# 26. *Relation between the Amplitude of Earthquake Motions and the Nature of Surface Layer. III.*

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The present paper is the continuation of our research work on the problem of earthquake movement of superficial stratum and deals with the case where the primary seismic waves are incident upwards normally to trebly stratified layers residing on the surface of a semi-infinite body.

Let, $\rho_1, \mu_1, V_1, H_1$; $\rho_2, \mu_2, V_2, H_2$; $\rho_3, \mu_3, V_3, H_3$ be the density, rigidity, velocity and the thickness of the first layer, the second layer and the third layer respectively, and $\rho_4, \mu_4, V_4$, be the density, rigidity, velocity of the bottom medium. Again, let $U_1, U_2, U_3, U_4$, be the respective displacements in the first layer, the second layer, the third layer and the bottom medium. And, $H$ represents the thickness between the free surface and the bottom medium, that is, $H=H_1+H_2+H_3$.

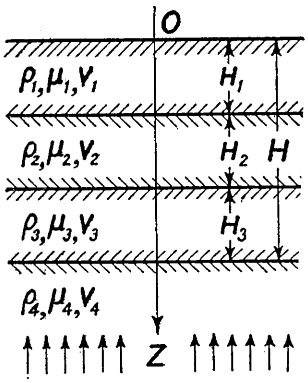


Fig. 1.

Then the equation of motion of each medium is written as follows;

$$\mu_n \frac{\partial^2 U_n}{\partial z^2} = \rho_n \frac{\partial^2 U_n}{\partial t^2} . \qquad [n=1,2,3,4] \tag{1}$$

The typical solutions of this equation are expressed by

$$\left.\begin{aligned} U_4 &= \mathfrak{A} e^{i(pt+f_4 z)} + B_4 e^{i(pt-f_4 z)}, \\ U_n &= A_n e^{i(pt+f_n z)} + B_n e^{i(pt-f_n z)}, \quad [n=1,2,3] \end{aligned}\right\} \tag{2}$$

where $\mathfrak{A}$ is the amplitude of the incident waves and $B_4, B_3, B_2, B_1$; $A_3, A_2, A_1$ are arbitrary constants to be determined from the boundary conditions. $2\pi/p$ and $2\pi/f$ mean the period and the wave length.

The boundary condition at the plane $z=0$ is

$$\frac{\partial U_1}{\partial z} = 0, \tag{3}$$

and the boundary conditions at the planes $z=H_1$, $z=H_1+H_2$, $z=H_1+H_2+H_3$ ($\equiv H$) are as follows;

$$U_n=U_{n+1}\,, \tag{4}$$

$$\mu_n\frac{\partial U_n}{\partial z}=\mu_{n+1}\frac{\partial U_{n+1}}{\partial z}\,. \qquad [n=1,2,3] \tag{5}$$

In the case of dilatational waves, it is necessary to replace $\mu$ by $\lambda+2\mu$. Putting equation (2) into equations (3)-(5), we find finally

$$\frac{|U_{1_{H_1=0}}|}{2|U_0|}=\frac{4}{\sqrt{\Phi_1^2+\Phi_2^2}}\,, \tag{6}$$

in which

$$\left.\begin{aligned}\Phi_1=&(1+\alpha_1)(1+\alpha_2)\cos X_1+(1+\alpha_1)(1-\alpha_2)\cos X_2\\&+(1-\alpha_1)(1-\alpha_2)\cos X_3+(1-\alpha_1)(1+\alpha_2)\cos X_4,\\\Phi_2=&\alpha_3\{(1+\alpha_1)(1+\alpha_2)\sin X_1-(1+\alpha_1)(1-\alpha_2)\sin X_2\\&+(1-\alpha_1)(1-\alpha_2)\sin X_3-(1-\alpha_1)(1+\alpha_2)\sin X_4\},\end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned}&\alpha_1=\frac{\rho_1V_1}{\rho_2V_2}\,,\quad \alpha_2=\frac{\rho_2V_2}{\rho_3V_3}\,,\quad \alpha_3=\frac{\rho_3V_3}{\rho_4V_4}\,,\\&X_1=\frac{pH_1}{V_1}\left(1+\frac{H_2V_1}{H_1V_2}+\frac{H_3V_1}{H_1V_3}\right),\quad X_2=\frac{pH_1}{V_1}\left(1+\frac{H_2V_1}{H_1V_2}-\frac{H_3V_1}{H_1V_3}\right),\\&X_3=\frac{pH_1}{V_1}\left(1-\frac{H_2V_1}{H_1V_2}+\frac{H_3V_1}{H_1V_3}\right),\quad X_4=\frac{pH_1}{V_1}\left(1-\frac{H_2V_1}{H_1V_2}-\frac{H_3V_1}{H_1V_3}\right).\end{aligned}\right\} \tag{8}$$

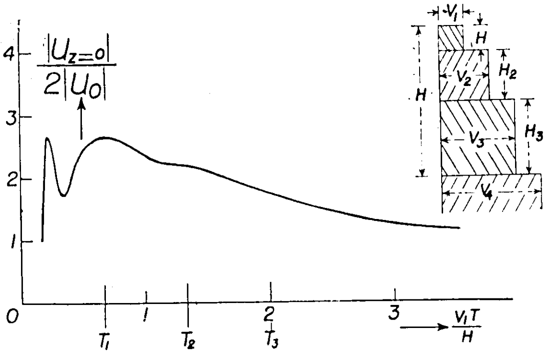


Fig. 2. $H_1:H_2:H_3=1:2:3$, $V_1:V_2:V_3:V_4=1:2:3:4$, $\rho_1=\rho_2=\rho_3=\rho_4$.

Some special cases are calculated using equations (6), (7) and (8), and are plotted in the following drawings. Figs. 2, 3 and 4 are the cases of (i) $H_1:H_2:H_3:=1:2:3$, $V_1:V_2:V_3:V_4=1:2:3:4$, (ii) $H_1:H_2:H_3=1:2:6$, $V_1:V_2:V_3:V_4=1:2:3:4$ and (iii) $H_1:H_2:H_3=1:3:6$, $V_1:V_2:V_3:V_4=1:2:4:8$ respectively, besides the conditions $\rho_1=\rho_2=\rho_3=\rho_4$ are in common. In these figures, $T_1$, $T_2$, and $T_3$ represent the period of free oscillations of the first layer, the upper two layers and the whole layers respectively, that is, $T_1=4H_1/V_1$, $T_2=4(H_1/V_1+H_2/V_2)$ and $T_3=4(H_1/V_1+$

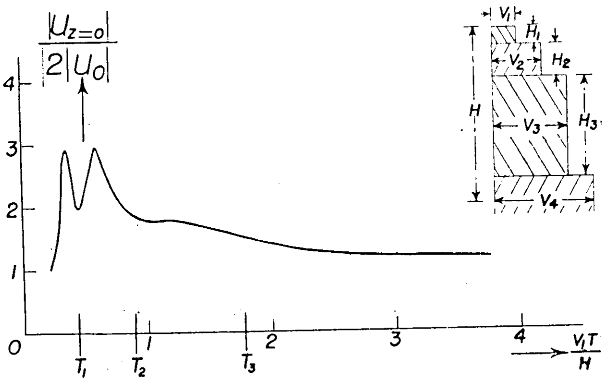


Fig. 3. $H_1:H_2:H_3=1:2:6$, $V_1:V_2:V_3:V_4=1:2:3:4$, $\rho_1=\rho_2=\rho_3=\rho_4$.

$H_2/V_2+H_3/V_3$). Figs. 2, 3 and 4 show that, even when the period of seismic waves synchronize with $T_2$ or $T_3$, the amplitude on the free surface becomes slightly larger than the cases of other periods. In other words, Figs. 2, 3 and 4 tell us that, neither in the upper two layers nor in the whole layers, resonance-like phenomena appear clearly.

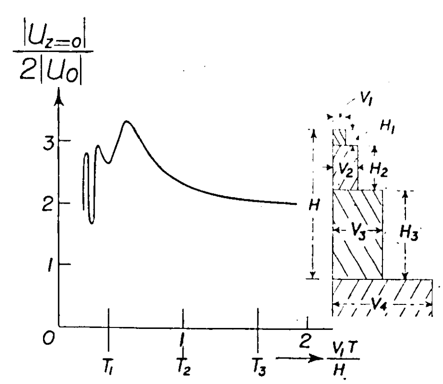


Fig. 4. $H_1:H_2:H_3=1:3:6$, $V_1:V_2:V_3:V_4=1:2:4:8$, $\rho_1=\rho_2=\rho_3=\rho_4$.

On the other hand, the resonance-like phenomenon of the first layer appeared clearly, and the maximum amplitude on the free surface is somewhat small compared with that of a superficial layer.

The cause of the phenomena mentioned above is the coherent effect which exists among the waves reflected at different boundary planes.

In order to ascertain the coherent effect of the seismic waves in the surface stratum, we illustrate the results of the numerical calculations of such cases as doubly stratified layers,[1] a stratified layer[2] and a surface layer of varying elasticity,[3] which were studied previously, in Figs. 5–8.

1) K. KANAI, "Relation between the Nature of Surface Layer and the Amplitude of Earthquake Motions. II", *Bull. Earthq. Res. Inst.*, **31** (1953), 219.

2) K. SEZAWA and K. KANAI, "Decay Constants of Seismic Vibrations of a Surface Layer", *Bull. Earthq. Res. Inst.*, **13** (1935), 251.

3) K. SEZAWA and K. KANAI, "The Rate of Damping in Seismic Vibrations of a Surface Layer of Varying Density or Elasticity", *Bull. Earthq. Res. Inst.*, **13** (1935), 484.

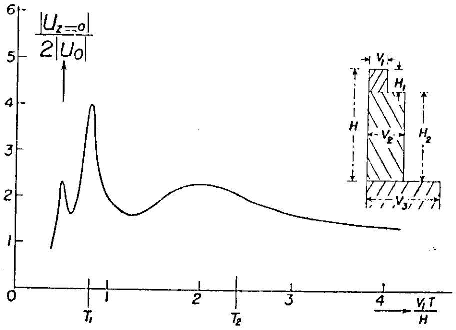


Fig. 5. The case of doubly stratified layers. $H_1:H_2=1:4$, $V_1:V_2:V_3=1:2:4$, $\rho_1=\rho_2=\rho_3$.

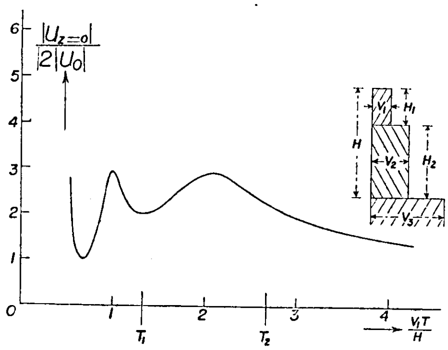


Fig. 6. The case of doubly stratified layers. $H_1:H_2=1:2$, $V_1:V_2:V_3=1:2:4$, $\rho_1=\rho_2=\rho_3$.

From Figs. 2–8, it appears that the differences of the amplitude on the free surface with respect to the period of incident waves decrease with the increase in the number of layers. And, it seems that the maximum amplitude on the free surface decreases with the increase in the number of layers, when the ratio of the velocity of the surface medium to that of the bottom medium is constant.

The results of the observational studies show that, when the superficial stratum consists of more than two layers as in the downtown of Tokyo, the relation of frequency to period of the ground movements which are caused by earthquake or ground noise is represented by a flat curve. On the other hand, when the superficial stratum is a layer such as in the up-town of Tokyo, the relation of frequency to period of the ground movements is described by a predominant sharp peak.

Now, it is noteworthy that the diagrammatical features of the results of the present theoretical investigations closely resembles the

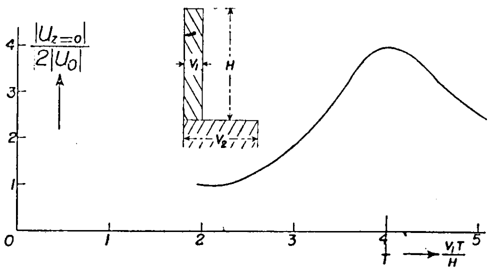


Fig. 7. The case of a stratified layer. $V_1:V_2=1:4$, $\rho_1=\rho_2$.

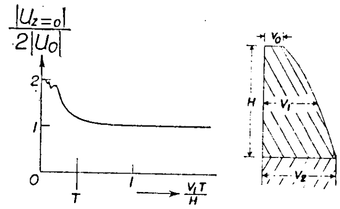


Fig. 8. The case of a surface layer of varying elasticity. $V_0:V_2=1:4$, $V_1\propto\sqrt{z}$, $\rho_1=\rho_2$.

diagramms regarding the frequency-period relation of the ground movements caused by earthquake or ground noise.

Though there are a number of subjects under consideration concerning the present investigation, they will be discussed later.

## 26. 地震動振幅と地表層の性質との関係 第4報

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弾性波地下探査の結果によると，冲積層が相当厚い土地では，弾性波速度の不連続面が，3つある場合が少くない．それで今回は，地表面近くに1層或は2層ある場合の前回の研究につづいて，3層ある場合の数理的研究を行つた．

地震波が2以上の不連続面で反射屈折をすると，それらの波が干渉し合つて，いわゆる重複反射によつて地表面の振幅が増大する度合は減らされることになる．

その結果，地表と第2層の下面の間，又は，地表と第3層の下面の間の自己振動周期にあたる周期の波が来た場合でも，地表面振幅の特別な増大はおこらず，それらの周期よりも，少し短い周期の波が来た場合に，振幅の多少の増大がある．しかし，地表第1層の自己振動周期にあたる周期の波の場合，地表面振幅は相当に増大する．

従つて，地表層の層数が増すほど，地表面振幅と震動周期との関係を示す曲線は平らになることになる．

厚い冲積層上の地震或は自然微動記録から，頻度周期曲線，振幅周期曲線を求めると，一般に，0.6秒とか1秒とかいう比較的に長い周期のところに，山ができる場合と，山がほとんどみとられない場合とがある．しかし，0.2秒とか0.4秒とかの比較的短い周期のところには，大抵の場合に，はつきりした山がみとめられる．地盤震動記録に対する，以上の解析結果は，今回の研究で，一応，理論的な説明がつく．