

18. *On the Vibration of an Elastic Globe with One Layer.* *The Vibration of the First Class.*

By Tosimatu MATUMOTO and Yasuo SATÔ,

Earthquake Research Institute.

(Read May 25, 1954.—Received June 30, 1954.)

1. Introduction.

It is one of the most important problems in the field of geophysics to make researches on the internal constitution of the earth, and since the 19th century many authorities have attacked this problem by means of statical, dynamical¹⁾⁻¹¹⁾ as well as a thermo-dynamical and other methods.

Recently M. Ewing and F. Press¹²⁾ suggested that it may be possible to make use of a long surface wave with a period of several hundred seconds for investigating the interior of the earth, especially with respect to the part of the mantle. This is a hopeful idea and it is reasonable

1) H. LAMB, "The Vibration of an Elastic Sphere," *Proc. Math. Soc. London*, **13** (1882), 189-212.

2) H. LAMB, "On the Vibration of a Spherical Shell," *Proc. Math. Soc. London*, **14** (1883), 50-56.

3) T. J. BROMWICH, "On the Influence of Gravity on Elastic Waves, and in Particular, on the Vibrations of an Elastic Globe," *ibid.*, **30** (1899), 98-120.

4) J. H. JEANS, "On the Vibrations and Stability of a Gravitational Planet," *Phil. Trans. Roy. Soc. London*, **201** (1903), 157-184.

5) J. H. JEANS, "The Propagation of Earthquake Waves," *Proc. Roy. Soc. London*, **102** (1923), 554-574.

6) K. SEZAWA, "On the Propagation of Rayleigh-waves on Plane and Spherical Surfaces," *Bull. Earthq. Res. Inst.*, **2** (1927), 21-28.

7) K. SEZAWA, "Dispersion of Elastic Waves Propagated on the Surface of Stratified Bodies on Rayleigh-wave having some Azimuthal Distribution," *ibid.*, **3** (1927), 1-18.

8) K. SEZAWA, "Further Studies on Rayleigh-wave having some Azimuthal Distribution," *ibid.*, **6** (1929), 1-18.

9) K. SEZAWA, "Propagation of Love-waves on a Spherical Surface and allied Problems," *ibid.*, **7** (1929), 473-455.

10) K. SEZAWA, *Sindôgaku*, (Iwanami, 1931), 147-152.

11) R. YOSHIYAMA, "Love-wave Propagation in Heterogeneous Elastic Sphere," *Zisin*, [i], **10** (1938), 272-276, (in Japanese).

12) M. EWING and F. PRESS, "An Investigation of Mantle Rayleigh Wave," *Bull. Seism. Soc. Amer.*, **44** (1954), 127-148.

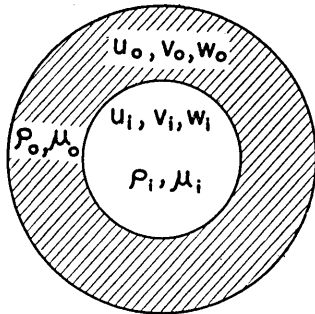


Fig. 1.

to expect that if the wave length of the surface wave becomes as long as the earth's radius, the dispersive property of the wave may be affected by the density and rigidity of the deeper part of the earth; therefore it may be possible to infer the state of the earth's interior by the dispersive property of the long surface wave.

In order to make clear whether such an analysis is possible or not, we calculated the vibration of the first class¹³⁾ of the earth model, which we assumed to be composed of two parts, namely the core and the mantle. In each part, the density and rigidity of the medium are assumed to be uniform, and are denoted by ρ_0, μ_0 in the mantle, and ρ_i, μ_i in the core respectively.

2. Vibration of the elastic sphere.

Fundamental equation of motion in a homogeneous, isotropic and perfectly elastic medium is given by the following expression

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda + \mu) \text{grad div } \mathbf{u} + \mu \nabla^2 \mathbf{u}, \tag{2.1}$$

where \mathbf{u} expresses the displacement vector.

If we take the divergence and the rotation of the above vectorial equation, and express them referring to the spherical coordinates, we get the equations of motion which give the dilatation Δ and the rotation components $\tilde{\omega}_r, \tilde{\omega}_\theta$ and $\tilde{\omega}_\phi$ as follows:

$$\left. \begin{aligned} \rho \frac{\partial^2 \Delta}{\partial t^2} &= (\lambda + 2\mu) \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Delta}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Delta}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Delta}{\partial \phi^2} \right], \\ \rho \frac{\partial^2 \tilde{\omega}_r}{\partial t^2} &= \mu \left[\frac{\partial^2 \tilde{\omega}_r}{\partial r^2} + \frac{4}{r} \frac{\partial \tilde{\omega}_r}{\partial r} + \frac{2}{r^2} \tilde{\omega}_r + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \tilde{\omega}_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \tilde{\omega}_r}{\partial \phi^2} \right], \\ \rho \frac{\partial^2 \tilde{\omega}_\theta}{\partial t^2} &= \mu \left[\frac{1}{r} \frac{\partial^2 (\tilde{\omega}_\theta r)}{\partial r^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \tilde{\omega}_\theta}{\partial \phi^2} - \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 (\tilde{\omega}_\phi \sin \theta)}{\partial \phi \partial \theta} - \frac{1}{r} \frac{\partial^2 \tilde{\omega}_r}{\partial r \partial \theta} \right], \\ \rho \frac{\partial^2 \tilde{\omega}_\phi}{\partial t^2} &= \mu \left[\frac{1}{r} \frac{\partial^2 (\tilde{\omega}_\phi r)}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial (\tilde{\omega}_\phi \sin \theta)}{\partial \theta} \right) \right] \end{aligned} \right\}$$

¹³⁾ "First (and second) class" of vibration is the term adopted by H. LAMB. The vibration of the first class corresponds to the transversal vibration.

$$-\frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial \tilde{\omega}_\theta}{\partial \phi} \right) - \frac{1}{r \sin \theta} \frac{\partial^2 \tilde{\omega}_r}{\partial r \partial \phi} \Big]. \quad (2.2)$$

The typical solutions of these equations were obtained by K. Sezawa in the following forms¹⁰⁾:

$$\left. \begin{aligned} \Delta &= A_{m \cdot n} \frac{J_{n+1/2}(kr)}{\sqrt{r}} P_n^m(\cos \theta) \frac{\cos}{\sin} \Big\} m\phi e^{i\mu t}, \\ 2\tilde{\omega}_r &= B_{m \cdot n} \frac{J_{n+1/2}(kr)}{r^{3/2}} P_n^m(\cos \theta) \frac{\sin}{-\cos} \Big\} m\phi e^{i\mu t}, \\ 2\tilde{\omega}_\theta &= \left[C_{m \cdot n} \frac{J_{n+1/2}(kr)}{\sqrt{r}} \frac{P_n^m(\cos \theta)}{\sin \theta} \right. \\ &\quad \left. + B_{m \cdot n} \frac{1}{n(n+1)} \frac{1}{r} \frac{d}{dr} (\sqrt{r} J_{n+1/2}(kr)) \frac{dP_n^m(\cos \theta)}{d\theta} \right] \frac{\sin}{-\cos} \Big\} m\phi e^{i\mu t}, \\ 2\tilde{\omega}_\phi &= \left[C_{m \cdot n} \frac{1}{m} \frac{J_{n+1/2}(kr)}{\sqrt{r}} \frac{dP_n^m(\cos \theta)}{d\theta} \right. \\ &\quad \left. + B_{m \cdot n} \frac{m}{n(n+1)} \frac{1}{r} \frac{d}{dr} (\sqrt{r} J_{n+1/2}(kr)) \frac{P_n^m(\cos \theta)}{\sin \theta} \right] \frac{\cos}{\sin} \Big\} m\phi e^{i\mu t}, \\ &\quad k^2 = \rho p^2 / (\lambda + \mu), \quad k^2 = \rho p^2 / \mu, \end{aligned} \right\} (2.3)$$

where $J_{n+1/2}$ means the Bessel function of the first kind and $P_n^m(\cos \theta)$ ($= \sin^m \theta (d/d \cos \theta)^m P_n(\cos \theta)$) the associated Legendre function.

Modifying the equation (2.3), we can obtain the expressions giving the displacement components, which we denote as u (r -component), v (θ -component) and w (ϕ -component) respectively¹⁴⁾.

The displacement components cited above are resolved into three parts¹⁵⁾, as for example, $u = u_1 + u_2 + u_3$.

We offer the set of displacement (u_2, v_2, w_2) as the solution of the first class, in which the dilatation Δ and the radial displacement u both vanish, and with which alone we can satisfy the boundary conditions at the spherical surface. The other set of displacement ($u_1 + u_3, v_1 + v_3, w_1 + w_3$) is given as the solution of the second class, in which the rotation component $\tilde{\omega}_r$ vanishes.

As a first step, we will consider only the solution of the first class in this paper, leaving that of the second class to the future.

It is easily proved that this solution of the first class is not affected

14) K. SEZAWA, "Amplitude of P - and S waves at Different Focal Distances," *Bull. Earthq. Res. Inst.*, **10** (1932), 299-334.

15) *Loc. cit.*, 10). See p. 556-562.

by gravity owing to the property $u=0$ and $\Delta=0$ ¹⁶⁾. This nature, which does not hold in the case of second class vibration, is convenient for our study which aims at making clear the change of the period caused by the variation of elastic constants and the density of the earth's interior.

3. Vibration of the first class generated in the layered earth's model.

In later sections, we will adopt the simplified model of the layered earth whose radius of the outer mantle is a and that of the internal surface of separation between the mantle and the core is b .

It is easily shown that the displacement components of the first class, which are denoted by suffix "o" in the mantle and "i" in the core, are as follows:

$$\left. \begin{aligned} u_o &= 0, \\ v_o &= \frac{m}{n(n+1)\sqrt{r}} \left\{ \frac{B_n J_{n+1/2}(k_0 r)}{B_n Y_{n+1/2}(k_0 r)} \right\} \left\{ \frac{D_m P_n^m(\cos \theta)}{D_m Q_n^m(\cos \theta)} \right\} \frac{1}{\sin \theta} \left\{ \begin{matrix} \cos \\ \sin \end{matrix} \right\} m \phi e^{i n t}, \\ w_o &= -\frac{1}{n(n+1)\sqrt{r}} \left\{ \frac{B_n J_{n+1/2}(k_0 r)}{B_n Y_{n+1/2}(k_0 r)} \right\} \left\{ \frac{D_m dP_n^m(\cos \theta)/d\theta}{D_m dQ_n^m(\cos \theta)/d\theta} \right\} \left\{ \begin{matrix} \sin \\ -\cos \end{matrix} \right\} m \phi e^{i n t}, \end{aligned} \right\} \text{for } b \leq r \leq a. \quad (3.1)$$

and

$$\left. \begin{aligned} u_i &= 0, \\ v_i &= \frac{m}{n(n+1)\sqrt{r}} E_n J_{n+1/2}(k_i r) \left\{ \frac{D_m P_n^m(\cos \theta)}{D_m Q_n^m(\cos \theta)} \right\} \frac{1}{\sin \theta} \left\{ \begin{matrix} \cos \\ \sin \end{matrix} \right\} m \phi e^{i n t}, \\ w_i &= -\frac{1}{n(n+1)\sqrt{r}} E_n J_{n+1/2}(k_i r) \left\{ \frac{D_m (dP_n^m(\cos \theta)/d\theta)}{D_m (dQ_n^m(\cos \theta)/d\theta)} \right\} \left\{ \begin{matrix} \sin \\ -\cos \end{matrix} \right\} m \phi e^{i n t}, \end{aligned} \right\} \text{for } 0 \leq r \leq b, \quad (3.2)$$

where $k_0^2 = p^2 \rho_0 / \mu_0$ and $k_i^2 = p^2 \rho_i / \mu_i$.

The boundary conditions are given as follows:

For the outer free surface (where $r=a$):

$$\frac{\partial}{\partial r} v_o = 0, \quad \left(\frac{\partial}{\partial r} w_o = 0 \right), \quad (3.3a)$$

16) H. TAKEUCHI, "On the Earth Tide," *Jour. Fac. Sci., Univ. Tokyo*, 7 (1951), 23-29.

and for the boundary surface between the mantle and the core (where $r=b$):

$$\mu_0 \frac{\partial v_0}{\partial r} = \mu_i \frac{\partial v_i}{\partial r}, \quad \left(\mu_0 \frac{\partial w_0}{\partial r} = \mu_i \frac{\partial w_i}{\partial r} \right), \quad (3.3b)$$

$$v_0 = v_i, \quad (w_0 = w_i). \quad (3.3c)$$

Introducing the equations (3.1) and (3.2) into the above conditions, we get three homogeneous, linear equations, from which by eliminating the coefficients B_n , B'_n and E_n , we can obtain the determinantal equation of the following form

$$\mathfrak{D}_n = \begin{vmatrix} \frac{\partial}{\partial a} \left\{ \frac{J_{n+1/2}(k_0 a)}{a^{3/2}} \right\} & \frac{\partial}{\partial a} \left\{ \frac{Y_{n+1/2}(k_0 a)}{a^{3/2}} \right\} & 0 \\ \frac{\partial}{\partial b} \left\{ \frac{J_{n+1/2}(k_0 b)}{b^{3/2}} \right\} & \frac{\partial}{\partial b} \left\{ \frac{Y_{n+1/2}(k_0 b)}{b^{3/2}} \right\} & -\frac{\mu_i}{\mu_0} \frac{\partial}{\partial b} \left\{ \frac{J_{n+1/2}(k_i b)}{b^{3/2}} \right\} \\ J_{n+1/2}(k_0 b) & Y_{n+1/2}(k_0 b) & -J_{n+1/2}(k_i b) \end{vmatrix} = 0. \quad (3.4)$$

This equation involves the variables n and $k_0 a (\equiv \xi)$, and parameters k_i/k_0 , μ_i/μ_0 and b/a .

3.1. Meaning of n and m .

To make clear the physical meaning of the equation (3.4), we will consider what sort of quantity n and m mean.

When the conditions $n \gg m$ and $n \gg 1$ are both satisfied, it is well known that the associated Legendre function has the approximation formulae

$$\left. \begin{aligned} P_n^m(\cos \theta) &\sim \left(\frac{1}{n \sin \theta} \right)^{1/2} \sin \{ (n+1/2)\theta + \delta \} + 0 \left(\frac{1}{n^{3/2}} \right), \\ Q_n^m(\cos \theta) &\sim \left(\frac{1}{n \sin \theta} \right)^{1/2} \cos \{ (n+1/2)\theta + \delta \} + 0 \left(\frac{1}{n^{3/2}} \right). \end{aligned} \right\} \quad (3.5)$$

Substituting the above equation into (3.1) and (3.2), and assuming the value of D'_m/D_m to be $-i$, we get, as R. Yoshiyama¹⁷⁾ has proved before,

$$\tilde{\omega}_r \sim R_{(r)} \begin{Bmatrix} \sin \\ -\cos \end{Bmatrix} m \phi e^{i\{pt - (n+1/2)\theta\}}. \quad (3.6)$$

(3.6) indicates that $p/(n+1/2)$ represents the angular velocity in the θ -direction in rough approximation, and therefore, $(n+1/2)$ represents the number of waves involved in a spherical surface in the direction of θ .

17) *loc. cit.*, 11).

However, if the condition $n \gg 1$ is not satisfied, $1/n^{3/2}$ cannot be neglected compared with $1/n^{1/2}$. Since we are now considering the case in which n takes small value, the approximation mentioned above is not admissible in our problem.

When n is not so large compared with unity, $p/(n+1/2)$ loses the meaning of angular velocity in the θ -direction, therefore we must consider what meaning the variable of n has. From the nature of Legendre's associated function P_n^m and Q_n^m , we know that $(n-m)$ represents the number of nodal lines in θ -direction, and m the number of nodal lines in ϕ -direction. Therefore we will regard n and m as the parameter by which we can define the mode of vibration.

Here it should be added that m must be an integer, while n can take any arbitrary positive real value. When n is an integer, (3.1) and (3.2) represent a stationary vibration, while if n does not take an integral value, (3.1) and (3.2) represent a vibration in which the nodal lines moves with the lapse of time, or a wave.

3.2. Modification of the equation of vibration.

Now, we will try a modification of fundamental equation of vibration (3.4) in order to get a more simplified form of equation.

First, we will introduce a new quantity

$$\xi = k_0 a = \frac{2\pi a}{V_{s0}} / T, \quad (3.7)$$

where V_{s0} means the velocity of S -waves in the mantle, and $\frac{1}{\xi} = T / \frac{2\pi a}{V_{s0}}$ the period of vibration, the time unit of which is the time necessary for the S -wave to travel around the earth's surface.

Introducing (3.7) into (3.4), we have

$$\begin{aligned} \mathfrak{D}_n \left(\xi, \frac{k_i}{k_0}, \frac{\mu_i}{\mu_0}, \frac{b}{a} \right) \\ = -J_{n+1/2}(k_i b) L_n \left(\xi, \frac{b}{a} \right) + \frac{\mu_i}{\mu_0} \frac{\partial}{\partial b} \left\{ \frac{J_{n+1/2}(k_i b)}{b^{3/2}} \right\} R_n \left(\xi, \frac{b}{a} \right), \quad (3.8) \end{aligned}$$

where

$$L_n = \begin{pmatrix} \frac{\partial}{\partial a} \left\{ \frac{J_{n+1/2}(k_0 a)}{a^{3/2}} \right\} & \frac{\partial}{\partial a} \left\{ \frac{Y_{n+1/2}(k_0 a)}{a^{3/2}} \right\} \\ \frac{\partial}{\partial b} \left\{ \frac{J_{n+1/2}(k_0 b)}{b^{3/2}} \right\} & \frac{\partial}{\partial b} \left\{ \frac{Y_{n+1/2}(k_0 b)}{b^{3/2}} \right\} \end{pmatrix},$$

$$R_n = \left. \begin{array}{cc} \frac{\partial}{\partial a} \left\{ \frac{J_{n+1/2}(k_0 a)}{a^{3/2}} \right\} & \frac{\partial}{\partial a} \left\{ \frac{Y_{n+1/2}(k_0 a)}{a^{3/2}} \right\} \\ J_{n+1/2}(k_0 b) & Y_{n+1/2}(k_0 b) \end{array} \right\} \quad (3.9)$$

4. Determination of the Period of Vibration.

Calculations were performed for two special models of the earth. In the first model, we assumed the core to be in rigid state, and in the second, the core to be in liquid state. In both models, we put the ratio of b (radius of the core) and a (radius of the free surface) as 0.5469 (=2900/6370). The numerical calculations were performed by means of series expansion.

Case I. When the core is perfectly rigid, the ratio k_0/k_i becomes infinite and μ_0/μ_i vanishes. Then in the right hand member of equation (3.8) the first term is neglected compared with the second term, and the period equation takes the following form

$$R_n(\xi, b/a) = s_n(\xi) \cos(1-b/a)\xi + t_n(\xi) \sin(1-b/a)\xi = 0, \quad (4.1)$$

or in a different expression

$$\tan\left(1 - \frac{b}{a}\right)\xi = -\frac{s_n(\xi)}{t_n(\xi)}, \quad (4.2)$$

in which $s_n(\xi)$ and $t_n(\xi)$ are obtained expanding the Bessel functions;

$$\left. \begin{aligned} s_n(\xi) &= \xi^{-(n+3)} \eta^{-(n+1/2)} \left\{ \sum_{r=0}^{\langle n \rangle} \sum_{p=0}^{\langle n \rangle} (-1)^{r+p} c_{2r,n} c_{2p,n} \xi^{2r+1} \eta^{2p} \right. \\ &\quad + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} c_{2r+1,n} c_{2p+1,n} \xi^{2r+2} \eta^{2p+1} \\ &\quad + \sum_r^{\langle n \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p+1} (n-2r+2) c_{2r,n} c_{2p+1,n} \xi^{2r} \eta^{2p+1} \\ &\quad \left. + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n \rangle} (-1)^r (n-2r+1) c_{2r+1,n} c_{2p,n} \xi^{2r+1} \eta^{2p} \right\}, \\ t_n(\xi) &= \xi^{-(n+3)} \eta^{-(n+1/2)} \left\{ \sum_{r=0}^{\langle n \rangle} \sum_{p=0}^{\langle n \rangle} (-1)^{r+p+1} (n-2r+2) c_{2r,n} c_{2p,n} \xi^{2r} \eta^{2p} \right. \\ &\quad + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p+1} (n-2r+1) c_{2r+1,n} c_{2p+1,n} \xi^{2r+1} \eta^{2p+1} \\ &\quad + \sum_r^{\langle n \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p+1} c_{2r,n} c_{2p+1,n} \xi^{2r+1} \eta^{2p+1} \\ &\quad \left. + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p} c_{2r+1,n} c_{2p,n} \xi^{2r+2} \eta^{2p} \right\}, \end{aligned} \right\} \quad (4.3)$$

where

$$\eta = \frac{b}{a} \xi,$$

and

$$\langle n \rangle = \begin{cases} n/2 & \text{when } n \text{ is even,} \\ (n-1)/2 & \text{when } n \text{ is odd.} \end{cases}$$

In this equation, the coefficients c_{pn} which are obtained from the coefficients of the expansion of Bessel Function are calculated as follows :

$$\begin{aligned} c_{00} &= 1 \\ c_{01} &= 1 & c_{11} &= 1 \\ c_{02} &= 3 & c_{12} &= 3 & c_{22} &= 1 \\ c_{03} &= 15 & c_{13} &= 15 & c_{23} &= 6 & c_{33} &= 1 \\ c_{04} &= 105 & c_{14} &= 105 & c_{24} &= 45 & c_{34} &= 10 & c_{44} &= 1 \\ c_{05} &= 945 & c_{15} &= 945 & c_{25} &= 420 & c_{35} &= 105 & c_{45} &= 15 & c_{55} &= 1 \end{aligned}$$

The roots of equation (4.2) are given by the ordinates of cross point of the curves of $\tan(1-b/a)\xi$ and $-s_n(\xi)/t_n(\xi)$. For any arbitrarily given n (of which the physical meaning is discussed in section 3), we can get a series of roots ξ_n , which are tabulated in Table I. The letter

Table I. The Roots of Characteristic Equation $R_n(\xi, b/a)=0$.

(Rigid Core, $1/\xi = T / \frac{2\pi a}{V_{s0}}$, $k_0/k_t = \infty$, $\mu_0/\mu_t = 0$, $b/a = 0.5469$)

N n	0		I		II		III	
	ξ	$1/\xi$	ξ	$1/\xi$	ξ	$1/\xi$	ξ	$1/\xi$
0	1.160	0.8261	9.963	0.1014	17.05	0.0587	24.08	0.0415
1	2.000	0.5000	10.14	0.0986	17.15	0.0583	24.2	0.0413
2	3.045	0.328	10.49	0.0953	17.4	0.0575	24.4	0.0410
3	4.30	0.233	10.8	0.0936	17.5	0.0570		
4	5.40	0.185	11.5	0.0870				

N stands for the number of nodal plane in r -direction, which should be explained in a latter section.

Case II. After the same method as the foregoing calculation, we can obtain the roots for the case of liquid core. In this case the ratio

k_0/k_i vanishes and μ_0/μ_i becomes infinite. Then the period equation (3.8) takes the following form

$$L_n(\xi, b/a) = g_n(\xi) \cos(1-b/a)\xi + h_n(\xi) \sin(1-b/a)\xi = 0, \quad (4.4)$$

in which $g_n(\xi)$ and $h_n(\xi)$ are obtained as before, namely

$$\begin{aligned} g_n(\xi) = & (\xi\eta)^{-n-3} \left\{ \sum_{r=0}^{\langle n \rangle} \sum_{p=0}^{\langle n \rangle} (-1)^{r+p+1} (n-2p+2) c_{2r,n} c_{2p,n} \xi^{2r+1} \eta^{2p} \right. \\ & + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p+1} (n-2p+1) c_{2r+1,n} c_{2p+1,n} \xi^{2r+2} \eta^{2p+1} \\ & + \sum_r^{\langle n \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p} (n-2r+2) c_{2r,n} c_{2p,n} \xi^{2r} \eta^{2p+1} \\ & + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} (n-2r+1) c_{2r+1,n} c_{2p+1,n} \xi^{2r+1} \eta^{2p+2} \\ & + \sum_r^{\langle n \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} (n-2r+2)(n-2p+1) c_{2r,n} c_{2p+1,n} \xi^{2r} \eta^{2p+1} \\ & + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p+1} (n-2r+1)(n-2p+2) c_{2r+1,n} c_{2p,n} \xi^{2r+1} \eta^{2p} \\ & + \sum_r^{\langle n \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} c_{2r,n} c_{2p+1,n} \xi^{2r+1} \eta^{2p+2} \\ & \left. + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p+1} c_{2r+1,n} c_{2p,n} \xi^{2r+2} \eta^{2p+1} \right\}, \\ h_n(\xi) = & (\xi\eta)^{-n-3} \left\{ \sum_{r=0}^{\langle n \rangle} \sum_{p=0}^{\langle n \rangle} (-1)^{r+p} c_{2r,n} c_{2p,n} \xi^{2r+1} \eta^{2p+1} \right. \\ & + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} c_{2r+1,n} c_{2p+1,n} \xi^{2r+2} \eta^{2p+2} \\ & + \sum_r^{\langle n \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p} (n-2r+2)(n-2p+2) c_{2r,n} c_{2p,n} \xi^{2r} \eta^{2p} \\ & + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} (n-2r+1)(n-2p+1) c_{2r+1,n} c_{2p+1,n} \xi^{2r+1} \eta^{2p+1} \\ & + \sum_r^{\langle n \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p+1} (n-2r+2) c_{2r,n} c_{2p+1,n} \xi^{2r} \eta^{2p+2} \\ & + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p} (n-2r+1) c_{2r+1,n} c_{2p,n} \xi^{2r+1} \eta^{2p+1} \\ & + \sum_r^{\langle n \rangle} \sum_p^{\langle n-1 \rangle} (-1)^{r+p} (n-2p+1) c_{2r,n} c_{2p+1,n} \xi^{2r+1} \eta^{2p+1} \\ & \left. + \sum_r^{\langle n-1 \rangle} \sum_p^{\langle n \rangle} (-1)^{r+p+1} (n-2p+2) c_{2r+1,n} c_{2p,n} \xi^{2r+2} \eta^{2p} \right\}. \end{aligned} \quad (4.5)$$

The roots of this equation are shown in Table II.

In the Tables I and II there exist many roots ξ_n for a given value of n (which are denoted by the letter $N=0, I, II, III, \dots$). These roots give the same distribution of displacement on the spherical surface, but

Table II. The Roots of Characteristic Equation $L_n(\xi, b/a)=0$.(Liquid Core. $1/\xi=T/2\pi a/V_{s0}$; $k_0/k_i=0$, $\mu_0/\mu_i=\infty$, $b/a=0.5469$.)

$n \backslash N$	0		I		II	
	ξ	$1/\xi$	ξ	$1/\xi$	ξ	$1/\xi$
0	—	—	7.362	0.1358	14.1	0.0709
1	—	—	7.62	0.131	14.2	0.0704
2	2.415	0.414	8.18	0.122	14.5	0.0690
3	3.79	0.264	8.9	0.112	14.9	0.0671
4	4.9	0.20	9.9	0.101		

differ in its distribution of amplitude in r -direction. For example, calculated distribution of amplitude in r -direction for the branch $N=0, I,$ and II ($n=2$, rigid core) are shown in Fig. 2. Here we can observe that the branch number 0, I and II indicate the number of nodal planes in r -direction. The branch 0 gives the lowest value of ξ_n , that is, the longest period of vibration.

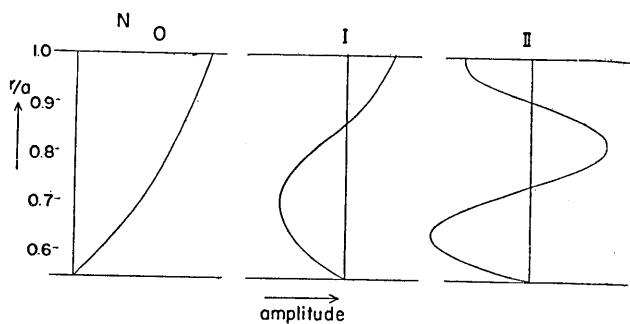


Fig. 2.

5. Conclusion.

The results of calculation are shown in Fig. 3. As stated above, the longest period of vibration is given by the reciprocal of the root on branch $N=0$. In the case of $n=2$, for instance, the longest period measured by the unit $2\pi a/V_{s0}$ (see (3.7), $\frac{1}{\xi} = T/2\pi a/V_{s0}$) are 0.328 for rigid core and 0.414 for liquid core respectively. If we adopt 6.5

km./sec. as the *S*-wave velocity in the mantle, the actual periods will be expressed by the relation

$$T = 1/\xi \times 6.16 \times 10^3 \text{ sec.}$$

and are about 33.7 min. for rigid core and 42.5 min. for liquid core.

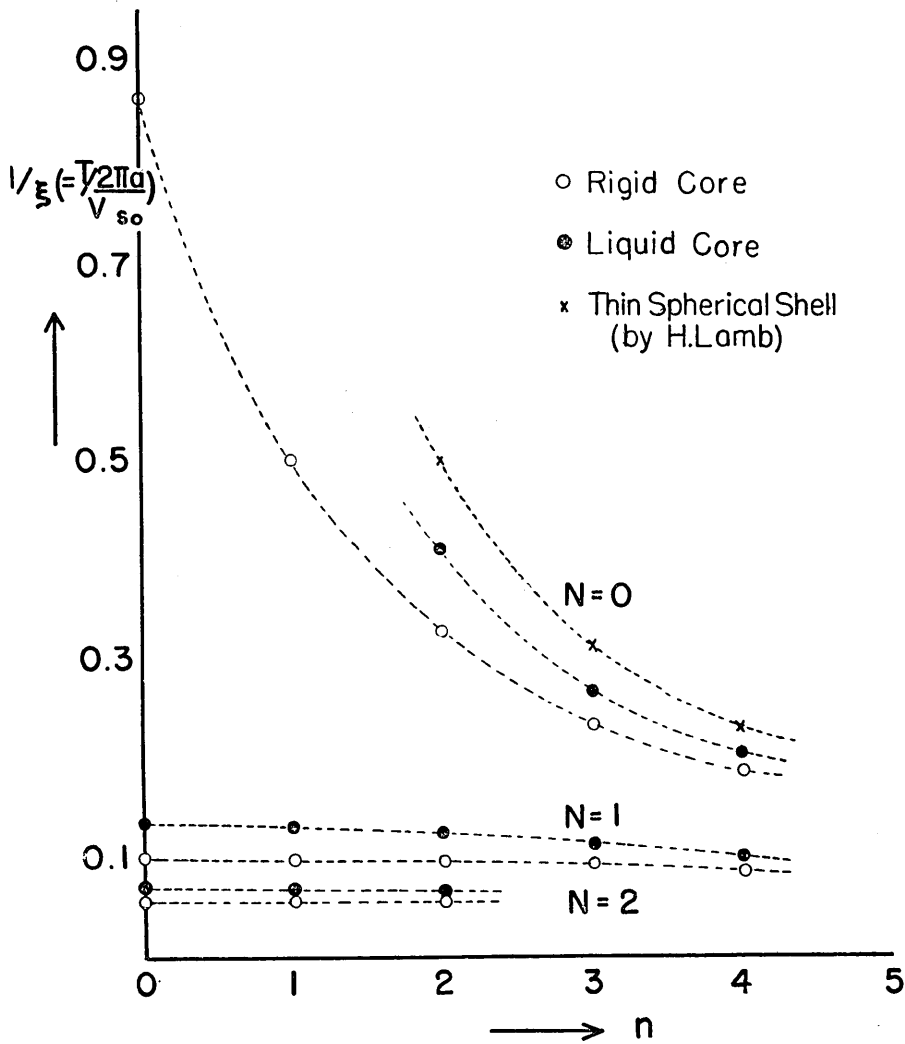


Fig. 3.

When the ratios k_0/k_i and μ_0/μ_i are finite and give the intermediate state of rigid and liquid core, the periods will also give the intermediate values of them.

Thus it may be possible to infer the ratios k_0/k_i and μ_0/μ_i if we have enough observations distributed over the earth's surface to make clear the period and the mode of vibration. But in actual observations such long period as cited above must be left to future consideration.

It is also necessary to calculate the period of the vibration of the second class, that is, the combined oscillation of the dilatational and rotational motion. This sort of vibration is conjectured to be more affected by the internal constitution than that of the first class, and its consideration is left to a future case.

18. 層のある地球の弾性振動 (第1種の振動の周期)

地震研究所 { 松 本 利 松
 { 佐 藤 泰 夫

弾性球としての地球の固有振動については H. Lamb のそれを始めとして多くの研究がなされている。ここでは Core と Mantle より成る地球に対して、第1種の振動の固有周期を求めた。振動の球面上における各 Mode についてそれぞれ一連の固有周期の群が存在するが、それらは半径方向に異なった振巾分布を示す振動に対応する。

この振動周期は核の内外における弾性常数及び密度の比の値によって変化するが、 $n=2$ (θ -又は φ -方向に2つの節平面が存在する)の場合には核が剛体であるか流体であるかによって、最も長い固有周期は、Mantle の S-波速度を 6.5km/sec として、約 33.7分及び 42.5分となる。実際にこのような振動が観測されれば、地球の Core の状態を推定する有力な手懸りを与えるであろう。
