6. A Note on the Elasticity of Marble.

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D. S. Hughes and others have recently measured the velocities of elastic waves in some igneous and sedimentary rocks under the conditions of elevated temperatures and pressures, and their results are given in several publications. In one of their results[1], velocities both of dilatational and rotational waves in marble are tabulated as the function of pressures and temperatures. Table I shows the values of pressures, velocities and temperatures after them.

Table I.

T°C	26	
P bars	V_p (m)	V_s (m)
70	5866	2823
140	6060	3071
345	6250	3104
690	6501	3147
1035	6551	3170
1725	6621	3195
2415	6651	3118
3100	6665	3209
4140	6673	3209
5170	6660	3254

In a previous paper[2], the author calculated the mean velocities of elastic waves in marble at 20°C from the known values of elastic constants of a calcite crystal, assuming that marble is a polycrystalline aggregate of calcite crystals only and that the crystals are oriented equally likely to all directions. The average velocities of dilatational and rotational waves can be calculated using the following relations,

$\bar{v}_p=\left(\frac{\overline{C}_{11}}{\rho}\right)^{1/2}$ and $\bar{v}_s=\left(\frac{\overline{C}_{44}}{\rho}\right)^{1/2}$ respectively, where $\overline{C}_{11}$ and $\overline{C}_{44}$ are mean elastic constants, and are given by the expressions

$$\overline{C}_{11}=\frac{1}{15}\left[8\hat{C}_{11}+3\hat{C}_{33}+4(C_{13}+2C_{44})+\frac{1}{2}C_{44}\right],$$

and

$$\overline{C}_{44}=\frac{1}{6}C_{11}-\frac{1}{6}C_{12}+\frac{2}{3}C_{44}-\frac{1}{6}C_{44}\,.$$

1) D. S. HUGHES, *Bull. Geol. Soc. Amer.*, **61** (1950), 843–856.
2) D. SHIMOZURU, *Bull. Earthq. Res. Inst.*, **33** (1955), 70.

Then, assuming that $\hat{C}_{11}=13.69\times10^{11}$dynes/cm^2, $\hat{C}_{12}=4.56\times10^{11}$, $\hat{C}_{33}=7.99\times10^{11}$, $\hat{C}_{13}=4.51\times10^{11}$, $\hat{C}_{44}=3.42\times10^{11}$, $\hat{C}_{14}=-2.08\times10^{11}$ as in the previous paper, the average velocities become

$$\bar{v}_p=\left(\frac{\overline{C}_{11}}{\rho}\right)^{1/2}=6609\ \text{m/sec and }\ \bar{v}_s=\left(\frac{\overline{C}_{44}}{\rho}\right)^{1/2}=3904\ \text{m/sec}\ ,$$

assuming that $\rho=2.715$ gr/cm^3.
These are the theoretically computed velocities of marble from the known elastic constants of calcite crystals.

Next, we shall consider the effect of hydrostatic pressure upon the velocities of elastic waves. F. Birch[3] has given the velocities of elastic waves of an isotropic medium which is subjected to hydrostatic pressure, using Murnaghan's theory of finite strain. According to him, the dilatational and rotational waves are calculated in very convenient expressions in terms of strain and Lame's constants as follows:

$$\left.\begin{aligned} v_p^2&=(1-2\varepsilon)^{5/2}[\lambda+2\mu-\varepsilon(11\lambda+10\mu)]/\rho\ ,\\ v_s^2&=(1-2\varepsilon)^{5/2}[\mu-\varepsilon(3\lambda+4\mu)]/\rho\ ,\\ \text{and}\qquad -p&=(1-2\varepsilon)^{5/2}\varepsilon(3\lambda+2\mu)\ ,\end{aligned}\right\}\qquad(1)$$

where
ε .... strain,
λ, μ .... Lame's elastic constants,
p .... pressure,
v_p .... velocity of dilatational wave,
v_s .... velocity of rotational wave.

For the purpose of comparison with laboratory measurements, it is sufficient to retain only the first powers of ε or p. Then the approximate expressions of equation (1) come to be,

$$\left.\begin{aligned} v_p^2&=v_{p0}^2\left[1+\frac{p}{3\lambda+2\mu}\,\frac{13\lambda+14\mu}{\lambda+2\mu}\right],\\ v_s^2&=v_{s0}^2\left[1+\frac{p}{\mu}\,\frac{3\lambda+6\mu}{3\lambda+2\mu}\right],\\ \text{and}\qquad p&=-\varepsilon(3\lambda+2\mu)\ ,\end{aligned}\right\}\qquad(2)$$

where the subscript zero refers to the state of zero pressure. Equa-

3) F. BIRCH, *Jour. Appl. Phys.*, **9** (1938), 279–288.

tion (2) gives the velocities of dilatational and rotational waves as a function of hydrostatic pressures as tabulated in Table II, where v_{l^0}, v_{s^0} and λ, μ were taken to be 6609 m/sec, 3904 m/sec and 3.58×10^{11} dynes/cm^2, 4.14×10^{11} dynes/cm^2 respectively. The theoretically computed velocities as given above, as well as the values of laboratory measurements by Hughes are shown in Fig. 1 and Fig. 2.

Table II.

P (bars)	V (m)	V (m)
0	6609	3904
1000	6624	3913
2000	6639	3922
3000	6655	3930
4000	6670	3939

It seems that theoretical and experimental values agree pretty well when the pressure is above 2000 bars. For pressures below 2000 bars, the discrepancy between the theory and the laboratory measurements is great. This discrepancy may be attributed to the pore space between crystals in the measured specimens. Owing to the existence of pore space, the velocity of dilatational wave at ordinally pressure must be much less in laboratory measurements than in the theory, in which pores are not taken into consideration. As the environing pressure becomes large, such void spaces tend to vanish being accompanied by the large increase in apparent elasticity. Therefore, the velocities of elastic waves increase in consequence of vanishment of such void spaces. Above 2000 bars, pore space seems to vanish completely and the compres-

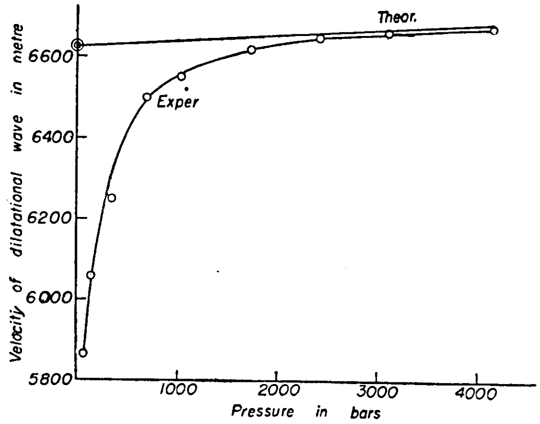

Fig. 1.

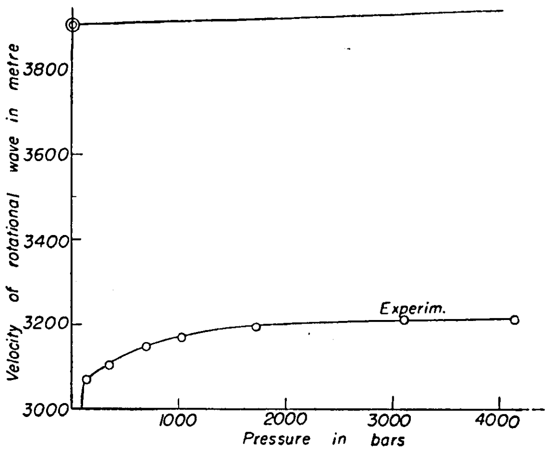

Fig. 2.

Velocities of dilatational and rotational waves versus pressure, after D. S. Hughes.
Description; Danby marble, Origin; Rutland.

sion of calcite itself begins to set in. In other words, agreement of velocity of dilatational wave between theory and experiment above 2000 bars indicates that crystal grains themselves are being compressed in such manner as assumed in the theory.

As for rotational wave, the propagation velocity obtained by laboratory measurements is much less than the theoretical one. Theoretical value, however, does not assume the effect of grain boundary such as cited in a previous paper, whereas such grain boundary is considered to affect the propagation velocity, especially in the case of shear. In the author's opinion, the reason cited above may be the cause of the discrepancy between theoretical and experimental values of propagation of rotational waves. The tendency of increase of velocity of rotational wave due to the compression of composing crystal when the pressure is more than 2000 bars is quite similar to those of dilatational wave.

N.B. According to the personal communication from Assist. Prof. Y. Satô of the Institute, large increase in velocity of laboratory measurement at low pressure does not seem to be merely due to the effect of disappearance of void space. His opinion, based on mathematical studies, is that the effect of void space upon the velocity of elastic wave up to 2000 bars is much less than that of laboratory measurement, accordingly another mechanism such as fissures, in addition to the above-cited effect, must be considered for the interpretation of large increase of velocity actually observable in the experiment.

I express my hearty thanks to Assist. Prof. Y. Satô for his helpful criticisms offered me.

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6. 大理石の弾性に関するノート

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最近 Hughes 一派は，超音波を用いて，高温，高圧下の岩石中を伝はる弾性波の速度を測定している．その結果によると一般的傾向として，圧力の小さい間は，速度の増加の割合が大きいが，次第に高圧になると，割合が少なくなる．そこで，前の論文で計算した大理石の理論的な速度と，Hughes の大理石の実験値を比較して見ると，圧力ゼロの所に於ては，理論値は実験値よりはるかに大きい．圧力が次第に増すと，Murnaghan の理論を用いて，計算した速度と，実験値を比べると，縦波の場合は，2000 bar あたりから，非常に良く一致する様になる．2000 bar 以下の圧力の低い所の理論値と実験値との差は実験に使用した大理石中の空隙が原因と考へられる．圧力が次第に高くなると，その様な空隙が次第に圧縮されて，見掛上，速度が急激に増すのであらうと考へら

れる．2000 bar になると，その様な空隙は完全に押しつぶされて，それ以上は，結晶の実体の圧縮が始まるのである．故に，2000 bar 以上では有限歪の理論を用いた理論値と良く一致する様になる．横波の場合は，理論値は実験値よりも非常に大きい値を示している．これは，実際の試料中の結晶の境界が，縦波に対しては，効果を現はさないが，横波に対しては大いに効いて来るためであらうと思はれる．