25. *Study on Surface Waves* XII. *Non-Dispersive Surface Waves.*

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(Read May 19, 1953.—Received Sept. 30, 1954.)

1. Introduction.

Among the various properties of Love-waves, the dispersion seems to be the most remarkable and well-known one, and makes the matter greatly complicated as well as interesting. Although Rayleigh-wave when propagated in a semi-infinite elastic medium does not show such a property, the actually observed waves of Rayleigh type are known to be dispersive. This paradox, after remaining unsolved for a pretty long time, was made clear by A. E. H. Love[1] some forty years ago. In his celebrated memoir he calculated the velocity of Rayleigh waves propagated upon the layered medium and proved that the velocity is the function of frequency. Now, the next question which naturally occurs is why Love- and Rayleigh-waves propagated in the layered structure show dispersive nature. Are there any surface waves, besides Rayleigh- and Stoneley-waves, which are non-dispersive? We will answer this question in the following sections.

2. Dimensional analysis.

We will employ the following notations;

V; Velocity of surface waves.
L; Wave-length.
λ, μ; Lamé's constants.
ρ; Density.

If the surface waves are propagated upon a semi-infinite body, its velocity must be expressed in the following form;

$$V=F_1(\lambda, \mu, \rho, L) . \qquad \text{(2·1)}$$

Now, from the consideration with respect to dimensions, V must have the form;

1) A. E. H. LOVE, *Some Problems on Geodynamics* (1911).

$$V=\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}F_2\left(\frac{\lambda}{\mu}\right). \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2.2)$$

Thus the wave-length L cannot have any relation with the velocity, or in other words, the waves are non-dispersive in this case.

If, however, the medium has a layered structure, which has some definite thickness, say H, we have the following relation

$$V=G_1(\lambda, \lambda', \mu, \mu', \rho, \rho', L, H)\,, \qquad \cdots\cdots\cdots\cdots(2.3)$$

where the new notations have the following meaning:

λ', μ'; Lamé's constants in the part of layer.
ρ'; Density in the part of layer.

By a similar consideration as with the former case, we have

$$V=\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}G_2\left(\frac{\lambda}{\mu}, \frac{\lambda'}{\mu'}, \frac{\mu'}{\mu}, \frac{\rho'}{\rho}, \frac{L}{H}\right). \qquad \cdots\cdots\cdots\cdots(2.4)$$

In this expression the function G_2 involves a variable L/H, which fact implies the dependency of V on L/H, namely the dispersive property of this case. Thus it was made clear that the existence of a layer makes the surface waves, which are propagated in this medium, dispersive.

3. Non-dispersive surface waves.

In the previous paper[2] we have described the examples of the types of surface waves with one- and two-phase. We will now pick out all the types of waves which are non-dispersive, i.e. the surface waves existent in the medium without layer structure. The symbolical expressions of the type of waves in a non-stratified medium are the following seven, the existence of which will be examined in the following section.

1) $[\overline{E}]$, 2) $[\overline{\overline{E}}]$,

3) $[\overline{EE}]$, 4) $[\overline{\overline{EE}}]$,

5) $\left[\frac{E}{E}\right]$, 6) $\left[\frac{EE}{E}\right]$, $\cdots\cdots\cdots\cdots(3.1)$

7) $\left[\frac{EE}{EE}\right]$.

2) Y. SATÔ, "Study on Surface Waves XI. Definition and Classification of Surface Waves," *Bull. Earthq. Res. Inst.*, **32** (1954), 161.

3.1 Examination of the seven types of waves.

We will now examine the above seven types of waves one by one.

3.11 $[\overline{E}]$

We have proved in our former paper that the waves of this type may be the SH-waves or the sound waves in an elastic liquid, but neither of them can satisfy the boundary condition of free surface.

3.12 $[\overline{\overline{E}}]$

In this case also no wave satisfying the fundamental wave equation and the boundary condition can exist.

3.13 $[\overline{EE}]$

This is the well-known Rayleigh-waves.

3.14 $[\overline{\overline{EE}}]$

Taking the displacement potential

$$\begin{cases} \phi=A\exp(\alpha z+ifx+ipt)\,, & \alpha=\sqrt{f^2-h^2}, \\ \psi=B\exp(\beta z+ifx+ipt)\,, & \beta=\sqrt{f^2-k^2}, \end{cases} \qquad \ldots\ldots(3.2)$$

(z-axis is taken upward and the medium is in the negative part of z.)

$$\begin{cases} h=p/V_P\,, & V_P=\sqrt{\{(\lambda+2\mu)/\rho\}}\,, \\ k=p/V_S\,, & V_S=\sqrt{(\mu/\rho)}\,, \end{cases}$$

we have

$$\begin{cases} u=\dfrac{\partial\phi}{\partial x}+\dfrac{\partial\psi}{\partial z}=Aif\cdot\exp(\alpha z+ifx+ipt) \\ \qquad\qquad +B\beta\cdot\exp(\beta z+ifx+ipt)\,, \\ w=\dfrac{\partial\phi}{\partial z}-\dfrac{\partial\psi}{\partial x}=A\alpha\cdot\exp(\alpha z+ifx+ipt) \\ \qquad\qquad -Bif\cdot\exp(\beta z+ifx+ipt)\,. \end{cases} \qquad \ldots\ldots\ldots(3.3)$$

If the boundary surface is expressed by $z=0$, the condition at this plane is

$$u=0, \quad w=0. \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.4)$$

Therefore

$$\begin{cases} Aif+B\beta=0, \\ A\alpha-Bif=0. \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

Eliminating A and B we have

$$f^2=\alpha\beta$$
$$=\sqrt{f^2-h^2}\sqrt{f^2-k^2}\,. \qquad \text{.................(3.6)}$$

By a glance it will become recognized that this equation cannot be satisfied by the real values of α and β. Consequently we must conclude that the waves of the type $[\overline{\overline{EE}}]$ also cannot exist actually.

3.15 $\left[\frac{E}{E}\right]$

If we take

$$\begin{cases} v=A\exp(-\beta z+ifx+ipt)\,, \\ v'=A'\exp(\beta' z+ifx+ipt)\,, \end{cases} \qquad \text{...............(3.7)}$$

where

$$\beta'=\sqrt{(f^2-k'^2)}\,,$$
$$k'=p/V_s'\,,$$
$$V_s'=\sqrt{(\mu'/\rho')}\,,$$

the next conditions must hold at the boundary surface $z=0$.

$$\begin{array}{ll} \text{Continuity of displacement;} & A=A'\,, \\ \text{Continuity of stress;} & A\mu(-\beta)=A'\mu'\beta'\,. \end{array} \qquad \text{.......(3.8)}$$

These conditions cannot be satisfied simultaneously, so we must conclude, in this case also, that the surface waves of the type $\left[\frac{E}{E}\right]$ do not exist.

3.16 $\left[\frac{EE}{E}\right]$

Next we will examine the type $\left[\frac{EE}{E}\right]$.

In the part of solid, we take

$$\begin{cases} \Delta=A\exp(-\alpha z+ifx+ipt)\,, \\ \tilde{\omega}=C\exp(-\beta z+ifx+ipt)\,, \end{cases} \qquad \text{................(3.9)}$$

where Δ is the dilatation and $\tilde{\omega}$ the component of rotation vector. Since

$$\begin{cases} \rho\dfrac{\partial^2 u}{\partial t^2}=(\lambda+2\mu)\dfrac{\partial\Delta}{\partial x}+\mu\dfrac{\partial\tilde{\omega}}{\partial z}\,, \\ \rho\dfrac{\partial^2 w}{\partial t^2}=(\lambda+2\mu)\dfrac{\partial\Delta}{\partial z}-\mu\dfrac{\partial\tilde{\omega}}{\partial x}\,, \end{cases} \qquad \text{.............(3.10)}$$

we have

$$\left\{\begin{aligned} u &= -\frac{1}{\rho p^2}\{(\lambda+2\mu)Aif\cdot\exp(-\alpha z+ifx+ipt) \\ &\qquad +\mu C(-\beta)\cdot\exp(-\beta z+ifx+ipt)\}\,, \\ w &= -\frac{1}{\rho p^2}\{(\lambda+2\mu)A(-\alpha)\cdot\exp(-\alpha z+ifx+ipt) \\ &\qquad -\mu C(if)\cdot\exp(-\beta z+ifx+ipt)\}\,, \end{aligned}\right. \qquad \ldots\ldots(3.11)$$

and

$$\begin{aligned} \widehat{zz} &= \lambda\Delta+2\mu\frac{\partial w}{\partial z} \\ &= \mu\left[\left\{\frac{\lambda}{\mu}-\frac{2\alpha^2}{h^2}\right\}A\cdot\exp(-\alpha z)+2\frac{-if\beta}{k^2}C\cdot\exp(-\beta z)\right] \\ &\qquad\qquad \cdot\exp(ifx+ipt)\,. \end{aligned} \qquad \ldots(3.12)$$

In the part of liquid we have

$$\left\{\begin{aligned} &\rho^*\frac{\partial^2 u^*}{\partial t^2}=\lambda^*\frac{\partial\Delta^*}{\partial x}\,, \\ &\rho^*\frac{\partial^2 w^*}{\partial t^2}=\lambda^*\frac{\partial\Delta^*}{\partial z}\,, \\ &\qquad \widehat{zz}^*=\lambda^*\Delta^*\,, \end{aligned}\right. \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.13)$$

where the asterisk implies the quantity in the liquid.

Putting $\qquad \Delta^*=B\exp(\alpha^*z+ifx+ipt)$

we have

$$\left\{\begin{aligned} &u^*=-\frac{if}{h^{*2}}B\exp(\alpha^*z+ifx-ipt)\,, \\ &w^*=-\frac{\alpha^*}{h^{*2}}B\exp(\alpha^*z+ifx-ipt)\,, \\ &\widehat{zz}^*=\lambda^*B\exp(\alpha^*z+ifx-ipt)\,,\quad \widehat{xz}^*=0. \end{aligned}\right. \qquad \ldots\ldots\ldots\ldots(3.14)$$

The boundary conditions are the continuity of the vertical component of displacement and that of the stress. Hence we have

$$\left\{\begin{aligned} &\frac{\alpha}{h^2}A+\frac{if}{k^2}C=-\frac{\alpha^*}{h^{*2}}B\,, \\ &\mu\left\{\frac{\lambda}{\mu}-\frac{2\alpha^2}{h^2}\right\}A+\mu 2\frac{-if\beta}{k^2}C=\lambda^*B\,, \\ &\frac{i2f\alpha}{h^2}A+\frac{k^2-2f^2}{k^2}C=0\,. \end{aligned}\right. \qquad \ldots\ldots\ldots(3.15)$$

We will introduce the following new notations :

$$\begin{aligned} &\gamma^2 \equiv V_p^2/V_s^2=(\lambda+2\mu)/\mu, \\ &\gamma^{*2} \equiv V^{*2}/V_s^2=k^2/h^{*2}, \\ &\Gamma \equiv \rho^*/\rho, \\ &\mathfrak{v} \equiv V/V_s=k/f, \ (V \text{ is the velocity of the surface waves.}) \end{aligned} \qquad \cdots\cdots(3.16)$$

Introducing these relations we have

$$\begin{aligned} &\lambda^*/\mu=\Gamma\gamma^{*2}, \\ &\beta/f=\sqrt{(1-\mathfrak{v}^2)}, \\ &\alpha/f=\sqrt{(1-\mathfrak{v}^2/\gamma^2)}, \\ &\alpha^*/f=\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}. \end{aligned} \qquad \cdots\cdots(3.17)$$

Then eliminating A, C and B from the expression (3.15), we have

$$\begin{vmatrix} \sqrt{(1-\mathfrak{v}^2/\gamma^2)} & 1 & \sqrt{(1-\mathfrak{v}^2/\gamma^{*2})} \\ \mathfrak{v}^2-2 & -2\sqrt{(1-\mathfrak{v}^2)} & -\Gamma\mathfrak{v}^2 \\ -2\sqrt{(1-\mathfrak{v}^2/\gamma^2)} & \mathfrak{v}^2-2 & 0 \end{vmatrix}=0, \quad ..(3.18)$$

or

$$\begin{aligned} &\frac{1}{4}\Gamma\mathfrak{v}^4\sqrt{(1-\mathfrak{v}^2/\gamma^2)}+\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}\cdot\left(1-\frac{1}{2}\mathfrak{v}^2\right)^2 \\ &=\sqrt{(1-\mathfrak{v}^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}. \end{aligned} \qquad \cdots\cdots(3.18)'$$

3.161 We will examine the equation (3.18)′ and investigate the range of existence and other properties of this wave.

First we will assume that the solid is perfectly rigid, and glance at the nature of the matter briefly. In this case

$$\gamma^2=\infty, \qquad \cdots\cdots(3.19)$$

and the equation (3.18)′ takes the following simple form

$$\frac{1}{4}\Gamma\mathfrak{v}^4+\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}\cdot\left(1-\frac{1}{2}\mathfrak{v}^2\right)^2=\sqrt{(1-\mathfrak{v}^2)}\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}. \qquad (3.20)$$

Now, this is an expression connecting Γ, γ^* and $\mathfrak{v}$, and since it does not contain any quantity such as period or wave length, the waves in question are, as have been expected, non-dispersive.

Solving the equation numerically we can obtain the relation velocity $\mathfrak{v}(=V/V_s)$ versus $\Gamma(=\rho^*/\rho)$ with a parameter $\gamma^*(=V^*/V_s)$. The following table and figure show this relation explicitly. As is clear from the figure, we can find a positive solution of $\mathfrak{v}$ corresponding to any set of values of γ^* and Γ. This is a very conspicuous property when compared with the Stoneley waves which can exist only under a very restricted conditions.

By a similar method we can get the solution when the solid is not rigid and γ has a finite value. The results of the numerical calculations when $\gamma^2=4$, or $\lambda=2\mu$ (cf. Table II and Fig. 2) and $\gamma^2=3$, or $\lambda=\mu$ (cf. Table III and Fig. 3) are shown here. Even if the parameter γ^2 takes other arbitrary values, we can easily infer, from

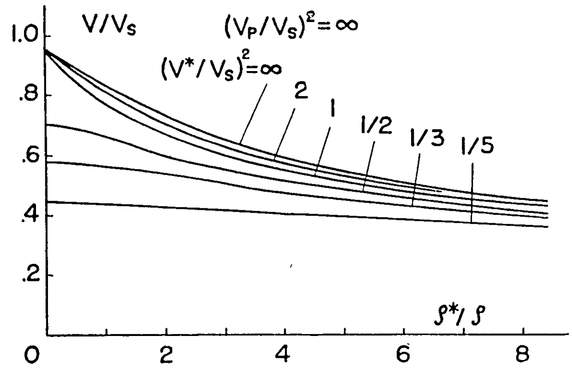

Fig. 1.

Table I. Values of Γ $(=\rho^*/\rho)$ when $\gamma^2=\infty$, (incompressible).

$\mathfrak{v}^2$	$\mathfrak{v}$ \ γ^{*2}	∞	10	2	1	1/2	1/3	1/5
0	0	∞	∞	∞	∞	∞	∞	∞
0.02	0.14142	108.5	108.4	108.0	107.4	106.3	105.2	102.9
0.10	0.31623	18.47	18.38	17.96	17.52	16.52	15.46	13.06
0.12	0.34641	15.13	15.04	14.67	14.20	13.19	12.11	9.572
0.16	0.4	10.866	10.779	10.403	9.959	8.961	7.836	4.860
0.18	0.42426	9.560	9.474	9.120	8.657	7.648	6.485	3.023
0.20	0.44721	8.443	8.358	8.010	7.552	6.540	5.334	0
0.25	0.5	6.426	6.345	6.011	5.565	4.544	3.213	
0.30	0.54772	5.074	4.998	4.678	4.245	3.209	1.605	
1/3	0.57735	4.394					0	
0.36	0.6	3.938	3.867	3.566	3.151	2.084		
0.40	0.63246	3.365	3.297	3.010	2.607	1.505		
0.46	0.67823	2.683	2.621	2.365	1.972	0.7590		
0.48	0.69282	2.492	2.431	2.171	1.797	0.4983		
0.50	0.70711	2.314	2.255	2.004	1.636	0		
0.6	0.77460	1.583	1.535	1.324	1.001			
0.7	0.83666	1.022	0.9858	0.8242	0.5599			
0.8	0.89443	0.5451	0.5228	0.4222	0.2438			
0.9	0.94868	0.06779	0.06467	0.0504	0.02144			
0.91	0.95394	0.01291	0.01231	0.0106	0.003873			
0.912618	0.95531	0	0	0	0			

Table II. Values of Γ $(=\rho^*/\rho)$ when $\gamma^2=4$, $(\lambda=2\mu)$.

$\mathfrak{v}^2$	$\mathfrak{v}$ \ γ^{*2}	∞	2	1	1/2	1/3	1/5
0	0	∞	∞	∞	∞	∞	∞
0.02	0.14142	73.89	73.52	73.14	72.40	71.64	70.10
0.1	0.31623	13.88	13.52	13.16	12.41	11.61	9.811
0.12	0.34641	11.37	10.98	10.66	9.909	9.093	7.189
0.16	0.4	7.878	7.556	7.220	6.496	5.681	3.523
0.18	0.42426	7.179	6.849	6.501	5.744	4.869	2.270
0.20	0.44721	6.338	6.013	5.669	4.910	4.009	0
0.25	0.5	4.820	4.509	4.175	3.408	2.401	
0.30	0.54772	3.798	3.501	3.177	2.402	1.201	
1/3	0.57735	3.282				0	
0.36	0.6	2.936	2.659	2.349	1.554		
0.40	0.63246	2.500	2.236	1.936	1.118		
0.46	0.67823	1.978	1.735	1.453	0.5593		
0.48	0.69282	1.830	1.595	1.319	0.3659		
0.50	0.70711	1.692	1.466	1.197	0		
0.6	0.77460	1.122	0.9387	0.7096			
0.7	0.83666	0.6740	0.5434	0.3691			
0.8	0.89443	0.2781	0.2165	0.1250			
0.869599	0.93252	0	0	0			

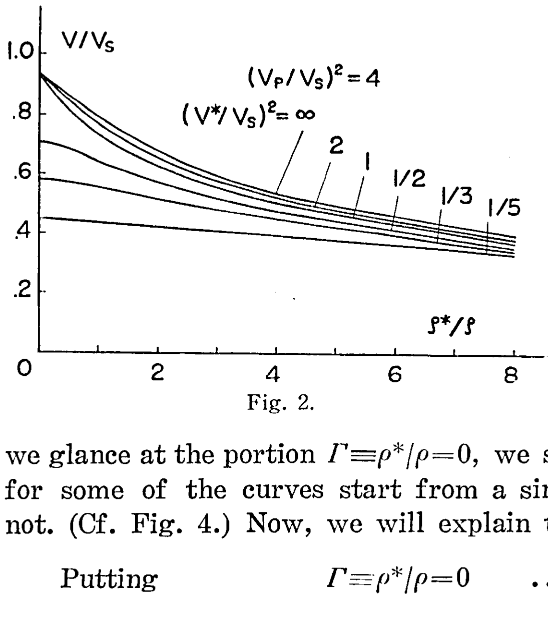

Fig. 2.

the form of the expression, that the equation has a solution, hence some special surface waves can propagate along the plane of separation of the solid and liquid medium for any given set of values of Γ and γ^*.

The curves in the three figures all show the general feature of monotonously decreasing. If, however, we glance at the portion $\Gamma\equiv\rho^*/\rho=0$, we shall find some peculiar aspect, for some of the curves start from a single point, while the others do not. (Cf. Fig. 4.) Now, we will explain this matter briefly.

Putting $$\Gamma\equiv\rho^*/\rho=0 \qquad\qquad (3.21)$$

Table III. Values of Γ $(=\rho^*/\rho)$ when $\gamma^2=3$, $(\lambda=\mu)$.

$\mathfrak{v}^2$	$\mathfrak{v}$ \ γ^{*2}	∞	2	1	1/2	1/3	1/5
0	0	∞	∞	∞	∞	∞	∞
0.02	0.141421	65.76	65.43	65.10	64.43	63.76	62.39
0.1	0.31623	12.30	11.99	11.67	11.00	10.29	8.700
0.12	0.34641	10.064	9.721	9.441	8.774	8.051	6.365
0.16	0.4	6.928	6.646	6.350	5.713	4.996	3.098
0.18	0.42426	6.323	6.032	5.726	5.059	4.289	2.000
0.20	0.44721	5.600	5.313	5.009	4.338	3.542	0
0.25	0.5	4.247	3.973	3.678	3.003	2.124	
0.30	0.54772	3.337	3.076	2.792	2.110	1.055	
1/3	0.57735	2.878				0	
0.36	0.6	2.568	2.326	2.055	1.359		
0.40	0.63246	2.178	1.948	1.687	0.9742		
0.46	0.67823	1.711	1.501	1.257	0.4839		
0.48	0.69281	1.578	1.376	1.138	0.3156		
0.50	0.70711	1.455	1.260	1.029	0		
0.6	0.77460	0.9403	0.7867	0.5947			
0.7	0.83666	0.5322	0.4291	0.2915			
0.8	0.89443	0.1676	0.1298	0.07497			
0.845306	0.919405	0	0	0			

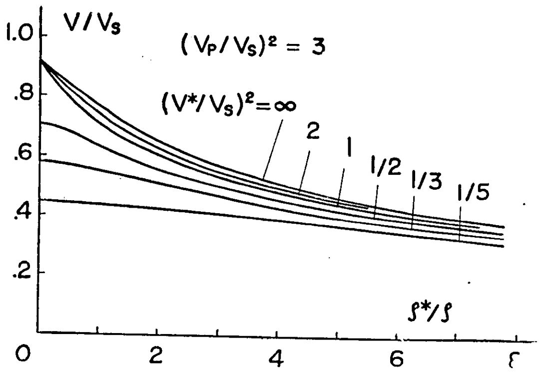

Fig. 3. $V/V_S=\mathfrak{v}$, $\rho^*/\rho=\Gamma$.

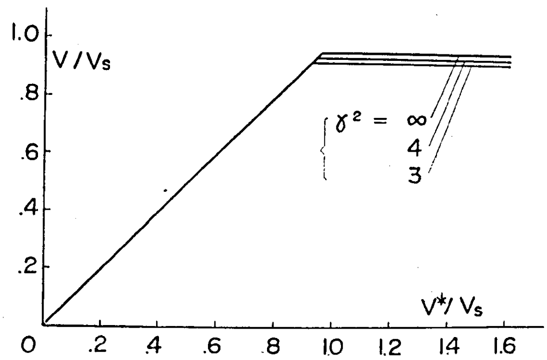

Fig. 4. $V/V_S=\mathfrak{v}$, $V^*/V_S=\gamma^*$.

into the equation (3.18)′, we have

$$\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}\cdot\{(1-\mathfrak{v}^2/2)^2-\sqrt{(1-\mathfrak{v}^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^2)}\}=0\,. \quad \ldots\ldots(3.22)$$

If the factor $\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}$ does not vanish, the expression in the parenthesis must be equal to zero, namely

$$(1-\mathfrak{v}^2/2)^2=\sqrt{(1-\mathfrak{v}^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^2)}\ , \qquad \text{...........(3.23)}$$

which gives the velocity of ordinary Rayleigh-waves. This is a very natural consequence, for the condition $\Gamma\equiv\rho^*/\rho=0$ implies that the liquid is infinitely rarefied. However, contrary to our common sense, if the sound velocity in the liquid is smaller than that of the Rayleigh-waves in the solid medium (viz. $V^*<V_{\text{Rayleigh}}$), the case is different.

Under such a condition we must abandon the equation (3.23) and adopt

$$\mathfrak{v}=\gamma^*. \qquad \text{.......................(3.24)}$$

for the velocity of surface waves of $[E]$-type must be smaller than any other velocity of bodily waves. The result of calculations is shown in the Fig. 4, which will readily show the above circumstances.

3.17 $\left[\dfrac{EE}{EE}\right]$

This is the well-known Stoneley waves, whose range of existence was examined precisely by K. Sezawa[3] and K. Kanai some fifteen years ago.

4. Conclusion.

Thus our examination of the non-dispersive surface waves was finished.

From the consideration through dimensional analysis, it was made clear that the non-dispersive surface waves are those which are propagated upon the non-stratified structure of medium. Possible types of surface waves which do not show the dispersive property are, as has been proved in the preceding section, the following three and no other type is conceivable.

$$[\overline{EE}]\ ,\quad \left[\frac{EE}{E}\right] \text{ and } \left[\frac{EE}{EE}\right]. \qquad \text{...........(4.1)}$$

Since the first and the third are already well-known waves, we examined the second type which always exists unconditionally, and obtained the velocity as the function of $\Gamma(\equiv\rho^*/\rho)$ and $\gamma^*(\equiv V^*/V_s)$.

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3) K. SEZAWA and K. KANAI, "The Range of Possible Existence of Stoneley-Waves, and Some Related Problems," *Bull. Earth. Res. Inst.*, **17** (1939), 1.

25. 表面波の研究 XII.
分散性のない表面波

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1. 半無限体を伝はるレーリー波は分散性を持たない事が知られてゐるにもかかはらず，実際に観測されるものは明らかに分散性を示す．これは地球が表面層を持つ為と考へられてゐるが，同様に層のある媒質を伝はるラブ波も亦分散を行ふ．然らば表面層と分散とは不可分の関係にあるのであろうか．又，レーリー波，ストンレー波の他には分散性のない表面波は存在しないのであらうか，これらの問題を以下に取り上げる．

2. 半無限体を伝はる表面波の速度 V は

$$V=F_1(\lambda, \mu, \rho; L) \qquad \ldots\ldots(2.1)$$

と書ける筈であるが（λ, μ は Lamé の常数，ρ は密度，L は波長），次元解析によれば，これは

$$V=\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}\cdot F_2\left(\frac{\lambda}{\mu}\right) \qquad \ldots\ldots(2.2)$$

の形に変形される事がわかる．この式には波長 L を含まない，従つて速度 V は波長に関係しない．この事は，とりもなほさず分散性のない事を示す．

これに反して，もし表面層（その厚さを H; Lamé 常数 λ', μ'; 密度 ρ' とする）がある場合には，速度はこれらの函数，即ち

$$V=G_1(\lambda, \lambda', \mu, \mu', \rho, \rho', L, H) \qquad \ldots\ldots(2.3)$$

の形をとる事になるが，上と同様の考へから，これは又

$$V=\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}\cdot G_2\left(\frac{\lambda}{\mu}, \frac{\lambda'}{\mu'}, \frac{\mu'}{\mu}, \frac{\rho'}{\rho}, \frac{L}{H}\right) \qquad \ldots\ldots(2.4)$$

と書かれなくてはならない，この式は L/H を変数として持つ．つまり V は波長の函数となり，分散性を示す．以上によつて，層の存在が表面波を分散性ならしめる所以である事が明らかとなつた．

3. 従つて，分散性のない表面波は，層を有しない媒質内を伝はるものの中に求められるべきであるが，その凡ての型を列挙すれば次の通りである．

$$1)\ [\overline{E}], \qquad 2)\ [\overline{\overline{E}}],$$
$$3)\ [\overline{EE}], \qquad 4)\ [\overline{\overline{EE}}],$$
$$5)\ \left[\frac{E}{E}\right], \qquad 6)\ \left[\frac{EE}{E}\right], \qquad \ldots\ldots(3.1)$$
$$7)\ \left[\frac{EE}{EE}\right].$$

この中で，1), 2), 4), 5) に対応するものは存在しない事が証明せられる．3) はレーリー波，7) はストンレー波としてよく知られているから，こゝでは従来調べられていない 6) の型のものについて立入つた議論を行つた．これは半無限の弾性固体と流体との接触面に沿つて伝はる波である．速度をきめる方程式は

$$\frac{1}{4}\Gamma \mathfrak{v}^4 \sqrt{(1-\mathfrak{v}^2/\gamma^2)}+\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}\cdot\left(1-\frac{1}{2}\mathfrak{v}^2\right)^2$$
$$=\sqrt{(1-\mathfrak{v}^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})} \qquad \ldots\ldots(3.18)'$$

但し $\mathfrak{v}\equiv V/V_s$,

$\gamma\equiv V_p/V_s$, $\gamma^*\equiv V^*/V_s$, $\Gamma\equiv\rho^*/\rho$

V_s, V_p, V^* は夫々固体中の P 波，固体中の S 波，流体中の音波の速度．

ストンレー波の存在が狭い範囲に限られているのに引きかへ，上述の表面波は殆ど無条件に存在し，その速度 V は Γ, γ^*, γ の函数として与へられる．(Table I, II, III; Fig. 1, 2, 3, 参照) 但しこゝで面白いことに，$\Gamma(=\rho^*/\rho)\to 0$ の極限では上の方程式は

$$\sqrt{(1-\mathfrak{v}^2/\gamma^{*2})}\cdot\{(1-\mathfrak{v}^2/2)^2-\sqrt{(1-\mathfrak{v}^2)}\cdot\sqrt{(1-\mathfrak{v}^2/\gamma^2)}\}=0 \qquad \ldots\ldots\ldots\ldots\ldots(3.22)$$

となり，この方程式の根によつて与へられる速度は，流体中の音波の速度が固体内のレーリー波のそれ (V_{Rayleigh}) より大きければ V_{Rayleigh} にひとしく，小さければ流体中の音波の速度にひとしくなる．(Fig. 4 参照).

以上によって分散性のない表面波は盡されたことになる．