents come from the conducting part of the deep interior along paths which are connected with the above-mentioned circuit. It is well known that the earth is conducting under the depth of several hundred *kilometers*. Therefore the electric currents induced there might come up near the earth's surface provided there is a conducting passage. Although it is rather artificial to suppose such a distribution of the conductivity, it is difficult to find any other explanation.

If we assume a suitable size and position of the circuit, the magnetic field due to the current in this hypothetical circuit may be consistent with the anisotropy examined in this paper. From the quantitative standpoint it is still unknown whether or not we have enough electric currents accounting for the anomalous magnetic field. T. Nagata and T. Oguti[23] made a preliminary experiment in order to examine the possibility of the present model. It is tentatively concluded that there is such a distribution of the conductivity as considered here from which we might expect an anomalous magnetic field as has been observed in

23) T. NAGATA and T. OGUTI, Unpublished.

Japan. It would perhaps be necessary to suppose a low conducting region wedged into the deep earth, at a depth of several hundred *kilometers*, just under the above-considered circuit, in order to make intense current flow along the circuit.

The speculation stated above should be criticized from geological and geophysical standpoints. The high conducting passage and circuit may correspond to some high temperature belts distributed in a special way. But there seems no serious objection to that sort of distribution though it is rather an extraordinary phenomenon. Until we carry out many more observations, we can not say anything definite about the cause of the anomaly considered here.

In Chapter II of this paper, the influence of the electric currents induced in the sea has been examined in detail. Since the effect of self-induction is fully taken into account, the result can be applied to rapid variations. It has been found that the influence of the sea is not of the type that reinforces the anomalous distribution of the magnetic variation in Japan. On the contrary, it diminishes the amplitude of ΔZ in the central part of Japan. So we need not hereafter worry about the influence of the sea.

In conclusion, the writers are grateful to Professor T. Nagata and Mr. T. Oguti who discussed this problem on the basis of their experiments. The writers are also indebted to Professor J. Bartels, Dr. U. Fleischer, Dr. H. Wiese, Sir Edward Bullard, Professor A.T. Price, Professor C. Tsuboi, and Professor H. Kuno with whom the writers had the pleasure of discussing the problem through personal talk or communication. A part of this study was carried out with the financial aid of the Research Grant from the Ministry of Education. The writers would like to extend their thanks to the Ministry.

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23. 日本に於ける地磁気短周期変化の異常と地下構造 (第6報)
(その後の観測結果及び地磁気変化におよぼす海の影響)

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日本における地磁氣短周期変化の異常に関連して，伊豆大島に臨時観測所を設けて約2ヶ月連続観測を行つた．大島においても ΔZ の ΔH に対する著しい相似性と，大きな比を認めた．即ち大島も油壷，柿岡を含む異常地域に属するものである．

油壷，間瀬においてその後えられた観測結果を加えて，新たに統計的に調べると，その地磁氣変化にそれぞれ異方性が認められる．両者とも，理論的に求められた正常分布からそれぞれの様相を以て偏倚している．

地磁氣変化に対する海の影響を，次の 2 つの場合に分けて論じた．すなわち地球が半球殻の海を有するとして，大洋の電磁感応的影響を球殻に関して論ずると同時に，陸，海と周期的に電氣伝導度が分布するとして，小規模な海岸線の電磁感応的影響を平板に関して論じた．両者とも明らかに陸地の海岸周辺にいわゆる“海岸線効果”とも稱すべき異常を生ずることを知つたが，この異常は吾々が日本中部において観測する異常とは正に反対向きのものである．以上からして，海の影響として観測事実を説明しようとする説は否定されるが，現段階において一応吾々が日本中部の地下構造に関して抱きうるであろう推測の一つを述べた．