3. *Analysis of Dispersed Surface Waves by means of Fourier Transform I.*

By Yasuo SATÔ,

Earthquake Research Institute.

(Read Dec. 22, 1953.—Received Jan. 6, 1955.)

1. Introduction

From the beginning of our investigation, it has been one of the main objects of seismometry to examine the mechanism of earthquakes and the nature of media through which the waves are propagated. Of course, we have several kinds of waves available for this purpose, and P-waves have been often used successfully[1], while the surface waves are not so easily employed as the bodily waves[2]. This is due chiefly to the complex structure of the earth's crust and the dispersive nature of Rayleigh- as well as Love-waves. If the analysis of the seismograms of the dispersed surface waves is done with success, the above object will be partly accomplished. The following procedure is merely an attempt in this direction of our investigations.

The analysis has its foundation upon the very simple principle that the amplitude and the phase angle of the Fourier transform of the curve observed at any station express the spectrum near the origin and the phase shift caused during the propagation respectively.

2. Fundamental concepts

At first we will consider the wave motion in one-dimensional space. Let us assume the movement at $x=0$ to be

$$f(t)=f(t\,;0)\,. \qquad \text{..............(2.1)}$$

This function is expressed as is well known from Fourier's double integral theorem,

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1) We can find an enormous number of papers and reports in seismic prospecting, which all utilize the P-waves.

2) References in this branch of seismology is found, for example, in T. Akima's paper. *Bull. Earthq. Res. Inst.*, **30** (1952), 237.

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f^*(p) \exp(ipt) dp \ ,$$

in which .... (2.2)

$$f^*(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\tau) \exp(-ip\tau) d\tau \ .$$

The physical meaning of this expression is as follows.

An arbitrary function $f(t)$ is (if some conditions are satisfied) represented as the sum of the waves with frequency p and amplitude $f^*(p)/\sqrt{2\pi}$, while the function $f^*(p)$, which is named as Fourier transform, is also expressible in a similar form as shown in the latter expression of (2.2).

Now, if a wave $A \exp(ipt)$ is propagated by the velocity $V(p)$, it takes the form $A \exp\{ip(t-x/V(p))\}$ at the distance x. Therefore, if the disturbance at $x=0$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f^*(p) \exp(ipt) dp$$

is propagated, the wave form at $x=x$ is

$$f(t\,; x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f^*(p) \exp\{ip(t-x/V(p))\} dp$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f^*(p) \exp\{-ipx/V(p)\}] \cdot \exp(ipt) dp \ . \quad (2.3)$$

The Fourier transform of this function is

$$f^*(p\,; x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\tau\,; x) \exp(-ip\tau) d\tau \ , \qquad \ldots\ldots (2.4)$$

which can be also expressed, by substituting the above expression, as follows.

$$f^*(p\,; x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-ip\tau) d\tau$$

$$\cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f^*(p') \exp\{-ip'x/V(p')\}] \cdot \exp(ip'\tau) dp'$$

$$= f^*(p) \exp\{-ipx/V(p)\} \ . \qquad \ldots\ldots\ldots\ldots\ldots (2.5)$$

Since $f(t\,; x)$ is the wave form observed at $x=x$, the above function can be calculated from the seismogram of that station, namely from (2.4)

$$f^*(p\,; x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(\tau\,; x) \cos(p\tau) d\tau$$

$$-i\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(\tau\,;x)\sin(p\tau)d\tau$$

$$\equiv c(p\,;x)-is(p\,;x)\,. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.6)$$

Now putting

$$f^*(p)=F(p)\exp\{-i\beta(p)\} \qquad \ldots\ldots\ldots\ldots\ldots(2.7)$$

we have

$$f^*(p\,;x)=F(p)\exp\{-i\beta(p)\}\cdot\exp\{-ipx/V(p)\}$$

and $$F(p)^2=c(p\,;x)^2+s(p\,;x)^2\,, \qquad \ldots\ldots\ldots\ldots\ldots(2.8)$$

where $F(p)$ and $\beta(p)$ are assumed both to be real.

From the expressions (2.6) and (2.7) we have easily

$$\mathfrak{C}(p)\equiv-\arg f^*(p\,;x)$$

$$=\arctan\{s(p\,;x)/c(p\,;x)\}$$

$$=\beta(p)+px/V(p)\,, \qquad \ldots\ldots\ldots\ldots(2.9)$$

where the inverse-trigonometric function does not necessarily take the principal value.

The function $F(p)$ is the spectrum of the wave observed at $x=x$, and also shows the spectrum of the vibration at the origin. The propagation velocity $V(p)$ with frequency p is from (2.9) expressed as

$$V(p)=\frac{px}{-\beta(p)+\arctan\{s(p\,;x)/c(p\,;x)\}} \qquad \ldots\ldots(2.10)$$

or, if we define ARCTAN z so as to satisfy

$$-\pi<\mathrm{ARCTAN}\, z\leq\pi\,, \qquad \ldots\ldots\ldots\ldots(2.11)$$

we have

$$V(p)=\frac{px}{-\beta(p)+\mathrm{ARCTAN}\{s(p\,;x)/c(p\,;x)\}+2n\pi} \qquad \ldots(2.12)$$

But we must add some remarks concerning the practical way of calculating the velocity. For the angle $\beta(p)$ can never be obtained from a single observation and therefore we cannot get $V(p)$ from the formula (2.12) if we do not neglect $\beta(p)$.

In order to perform an exact calculation, we are to choose at least two observations at different places, namely

$$\mathfrak{C}(p)^{(1)}=\beta(p)+px^{(1)}/V(p)$$

and $$\mathfrak{C}(p)^{(2)}=\beta(p)+px^{(2)}/V(p) \qquad \ldots\ldots\ldots\ldots(2.13)$$

Taking the difference of two expressions we have

$$\mathfrak{C}(p)^{(1)}-\mathfrak{C}(p)^{(2)}=p(x^{(1)}-x^{(2)})/V(p) \qquad \ldots\ldots.(2.14)$$

from which we get

$$V(p)=\frac{p(x^{(1)}-x^{(2)})}{\mathfrak{C}(p)^{(1)}-\mathfrak{C}(p)^{(2)}}, \qquad \ldots\ldots\ldots\ldots\ldots.(2.15)$$

or

$$V(p)=\frac{p(x^{(1)}-x^{(2)})}{\mathrm{ARCTAN}\left\{\dfrac{s(p\,;x^{(1)})}{c(p\,;x^{(1)})}\right\}-\mathrm{ARCTAN}\left\{\dfrac{s(p\,;x^{(2)})}{c(p\,;x^{(2)})}\right\}+2m\pi}. \quad (2.16)$$

If three or more observations are available and the dispersion formula can be assumed to be common to all of them, we can apply the method of least squares to the expressions of the type (2.13), and determine the unknown parameters $\beta(p)$ and $1/V(p)$, or $V(p)$.

3. Cylindrically-spreading waves

In the previous section we have treated the analysis of waves propagated one-dimensionally; we will now modify this theory to discuss the waves spreading in a two-dimensional space.

Cylindrically-spreading waves of course decrease their amplitude because their energy is distributed over the enlarging wave front. Since, in the two-dimensional propagation, the wave front increases proportionally to the epicentral distance, $f(t\,;r)$ must involve a factor $1/\sqrt{r}$. (Where r is the epicentral distance.) Consequently, if some wave train, whose amplitude at $r=r_1$ is A, is propagated and arrives at a point $r=r_2$, its amplitude becomes $A\sqrt{r_1/r_2}$.

Taking the above nature into consideration we will formulate expressions similar to the case of one-dimensional propagation.

Let us assume at first the movement at a point $r=r_0$, which is situated near the origin, to be

$$f(t\,;r_0)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f^*(p\,;r_0)\exp(ipt)dp\,, \qquad \ldots.(3.1)$$

in which

$$f^*(p\,;r_0)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;r_0)\exp(-ip\tau)d\tau\,, \qquad ..(3.2)$$

then the expression corresponding to the equation (2.3) is

$$f(t\,;r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sqrt{\frac{r_0}{r}}\,f^*(p\,;r_0)\exp\left\{ip(t-\frac{r-r_0}{V(p)})\right\}dp$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sqrt{\frac{r_0}{r}}\,f^*(p\,;r_0)\exp\left\{-ip\frac{r-r_0}{V(p)}\right\}\right]\cdot\exp(ipt)dp \qquad \ldots(3.3)$$

Now the Fourier transform of this function is

$$f^*(p\,;r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;r)\exp(-ip\tau)d\tau$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(-ip\tau)d\tau$$

$$\cdot\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\sqrt{\frac{r_0}{r}}f^*(p'\,;r_0)\exp\left\{-ip'\frac{r-r_0}{V(p')}\right\}\right]\cdot\exp(ip'\tau)dp'$$

$$=\sqrt{\frac{r_0}{r}}f^*(p\,;r_0)\exp\left\{-ip\frac{r-r_0}{V(p)}\right\} \qquad \ldots\ldots\ldots\ldots\ldots(3.4)$$

If we compare (3.2) with (2.4) and (3.4) with (2.5), we find at a glance that the modification is very slight and we can immediately obtain the formula applicable to the analysis of the problem in two dimensions.

We put as before (cf. (2.6)~(2.9))

$$f^*(p\,;r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;r)\exp(-ip\tau)d\tau$$

$$\equiv c(p\,;r)-is(p\,;r)\,, \qquad \ldots\ldots\ldots\ldots\ldots(3.5)$$

$$F(p\,;r)^2\equiv c(p\,;r)^2+s(p\,;r)^2\,,$$

and

$$\mathfrak{C}(p)\equiv-\arg f^*(p\,;r)$$

$$=\arctan\{s(p\,;r)/c(p\,;r)\}$$

$$=\beta(p)+\frac{p(r-r_0)}{V(p)}\,, \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.6)$$

then the spectrum distribution at the place with the epicentral distance $r=r_0$ is easily obtained.

$$F(p\,;r_0)=\sqrt{\frac{r}{r_0}}F(p\,;r)\,. \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.7)$$

The following treatments concerning the velocity are quite same with the process shown in the equations (2.10)~(2.12), and the formulae (2.13)~(2.16) can be applied in this case, too.

If we neglect the phase angle $\beta(p)$ and $pr_0/V(p)$ compared with $pr/V(p)$ we have immediately the next simple formula

$$V(p)=\frac{pr}{\mathfrak{C}(p)}=\frac{pr}{\text{ARCTAN}\{s(p\,;\,r)/c(p\,;\,r)\}+2n\pi}, \qquad (3.8)$$

while, if a more presise estimation is required, we must employ the formulae (2.15) and (2.16), where $x^{(1)}$ and $x^{(2)}$ should be replaced by $r^{(1)}$ and $r^{(2)}$ respectively. Namely

$$V(p)=\frac{p(r^{(1)}-r^{(2)})}{\text{ARCTAN}\left\{\dfrac{s(p\,;\,r^{(1)})}{c(p\,;\,r^{(1)})}\right\}-\text{ARCTAN}\left\{\dfrac{s(p\,;\,r^{(2)})}{c(p\,;\,r^{(2)})}\right\}+2m\pi}. \qquad (3.9)$$

4. Approximation formula for the integration

In the previous sections the integral of the form

$$f^*(p\,;\,r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau\,;\,r)\exp(-ip\tau)d\tau$$
$$=c(p\,;\,r)-is(p\,;\,r) \qquad \ldots\ldots\ldots\ldots\ldots(4.1)$$

often appeared, which must be calculated in order to perform the treatment stated. Numerical integration by means of trapezoidal or Simpson rule can be of course utilized, but when the wave takes some simple form, we can approximate to this curve by some simple function and perform the integration more conveniently. For example, if the wave is, as shown in Fig. 1, consisted of sinusoidal curve with slowly varying amplitude and period, then we can approximate to it by

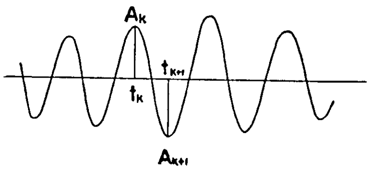

Fig. 1.

$$f(t\,;\,r)=\begin{cases}0 & \ldots\ldots\ldots t<t_0\,,\\ \frac{1}{2}(A_k+A_{k+1})+\frac{1}{2}(A_k-A_{k+1})\cos\left\{\pi\frac{t-t_k}{t_{k+1}-t_k}\right\} & \\ & \ldots\ldots\ldots t_k<t<t_{k+1},\ (k=0,\,1,\,..,n-1),\\ 0 & \ldots\ldots\ldots t_n<t\,. \end{cases} \qquad \ldots\ldots\ldots(4.2)$$

Hitherto, in such a case, we customarily used the method to assume the time interval $(t_{k+1}-t_k)$ as an approximate value of the period of waves, and obtained the velocity corresponding to this assumed period.

But here, we introduced the expression (4.2) into the above formula and obtained

$$\left\{\begin{aligned} & c(p\,;r)=\sum_{k=0}^{n-1} I_{Ck}, \quad s(p\,;r)=\sum_{k=0}^{n-1} I_{Sk} \\ & I_{Ck}=\int_{t_k}^{t_{k+1}}\left[\frac{1}{2}(A_k+A_{k+1})+\frac{1}{2}(A_k-A_{k+1})\cos\left\{\pi\frac{\tau-t_k}{t_{k+1}-t_k}\right\}\right]\cos p\tau\, d\tau\,, \\ & I_{Sk}=\int_{t_k}^{t_{k+1}}\left[\frac{1}{2}(A_k+A_{k+1})+\frac{1}{2}(A_k-A_{k+1})\cos\left\{\pi\frac{\tau-t_k}{t_{k+1}-t_k}\right\}\right]\sin p\tau\, d\tau\,. \end{aligned}\right.$$

.....(4.3)

Then I_{Ck} and I_{Sk} can be integrated and after a somewhat complicated but easy calculation

$$\begin{aligned} 2pI_{Ck} &= -2A_k \sin pt_k + 2A_{k+1}\sin pt_{k+1} \\ &\quad -\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\sin pt_k+\sin pt_{k+1})\,, \\ 2pI_{Sk} &= 2A_k\cos pt_k - 2A_{k+1}\cos pt_{k+1} \\ &\quad +\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\cos pt_k+\cos pt_{k+1})\,, \end{aligned}$$

...........(4.4)

where $\quad c_k=t_{k+1}-t_k$.

Therefore, summing up the above expressions,

$$\begin{aligned} 2p\sum_{k=0}^{n-1} I_{Ck} &= -2A_0\sin pt_0+2A_n\sin pt_n \\ &\quad -\sum_{k=0}^{n-1}\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\sin pt_k+\sin pt_{k+1}) \\ 2p\sum_{k=0}^{n-1} I_{Sk} &= 2A_0\cos pt_k-2A_n\cos pt_k \\ &\quad +\sum_{k=0}^{n-1}\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\cos pt_k+\cos pt_{k+1}) \end{aligned}$$

..............(4.5)

If we can assume that

$$A_0=A_n=0 \qquad \text{...(4.6)}$$

as is in the Fig. 2, the above expression may be reduced to the next simple form.

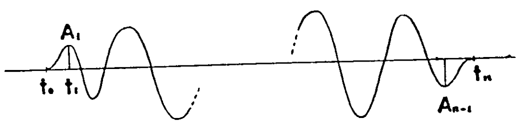

Fig. 2

$$\left\{\begin{aligned} & \sum_{k=0}^{n-1} I_{Ck}=c(p\,;r)=-\frac{1}{2p}\sum_{k=0}^{n-1}\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\sin pt_k+\sin pt_{k+1}) \\ & \sum_{k=0}^{n-1} I_{Sk}=s(p\,;r)=+\frac{1}{2p}\sum_{k=0}^{n-1}\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\cos pt_k+\cos pt_{k+1}) \end{aligned}\right.$$

..........(4.7)

If

$$pt_{k+1}=pt_k+\pi, \text{ or } pc_k/\pi=1 \ , \qquad \ldots\ldots\ldots\ldots\ldots(4.8)$$

$(\sin pt_k+\sin pt_{k+1})$, $(\cos pt_k+\cos pt_{k+1})$ and $\{1-(pc_k/\pi)^2\}$ all vanish and the $(k+1)$-th term in the above summation becomes indefinite. In this case we must employ the expressions

$$-(A_k-A_{k+1})\frac{\pi}{2}\cos pt_k \text{ and } (A_k-A_{k+1})\frac{\pi}{2}\sin pt_k \qquad \ldots\ldots(4.9)$$

instead of the terms

$$\frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\sin pt_k+\sin pt_{k+1}) \text{ and } \frac{A_k-A_{k+1}}{(pc_k/\pi)^2-1}(\cos pt_k+\cos pt_{k+1})$$

respectively. These formulae are easily obtained as the limit

$$p(t_{k+1}-t_k)\to\pi \ .$$

In some case the next formulae may be more convenient than the expression (4.7)

$$\left\{\begin{aligned} c(p\,;r)&=-\frac{1}{p}\sum(A_k-A_{k+1})\frac{\cos(pc_k/2)}{(pc_k/\pi)^2-1}\sin pt_{k+1/2} \ ,\\ s(p\,;r)&=+\frac{1}{p}\sum(A_k-A_{k+1})\frac{\cos(pc_k/2)}{(pc_k/\pi)^2-1}\cos pt_{k+1/2} \ , \end{aligned}\right. \qquad \ldots(4.10)$$

where $\qquad t_{k+1/2}=(t_k+t_{k+1})/2.$

5. Analysis of a practical record of explosion

We applied the above theory in the analysis of the practical record made by F. Kishinouye and reported some ten years ago.[3] It is a small explosion which took place on February, 1942 at Lake Haruna, and the details of the experiment can be found in the original paper. But we must notice here that the actual record of this experiment contains some regular oscillation with a constant period and amplitude, such as hum,[4] so we omitted this vibration and obtained the curves by hand-writing, which are given in Fig. 3. The epicentral distance is 141.75 m for A and 76.2 m for B respectively.

We did not integrate these curves in order to get the true vibration of the particle. For the records were obtained using an oscillograph

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3) F. KISHINOUYE, "Studies on Lake-Ice," *Bull. Earthq. Res. inst.*, **21** (1943), 298.

4) F. KISHINOUYE, *ibid.* See Fig. 3 on Page 304 in this paper.

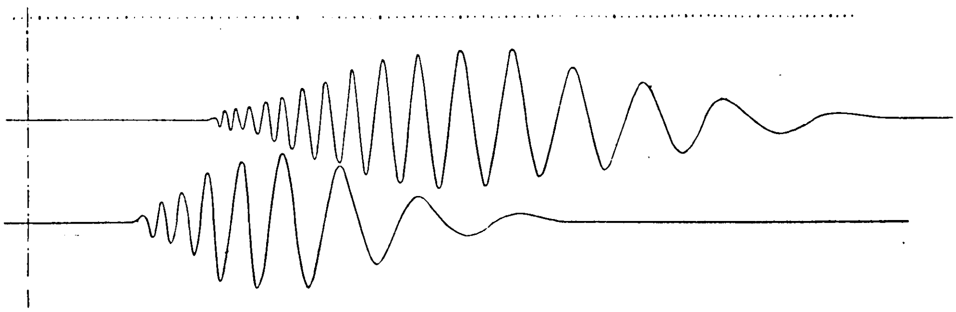

Fig. 3. The upper dotted line is the time mark. One interval is 0.01 sec. At every 0.1 sec. there is a large mark.
The middle curve shows the observation A. ($r=141.75$ m.)
The lower curve shows the observation B. ($r=76.2$ m.)
The chain line at the left indicates the shot time.

and an amplifier involving vacuum tubes, and thus their precise characteristics now cannot be clarified enough to use for the reduction of the records. Consequently we gave up such treatment and used them as they were. The results, however, seem not to contain so much error as to affect the conclusion perceptively.

We utilized the formulae introduced in the previous section for the numerical integration of the curves in Fig. 3. The data used for the calculation is given in Table I. Introducing the numerical values into (4.7), and if necessary by the aid of (4.8) and (4.9), we can obtain

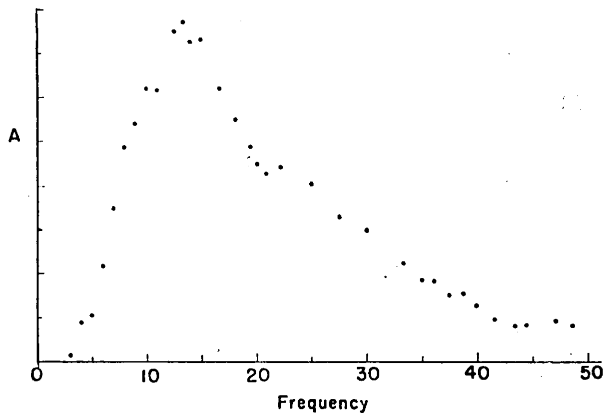

Fig. 4a.

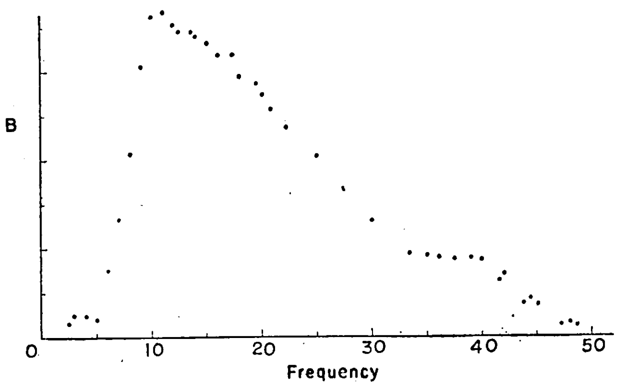

Fig. 4b. Ordinate; Amplitude in arbitrary scale.
Abscissa; Frequency per second.

Table I.

	A (r=141.75 m)				B (r=76.2 m)			
k	t_k	$t_{k+1}-t_k$ $=c_k$	A_k	A_k-A_{k+1}	t_k	$t_{k+1}-t_k$ $=c_k$	A_k	A_k-A_{k+1}
0	21.1	0.5	0	—0.9	12.1	1.2	0	—2.0
1	21.6	0.6	0.9	3.0	13.3	1.0	2.0	6.5
2	22.2	0.6	—2.1	—5.3	14.3	1.1	—4.5	—10.6
3	22.8	0.6	3.2	5.7	15.4	1.0	6.1	12.2
4	23.4	0.7	—2.5	—6.3	16.4	1.2	—6.1	—14.9
5	24.1	0.8	3.8	6.3	17.6	1.6	8.8	17.9
6	24.9	0.8	—2.5	—6.7	19.2	1.5	—9.1	—23.3
7	25.7	0.9	4.2	8.2	20.7	1.5	14.2	30.9
8	26.6	0.9	—4.0	—9.7	22.2	2.4	—16.7	—34.3
9	27.5	0.9	5.7	11.7	24.6	2.0	17.6	36.3
10	28.4	1.0	—6.0	—13.0	26.6	2.8	—18.7	—38.7
11	29.4	1.3	7.0	15.7	29.4	3.2	20.0	38.5
12	30.7	1.2	—8.7	—17.7	32.6	3.7	—18.5	—35.1
13	31.9	1.4	9.0	20.0	36.3	4.8	16.6	28.6
14	33.3	1.3	—11.0	—22.3	41.1	4.7	—12.0	—19.9
15	34.6	1.7	11.3	23.7	45.8	6.5	7.9	11.9
16	36.3	1.6	—12.4	—27.0	52.3	6.3	—4.0	—6.2
17	37.9	1.7	14.6	30.5	58.6	7.4	2.2	2.2
18	39.6	2.1	—15.9	—33.8	66.0		0	
19	41.7	2.0	17.9	36.0				
20	43.7	2.2	—18.1	—36.8				
21	45.9	2.6	18.7	38.1				
22	48.5	2.8	—19.4	—39.4				
23	51.3	3.3	20.0	39.0				
24	54.6	3.5	—19.0	—39.0				
25	58.1	3.6	20.0	36.8				
26	61.7	4.1	—16.8	—31.7				
27	65.8	4.4	14.9	29.3				
28	70.2	5.2	—14.4	—25.3				
29	75.4	5.6	10.9	20.2				
30	81.0	6.2	—9.3	—13.3				
31	87.2	7.3	4.0	8.1				
32	94.5	7.3	—4.1	—5.8				
33	101.8	9.3	1.7	1.7				
34	111.1		0					

Unit of time is 1/100 sec.
Unit of length is mm. on the original seismogram.

Table II.

		A ($r=141.75$ m)				B ($r=76.2$ m)				*
p(cycle)	T(sec)	$-p\cdot c(p;r)$	$p\cdot s(p;r)$	$\|f^*(p;r)\|$	arg $f^*(p;r)$	$-p\cdot c(p;r)$	$p\cdot s(p;r)$	$\|f^*(p;r)\|$	arg $f^*(p;r)$	
2.5	0.40	-4.80	-1.60	2.00		0.27	-0.82	0.32	-1.89	
3	0.333	0.45	0	0.17	3.14	-1.45	-0.19	0.49	-0.13	3.27
4	0.25	-2.05	4.20	1.18	1.12	2.13	-0.37	0.52	-2.97	4.09
5	0.2	6.70	2.40	1.42	2.79	0.64	1.86	0.40	1.90	1.89
6	0.167	15.5	8.2	2.91	2.65	-6.30	-7.8	1.67	-0.89	3.54
7	0.143	2.6	32.6	4.67	1.65	10.9	15.1	2.66	2.20	2.47
8	0.125	-39.2	34.5	6.52	0.72	-20.6	-23.8	4.14	-0.86	1.58
9	0.111	-55.5	-33.7	7.21	-0.55	24.2	49.0	6.08	2.03	-2.58
10	0.1	48.2	-67.4	7.28	-2.19	9.0	-71.9	7.25	-1.70	-0.25
11	0.0909	41.8	80.4	8.28	2.05	-60.1	53.9	7.34	0.73	1.32
12	0.0833					84.4	6.4	7.06	3.07	
12.5	0.08	62.4	109.1	10.05	2.09	41.4	-75.4	6.88	-1.97	4.06
13.333	0.075	-60.0	-123.8	10.31	-1.12	-86.4	-20.0	6.58	-0.23	-0.89
13.888	0.072	-61.4	120.1	9.71	1.10	-52.2	78.8	6.82	0.99	0.11
15	0.0667	-43.4	-139.6	9.75	-1.27	98.6	-14.0	6.64	-3.00	1.73
16	0.0625					-50.4	-89.4	6.42	-1.06	
16.666	0.06	45.3	-128.1	8.17	-1.91	-109.2	10.3	6.59	0.09	-2.00
18.055	0.0554	103.3	82.2	7.31	2.47	93.5	51.2	5.91	2.64	-0.17
19.444	0.0514	-116.6	46.3	6.45	0.38	-43.7	-102.2	5.73	-1.17	1.55
20	0.05	52.6	107.0	5.96	2.03	-108.3	-19.5	5.50	-0.18	2.21
20.833	0.048	14.6	-118.2	5.72	-1.69	-30.1	103.0	5.15	1.29	-2.98
22.222	0.045	65.2	114.2	5.92	2.09	101.0	-30.6	4.75	-2.85	4.94
25	0.04	134.0	-23.1	5.44	-2.97	-30.9	97.8	4.10	1.26	-4.23
27.5	0.0364	121.8	-27.4	4.40	-2.92	-10.0	-91.6	3.35	-1.46	-1.46
30	0.0333	118.5	-0.4	3.95	-3.14	35.4	71.3	2.65	2.03	-5.17
33.333	0.03	34.0	-92.6	3.34	-1.92	-62.7	7.0	1.89	0.11	-2.03
35	0.0286	0.6	86.1	2.46	1.58	24.5	59.8	1.85	1.96	-0.38
36.111	0.0277	68.4	-57.2	2.47	-2.44	64.0	5.7	1.78	3.05	-5.49
37.5	0.0267	-65.1	38.4	2.02	0.53	4.0	-66.0	1.76	-1.63	2.16
38.888	0.0257	78.9	-17.3	2.08	-2.92	-69.4	-6.1	1.79	-0.09	-2.83
40	0.025	-54.1	-41.0	1.70	-0.65	-25.6	64.2	1.73	1.19	-1.84
41.666	0.024	43.6	31.0	1.28	2.52	52.5	-5.9	1.27	-3.03	5.55
42	0.0238					56.0	-15.1	1.38	-2.88	
43.555	0.0230	-23.3	-43.7	1.14	-1.08					
44.444	0.0225	-0.1	52.5	1.18	1.57	-37.8	-6.5	0.86	-0.17	1.74
45	0.0222					-28.2	12.5	0.68	0.42	
47.222	0.0212	-56.2	18.5	1.25	0.32	2.6	10.6	0.23	1.81	-1.49
48	0.0208					6.5	13.8	0.32	2.01	
48.611	0.0206	53.4	12.0	1.12	2.92	1.5	12.8	0.27	1.69	1.23

*ARCTAN$\{s(p;r_A)/c(p;r_A)\}$ − ARCTAN$\{s(p;r_B)/c(p;r_B)\}$

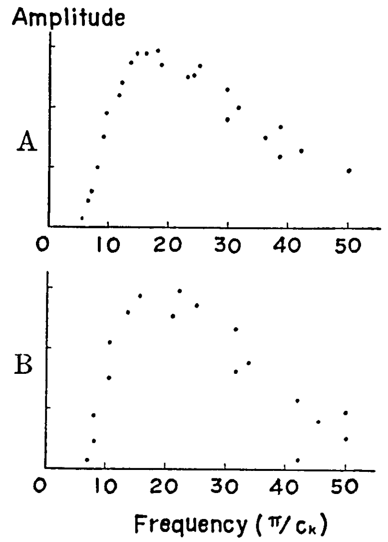

Fig. 5.

Ordinate; Amplitude in arbitrary scale.

Abscissa; Frequency per second.

Notice that the form indicated by this method is similar to that in Fig. 4.

$c(p;r)$ and $s(p;r)$, which are shown in Table II.

By means of these results, we can determine the spectrum distribution function $F(p;r)$ and the propagation velocity $V(p)$. $F(p;r)$ is obtained from (3.4) and (3.5) and is shown in Fig. 4, while in Fig. 5 the graph A_k versus π/c_k is given, which may also serve as a kind of spectrum distribution for a rough purpose. $V(p)$ is derived from (3.9). But here occures some unexpected difficulty in determining the numerical value, which is caused by the many-valuedness of the inverse trigonometric function. Although we can determine ARCTAN $\{s(p;r)/c(p;r)\}$ uniquely, there is uncertainty of $2m\pi$ in the determination of $\mathfrak{C}(p)$ which is necessary for obtaining the velocity $V(p)$. (Cf. (2.16) and (3.9). ARCTAN $\{s(p;r_A)/c(p;r_A)\}$ $-$ ARCTAN $\{s(p;r_B)/c(p;r_B)\}$ is involved in Table II.) Hence we adopted the following method. We gave various values of m in the formula (3.9) for each p, and plotted all the points in the same figure. Thus we obtained Figs. 6a and 6b, which have same

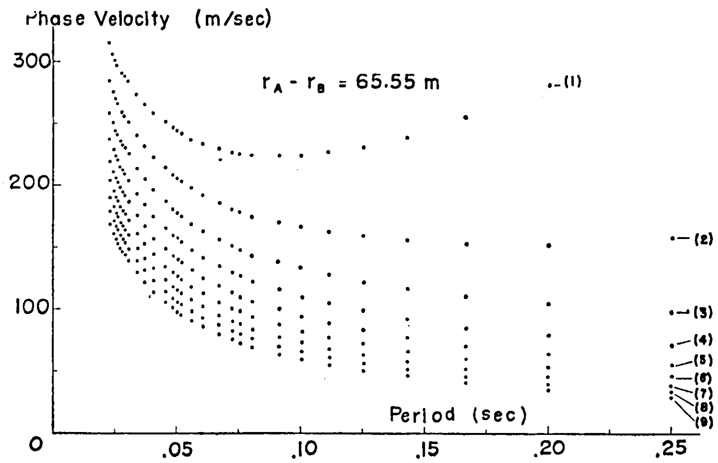

Fig. 6a.

contents, only the latter is plotted in log-log-scale. The points in this figure are naturally divided into several groups, each one describing a smooth curve. Of course, only one of them stands for the true dispersion curve, but we cannot easily determine which is the true one. To get rid of the false curves and discern the true one, we employed the following methods.

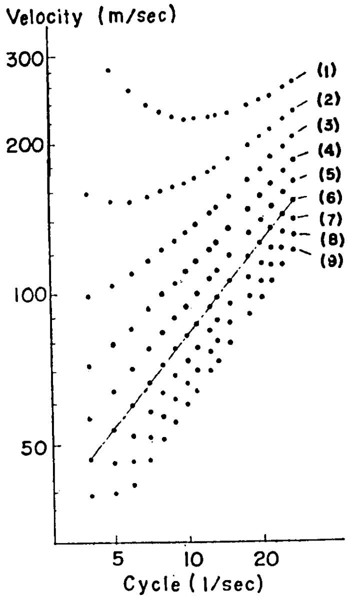

Fig. 6b.

At first we made a graph showing the relation group velocity versus period by means of a conventional way of measuring the interval of crest to crest. This is easily performed without special trouble, for we can use $2(t_{k+1}-t_k)$ as a period and r/t_k as a group velocity. Fig. 7 shows this relation. On the other hand we obtained the group velocity (U) from the data of the phase velocity. For this purpose we can conveniently use Fig. 6b. As we can notice easily in this figure log $V(p)$ and log p are approximately connected by a linear relation, namely,

$$\log V(p)=a\log p+b\,. \qquad \ldots\ldots\ldots\ldots(5.1)$$

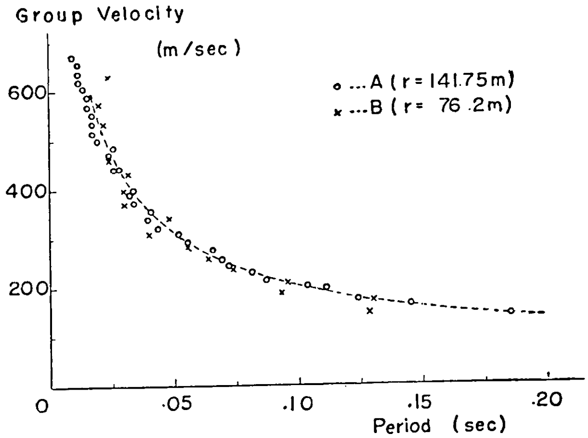

Fig. 7. Broken line is the group velocity of the curve (6) in Fig. 6a or 6b.

Now,

$$\frac{U}{V}=\frac{1}{V}\frac{dp}{df}$$

$$=\frac{1}{V}\frac{dp}{dV}\Big/\frac{d(p/V)}{dV}$$

$$=V\frac{dp}{dV}\Big/\left(V\frac{dp}{dV}-p\right). \qquad \ldots\ldots(5.2)$$

On the other hand, differentiating the expression (5.1) by V, we get

$$V\frac{dp}{dV}=\frac{p}{a}\,,$$

therefore we have at once

$$U/V=1/(1-\alpha)\ . \qquad \dots\dots\dots\dots\dots\dots(5.3)$$

Thus we can easily deduce the group velocity from Figs. 6a and 6b corresponding to every curve in the figures. We chose a curve that fits best for the one conventionally obtained in the Fig. 7. That is the curve (6) in the Figs. 6a and 6b. Moreover the slope of this curve 0.60 coincides well with the theoretical value[5].

Now, if we assume that the curve (6) in Fig. 6b represents true dispersion curve,

$$\log V(p)=0.60 \log p+3.316 \text{ (c.g.s. unit)}\ . \qquad \dots\dots(5.4)$$

Since the material constants have been determined by Kishinouye[6], we will employ those values given in his paper, viz.,

$$V_S=1.51 \text{ km/sec}\ ,$$

$$V_P=2.82 \text{ km/sec}\ ,$$

$$\gamma^2\equiv(V_P/V_S)^2=3.49=1.87^2\ ,$$

$$\text{Poisson's ratio}=0.30\ ,$$

$$\mathfrak{v}_0\equiv 2\sqrt{-1+\gamma^2}/\gamma=1.69\ .$$

Putting these values into the formula

$$\mathfrak{v}=\omega^{3/5}\left\{\frac{1}{12\Gamma}\mathfrak{v}_0^2\right\}^{1/5}\ ,$$ [7]

where
$$\begin{cases} \mathfrak{v}=V/V_S\ , \\ \omega=pH/V_S=2\pi nH/V_S\ ,\ (n\text{: frequency per sec.}) \\ \Gamma=(\text{density of water/density of ice}) \\ \quad =1.09 \end{cases}$$

we have as the thickness of the ice plate

$$H=31.5 \text{ cm}\ ,$$

which is in good agreement with the directly observed value by Kishinouye (34cm.).

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5) Y. SATÔ, "Study on Surface Waves II. Velocity of Surface Waves Propagated upon Elastic Plates," *Bull. Earthq. Res. Inst.*, **29** (1951), 237. Table VI.

6) *loc. cit.*, 3).

7) *loc. cit.*, 5). Table VI or expression (7.3).

6. Conclusion

By applying the method of Fourier transform to the dispersed seismograms, we can obtain the dispersion formula and the spectrum of the vibration near the origin. For, the argument of the Fourier transform indicates the velocity of the waves, while its absolute value gives the spectrum of the given disturbance. The obtained dispersion curve agrees well with those arrived at by the conventional method of measuring the interval of crest to crest, which gives only several isolated points and its precision is decidedly inferior to that of the former. Forms of the spectrum curves calculated by means of two independent observations coincide pretty well.

Thickness of the ice plate estimated from the dispersion curve, which was obtained by the method here proposed, is 31.5 cm., while the value given by Kishinouye from the direct measurement is 34 cm.[8] The fact that the discrepancy is quite small implies the good agreement of the theory on the elastic plates, which was developed several years ago, and the observation made upon the lake ice. Also, the author believes, it assures the utility of the method here proposed.

The expense of the present study was partly defrayed from the Fund of the Scientific Research from the Ministry of Education.

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3. 分散した地震波の解析 I

地震研究所　佐　藤　泰　夫

1. 発震機構をしらべ，地殻構造を決定する事は，地震計測学におけるもつとも重要な課題の一つであるが，実体波は従来この目的の為に利用されて見るべき成果をあげてゐる．これに反して表面波は，さまざまの試みがあるにもかかはらず，実体波ほどには用ゐられてゐないが，これは一つには，Rayleigh 波にも，Love 波にも分散性があり，取扱ひに困難があるからであらう．

本論文は，分散した表面波の記象を解析して，その分散公式，できうれば発震機構に関する推測を行はうとするものであり，その原理は，“任意の観測点において得られた波形の Fourier transform を求めれば，その振幅は震源附近での spectrum を，その位相は伝播によつて生じた位相の変りを示す”，との簡単な考へに基づくものである．

2. 先づ 1 次元の波動を扱ふ．

原点での動きを $f(t)$ とすれば

$$f(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f^*(p)\exp(ipt)dp \qquad \ldots.(2.2)$$

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8) *loc. cit.*, 5). p. 303.

又 x の所では

$$f(t;\ x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}[f^*(p)\exp\{-ipx/V(p)\}]\exp(ipt)dp \qquad \ldots.(2.3)$$

であるから，その Fourier transform は

$$f^*(p;\ x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau;\ x)\cdot\exp(-ip\tau)d\tau \qquad \ldots.(2.4)$$

$$=f^*(p)\exp\{-ipx/V(p)\} \qquad \ldots.(2.5)$$

従つて

$$f^*(p;\ x)=c(p;\ x)-is(p;\ x) \qquad \ldots.(2.6)$$

$$f^*(p)=F(p)\exp\{-i\beta(p)\} \qquad \ldots.(2.7)$$

とおけば

$$F(p)^2=c(p;\ x)^2+s(p;\ x)^2 \qquad \ldots.(2.8)$$

$F(p)$ は震源での spectrum をあらはし，

$$-\arg f^*(p;\ x)=\beta(p)+px/V(p) \qquad \ldots.(2.9)$$

からは速度 $V(p)$ が求められる．ここに $F(p)$, $\beta(p)$ は実数とする．(公式 (2·10))．

$\beta(p)$ は，分散公式が共通である 2 個の観測点での記録が得られれば，消去する事ができる．又 $f(t;\ x)$ から逆に $f(t)$ を求める事も可能な筈である．

3．波が 2 次元的にひろがる場合にも，上の公式をいくらか変形する事により，同様の式を導く事ができる．

即ち，震源に近い $r=r_0$ での spectrum としては，

$$F(p;\ r_0)=\sqrt{\frac{r}{r_0}}F(p;\ r) \qquad \ldots.(3.7)$$

ここに

$$F^2(p;\ r)=c(p;\ r)^2+s(p;\ r)^2$$

$$f^*(p;\ r)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(\tau;\ x)\exp(-ip\tau)d\tau=c(p;\ r)-is(p;\ r) \qquad \ldots.(3.5)$$

又速度は

$$-\arg f^*(p;r)=\beta(p)+p(r-r_0)/V(p)=\arctan\{s(p;r)/c(p;r)\} \qquad \ldots.(3.6)$$

から求められる．

即ち

$$V(p)=\frac{p(r^{(1)}-r^{(2)})}{\mathrm{ARCTAN}\left\{\dfrac{s(p;\ r^{(1)})}{c(p;\ r^{(1)})}\right\}-\mathrm{ARCTAN}\left\{\dfrac{s(p;\ r^{(2)})}{c(p;\ r^{(2)})}\right\}+2m\pi} \qquad \ldots.(3.9)$$

前節の場合にも起つた事であるが，arctan の多価性の為に生ずる不確定性は，他の条件を考慮して取除かなくてはならない．

4．もし波形が Fig. 1, 2, 3 に示すやうに，振幅と周期がゆつくりと変る正弦形の波からできてゐる時には，適当な近似式で表現する事により (式 (4.2))，積分を簡単に行う事ができる (公式 (4.7)～(4.10))．

5．以上の理論を，10 年余り前に岸上が榛名湖において行つた実験の記録に適用した．Hum とおぼしき雑音を除く為，適当に曲線をならした上で，さきに求めた公式によつて数値計算を行つた．この結果を整理して，位相速度を表はす曲線としては Fig. 6a 又は 6b の曲線 (6) が得られた．この線は，以前筆者が水の上におかれた薄い弾性板を伝はる表面波について求めた理論式とよく一致する．観測から得られた物理常数を入れて氷の厚さを求めて見ると，31.5 cm. なる値が得られる．実際に現地において測定した値は 34 cm. である．

本研究には，岸上冬彦，高橋竜太郎両先生に種々御指導を賜はつた．記して感謝を捧げる次第である．

Errata and Appendix to

Study on Surface Waves II.

Velocity of Surface Waves Propagated upon Elastic Plates.

By Yasuo SATÔ,

Earthquake Research Institute. (Published 29 (1951), 223.)

Page	Line or expression			
224, 225		$\mathfrak{B}$	read	$\mathfrak{B}$
231, 246				
227	6 from bottom	(3.10)	〃	(3.9)
231	1	farely	〃	fairly
〃	Fig. 5	u/w	〃	w/u
237	(5.4)	u^*, w^*	〃	$\dot{u}^*$, $\dot{w}^*$
	(5.7)	$-\rho^*\partial\phi/\partial t$	〃	$+\rho^*\partial\phi/\partial t$
	5)	$\frac{\alpha}{h^2}B$	〃	$-ip\frac{\alpha}{h_2}B$
249	(8.4)	u	〃	u^*
250	(8.6), w	$[\{-\xi\zeta(-1+\frac{1}{2}\gamma^2)+$	〃	$[-\xi\zeta\{(-1+\frac{1}{2}\gamma^2)+$
		$\mathfrak{v}^2(\frac{1}{4}-\frac{1}{6}\zeta^2(-1+\frac{1}{2}\gamma^4)/\gamma^2)$	〃	$\mathfrak{v}^2(\frac{1}{4}-\frac{1}{6}\zeta^2(-2+\frac{1}{2}\gamma^4)/\gamma^2)$
221	last	the reverse of	〃	same with
253	Figs. 9, 10, 11	see below		
254	last, w	$-iH\xi^2\frac{1}{2\gamma^3}$	〃	$-iH\xi\frac{2}{\gamma^3}$
255	11	same with that of	〃	reverse to
	(8.21)	$\mathfrak{v}_0^2/(-4+3\gamma^2)$	〃	$\mathfrak{v}_0^2/V(-4+3\gamma^2)$
256	Figs. 13b, 14b	see below		

The upper part of Figs. 9, 10 and 11 must be replaced by the following figure.

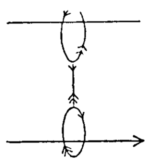

Fig. 9

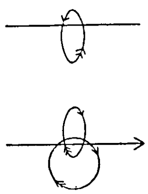

Fig. 10

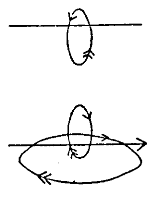

Fig. 11

And the upper part of Figs. 13b and 14b also must be corrected.

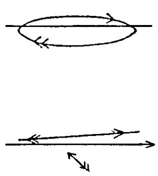

Fig. 13b

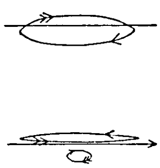

Fig. 14a

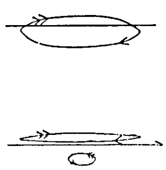

Fig. 14b

In the following table we will show the sense of rotation of particle orbit and the order of magnitude of displacement amplitude when the wave lengths are sufficiently long.

Π_0^- – branch

	(I) Plate	(II) Plate and infinitely deep water	(III) Plate and water with finite depth
Upper surface of the elastic plate	$u:w=O(\xi)$ ↺	$u:w=O(\xi)$ ↺	$u:w=O(\xi)$ ↺
Lower surface of the elastic plate	$u:w=O(\xi)$ ↻	$u:w=O(\xi)$ ↻	$u:w=O(\xi)$ ↻
Upper surface of the liquid		$u^*:w=O(1)$ $u^*:w=O(1)$ ↻	$u^*:w=O(1/\xi)$ $u^*:w=O(1)$ ↻
Bottom of the liquid		Decreases exponentially	$u^*:w=O(1/\xi)$ $w^* \quad =0$ ⇄

Direction of propagation of waves is assumed to be →. ($\xi=2\pi$/wave length)

Π_0^+ – branch

	(I) Plate	(II) Plate and infinitely deep water		(III) Plate and water with finite depth	
		$\mathfrak{v}_0<\gamma^*$	$\mathfrak{v}_0>\gamma^*$	$\mathfrak{v}_0<\gamma^*$	$\mathfrak{v}_0>\gamma^*$
Upper surface of the elastic plate	$u:w=O(1/\xi)$ ↻	$u:w=O(1/\xi)$ ↻		$u:w=O(1/\xi)$ ↻	
Lower surface of the elastic plate	$u:w=O(1/\xi)$ ↺	$u:w=O(1/\xi^2)$ ↺	⇵	$u:w=O(1/\xi_3)$ ↺	↻
Upper surface of the water		$u^*:w$ (lower surface) $=O(1)$ $w^*:w \quad =O(1)$ ↻	⇅	$u^*:w$ (lower surface) $=O(1)$ $w^*:w \quad =O(1)$ ↻	↺
Bottom of the water		Decreases exponentially	Not decreases	$u^*:w=O(1)$ $w^* \quad =0$ ⇄	

Direction of propagation of waves is assumed to be →. ($\xi=2\pi$/wave length)

When $\mathfrak{v}_0>\gamma^*$, or the limiting velocity of Π_0^+ is larger than that of the sound waves in water, the surface waves in the true sense cannot exist in a plate covering infinitely deep water. ($\mathfrak{v}_0^2=4(-1+\gamma^2)/\gamma^2$, $\gamma=V_P/V_S$ and $\gamma^*=V^*/V_S$.)