20. *Observational Study of Earthquake Motion in the Depth of the Ground. IV.* *(Relation between the Amplitude at Ground Surface and the Period.)*

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1. Introduction.

Since the problem of comparing the earthquake motions in the depth with the motions on the surface of ground has attracted the attention of many researchers, many mathematical studies[1] as well as observational ones[2] have been made till now.

Recently this problem has been given so much importance even in the fields of architecture and civil engineering, that the ground survey is under way at various places in Japan.

This paper, in comparing the earthquake-motions in the depth with the motions at two points on the ground which are of different ground property and very adjacent to each other, concerns the vibration at surface layer.

2. Observational results.

Observations were made at the following three places: 300 m in depth under the ground of Hitachi Mine (paleozoic system) in Ibaraki Prefecture; the ground surface of the same mine; Hitachi First High School (on alluvium) about 6 km east-south of Hitachi Mine.

These places and the epicentres treated in this paper are shown in Fig. 1. The representative seismograms of the earthquakes used in this investigation are illustrated in Figs. 2–5. And Table I shows the maxi-

1) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **8** (1930), 1;
R. TAKAHASI and K. HIRANO, *Bull. Earthq. Res. Inst.*, **19** (1941), 534;
K. KANAI, *Bull. Earthq. Res. Inst.*, **28** (1950), 31; **30** (1952), 31.

2) N. NASU, *Bull. Earthq. Res. Inst.*, **9** (1931), 454;
U. INOUYE, *Bull. Earthq. Res. Inst.*, **12** (1934), 712;
T. SAITA and M. SUZUKI, *Bull. Earthq. Res. Inst.*, **12** (1934), 517;
K. KANAI and T. TANAKA, *Bull. Earthq. Res. Inst.*, **29** (1951), 107.

mum amplitude of S-waves and the average period of those waves of 14 earthquakes treated in this paper. This average period was struck with 10–20 waves in the vicinity of the maximum amplitude (the waves within the range, containing the wave of maximum amplitude, limited by the waves of a half amplitude of the maximum). The amplitude shown in Table I is the average of 3 waves in the vicinity of the maximum.

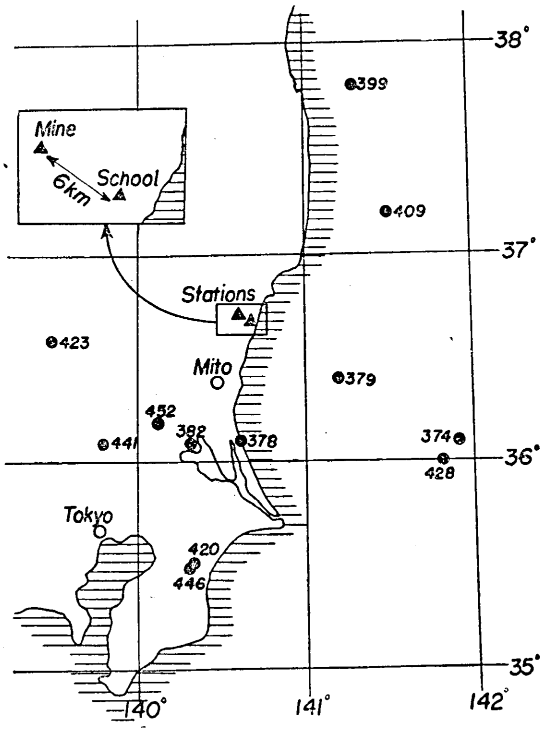

Fig. 1.

From Table I, we get the ratios of the amplitudes at the surfaces of paleozoic system (Mine) and alluvium (School) to the amplitude at the point of 300 m underground. The relation between the ratios thus obtained and the period of earthquake-motions at 300 m depth is represented in Table II and Figs. 6 and 7.

Then, from the seismograms of the surfaces of paleozoic system (Mine) and alluvium (School), the frequency of the number of waves concerned with the period of earthquake-motions is plotted in Figs. 8–11.

Table I.

No.	Date			Hypocenter			Average Period (sec)			Amplitude (μ)		
				φ	λ	H (km)	300 m Depth	Surface Mine	Surface School	300 m Depth	Surface Mine	Surface School
374	1952	XI	16	36.1	141.9	40	0.68	0.38	0.38	10	17	53
378	1952	XI	18	36.1	140.6	60	0.29	0.15	0.31	4.4	16	16
379	1952	XI	19	36.4	141.2	40	0.25	0.12	0.19	2.2	10	12
382	1952	XI	19	36.1	140.3	40	0.37	0.15	0.24	5.0	10	15
390	1952	XI	22	31.0	137.0	300	0.29	0.25	0.32	2.9	7.5	13
399	1952	XI	27	37.8	141.3	60	0.30	0.12	0.17	1.9	4.6	7.4
409	1952	XII	11	37.2	141.5	30	0.22	0.14	0.21	4.5	15	40
414	1952	XII	16	36.1	139.9	40	0.13	0.10	0.16	1.0	6.0	3.4
420	1952	XII	25	35.5	140.3	50	0.30	0.21	0.24	4.0	12	18
423	1953	I	12	36.6	139.5	10	0.21	0.13	0.24	1.2	4.7	10
428	1953	I	14	36.0	141.8	60	0.41	0.31	0.48	1.7	27	70
441	1953	I	26	36.1	139.8	40	0.28	0.14	0.21	6.5	16	50
446	1953	II	3	35.5	140.3	50	0.26	0.19	0.28	8.5	19	70
452	1953	II	14	36.2	140.1	50	0.22	0.11	0.24	2.5	6.0	11

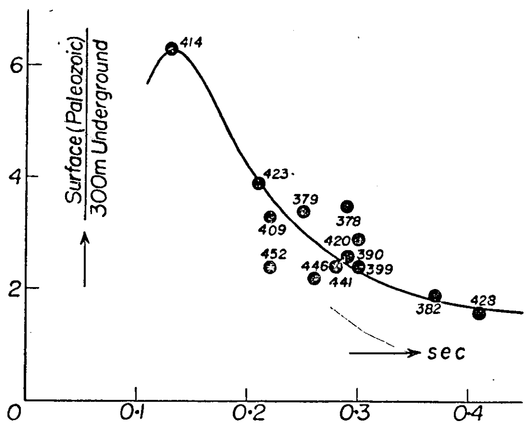

Fig. 6. Relation between the amplitude ratio of Mine to 300 m depth and the period at 300 m depth.

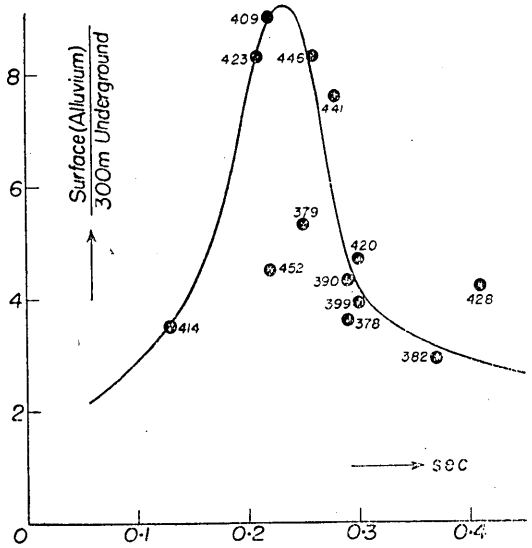

Fig. 7. Relation between the amplitude ratio of School to 300 m depth and the period at 300 m depth.

Table II.

No.	Average period at 300 m depth (sec)	Amplitude ratio	
		Mine/300 m	School/300 m
374	0.68	1.7	5.3
378	0.29	3.5	3.6
379	0.25	3.4	5.3
382	0.37	1.9	2.9
390	0.29	2.6	4.3
399	0.30	2.4	3.9
409	0.22	3.3	9.0
414	0.13	6.3	3.5
420	0.30	2.9	4.7
423	0.21	3.9	8.3
428	0.41	1.6	4.2
441	0.28	2.4	7.6
446	0.26	2.2	8.3
452	0.22	2.4	4.5

Figs. 6, 7 and Figs. 8–11 make it clear that the amplitude at ground surface becomes maximum when the period of earthquake-motions in the depth of 300 m coincides with the period which corresponds to the peak of period frequency. Supposing that the period corresponding to the peak

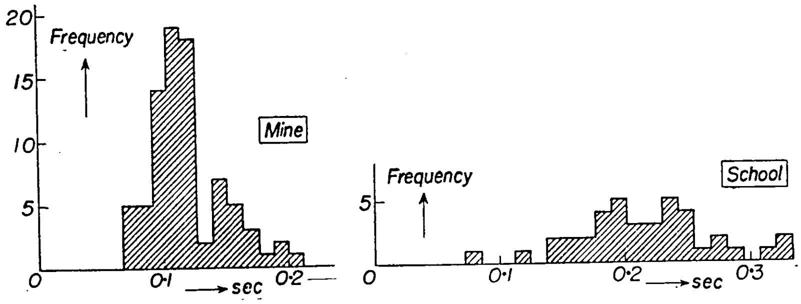

Fig. 8. Period frequency of No. 378 earthquake.

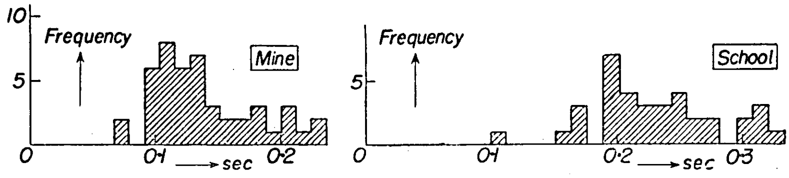

Fig. 9. Period frequency of No. 382 earthquake.

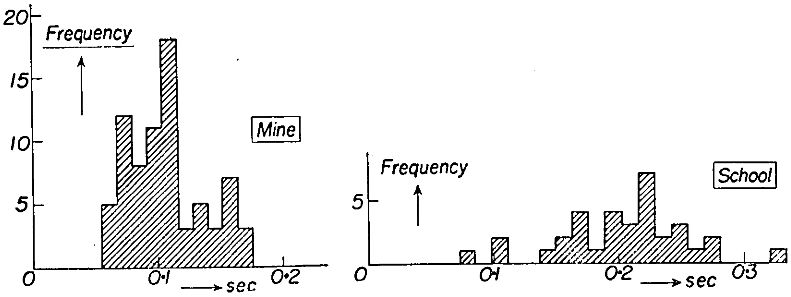

Fig. 10. Period frequency of No. 409 earthquake.

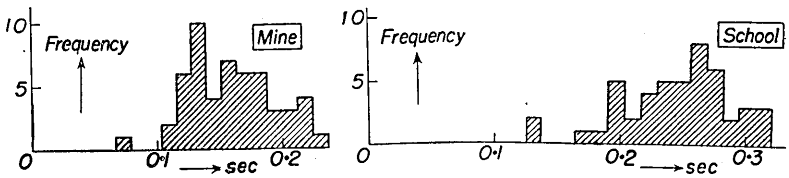

Fig. 11. Period frequency of No. 446 earthquake.

frequency of the period at the ground surface indicates the natural period of the ground, the relation between the amplitude at the surface and the period of earthquake-motions shown in Figs. 6 and 7 coincides very well with the results of mathematical study.

3. Theoretical examination.

As this paper treats only the waves of short period, about 0.1–0.3 sec, waves which are reflected at the surface come back to the point of 300 m deep under the ground with the delay of more than one period compared with waves which come directly from the origin. Consequently it can be considered that the amplitude and the period of waves in the vicinity of the maximum amplitude of S-waves at the point of 300 m in the depth are under little influence of the waves reflected at the free surface.

In other words, even if the waves in the depth of 300 m are the ones incident upon the under surface of surface layer, the error may be small as far as it concerns few waves in the vicinity of the maximum of S-waves.

When the periodical waves continue infinitely, advance vertically upward and synchronize with the natural period of the surface layer, the ratio between the amplitude at the free surface and that of primary waves, that is A_s/A_0, will be

$$A_s/A_0 = 2V_2\rho_2/V_1\rho_1 , \qquad (1)$$

in which V_1, ρ_1 and V_2, ρ_2 represent the velocities and the densities of surface layer and subjacent medium respectively. In that case the damping due to the solid viscosity of media is neglected.

From the maximum value in Figs. 6, 7 and equation (1), we obtain the value of $V_2\rho_2/V_1\rho_1$. At Mine $V_2\rho_2/V_1\rho_1=3$, while at School $V_2\rho_2/V_1\rho_1=4.5$.

Then, in case the waves of finite train which synchronize with the natural period of the surface layer come, the ratio between the maximum amplitude at the free surface and amplitude of incident waves, A_s/A_0, becomes

$$\frac{A_s}{A_0}=\frac{2}{\alpha}\left\{1-\left(\frac{1-\alpha}{1+\alpha}\right)^n\right\}. \qquad (2)$$ [3]

Where $\alpha=V_1\rho_1/V_2\rho_2$ and n represents the number of successive waves (See Fig. 12). From equation (2) the relation between the amplitude

3) K. SEZAWA, *loc. cit.*, 1).

ratio and wave number in the case of $\alpha=1/3$ (Mine) and $\alpha=1/4.5$ (School) is plotted in Fig. 12.

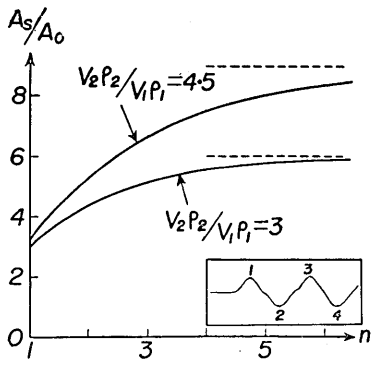

Fig. 12.

Fig. 12 makes it known that, in the case of observation at such places as treated here, if 3 or 5 waves of which the periods are nearly equal to the natural period of the surface layer appear in succession, the amplitude at free surface may become approximate to the amplitude in the case of perfect synchronization.

From equation (2) and Fig. 12, it is also seen that with the surface layer of soft substance the amplitude at the free surface becomes very large if there come such a seismic-waves as will cause synchronization in the surface layer. But even if the period of primary waves is very approximate to the natural period of surface layer, the amplitude at the free surface may be almost constant, regardless of hardness of the substance of layer, unless the number of primary waves appearing in succession is satisfactory.

In brief, such a property is the same with the forced vibration of pendulum in consideration of damping.

4. Conclusion.

The results of the comparative observations made at Hitachi Mine, both on the surface and in the depth of 300 m under the ground, and at Hitachi First High School (on alluvium) about 6 km from Hitachi Mine, proved the followings more evidently.

(1) In case where the period of the earthquake-motions in the depth of 300 m coincides with the period which corresponds to the peak of period frequency at the surface, the amplitude at the ground surface becomes maximum. Suppose such period as corresponds to the peak of period frequency to be the natural period of surface layer. Then it is possible to admit the above-mentioned fact to be a synchronization (resonance) of surface layer.

(2) If the surface layer is of soft substance, the amplitude at the surface becomes very large if there come seismic-waves which may cause a synchronization (resonance) at surface layer. But if the waves which appear in succession are few, the difference of ground property have

little influence upon the amplitude at the surface, even if the period of primary waves may be approximate to the natural period of surface layer.

In conclusion, we wish to express our hearty thanks to the members of Motoyama Office, Hitachi Mine and the authorities of Hitachi First High School for their contributions to this investigation. We also wish to express our thanks to Messrs. T. Tanaka and T. Suzuki who took part in the observation.

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20. 地下深所における地震動の観測結果 (第4報)

地震研究所 {金井清 / 長田甲斐男 / 吉沢静代}

地盤の差違によって，地震動の性質が異る問題は，古くから多くの地震学者の注意をひき，数多くの験震学的並びに数理的な研究が行われて来た．

この問題は，耐震構造上大切な事柄なので，近年になって，日本の各都市でいろんな方法による地盤調査が行われようとしている．

この研究は，日立鉱山の地下 300 m と地表面及び日立鉱山から約 6 km 離れた冲積層上とで地震動の比較観測を行い，地表附近の振動的性質の究明を試みたものである．

研究の結果，地下 300m の地震動の周期が，地表面の地震動の周期頻度の山にあたる周期に一致する場合に，地表面の振幅が最も大きくなる．しかして，S 波の最大振幅附近では，地下 300m の地震動は，一応，地表層と下層との境界面に入射する波と考えてよさそうである．従って，周期頻度の山にあたる周期を，表面層の固有周期と考えれば，地表面の振幅と周期との関係は表面層の同期現象に関係するものと解釈される．

次に，表面層に同期現象を起すような地震波が来た場合には，表面層の物質が軟い程，地表面の振幅の大きさは増す．しかし，たとえ入射波の周期が表面層の固有周期に近いものであっても，入射波の連続する波数がごく少い場合には，地盤の硬軟の差違は地表面の振巾にあまり影響を与えないことも明かになった．

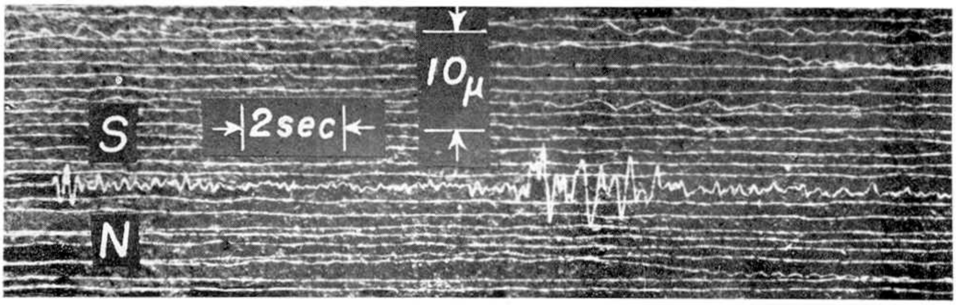

Fig. 2-a. Earthquake No. 378. 300 m depth.

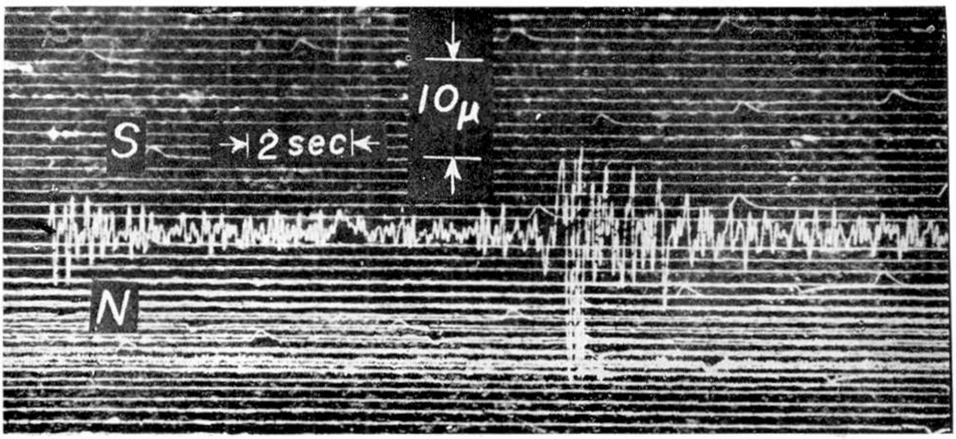

Fig. 2-b. Earthquake No. 378. Mine.

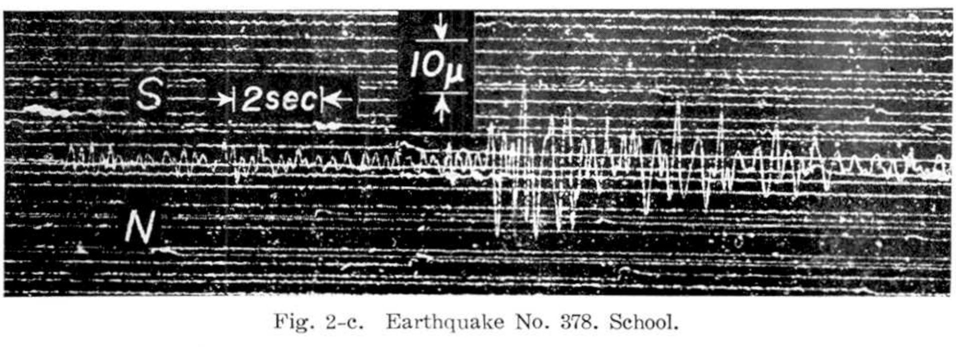

Fig. 2-c. Earthquake No. 378. School.

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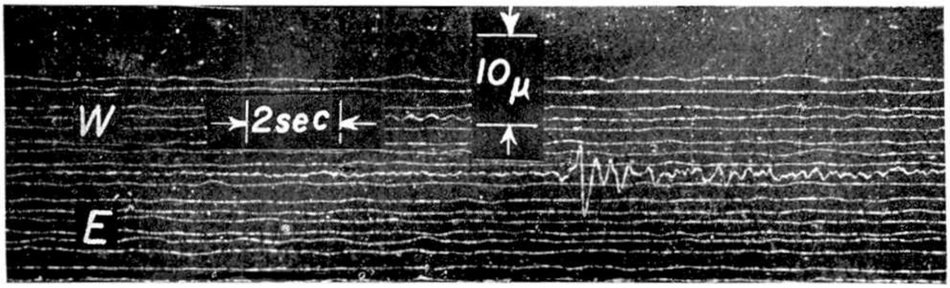

Fig. 3-a. Earthquake No. 382. 200 m depth.

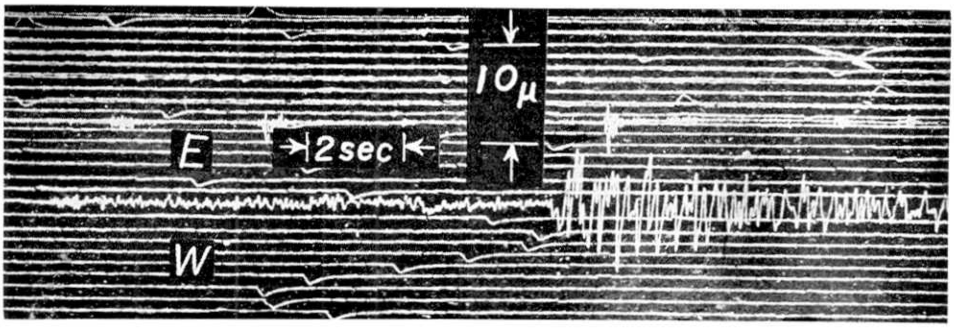

Fig. 3-b. Earthquake No. 382. Mine.

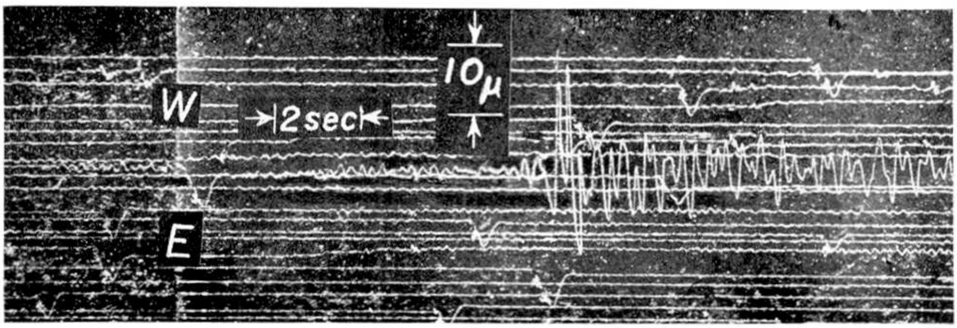

Fig. 3-c. Earthquake No. 382. School.

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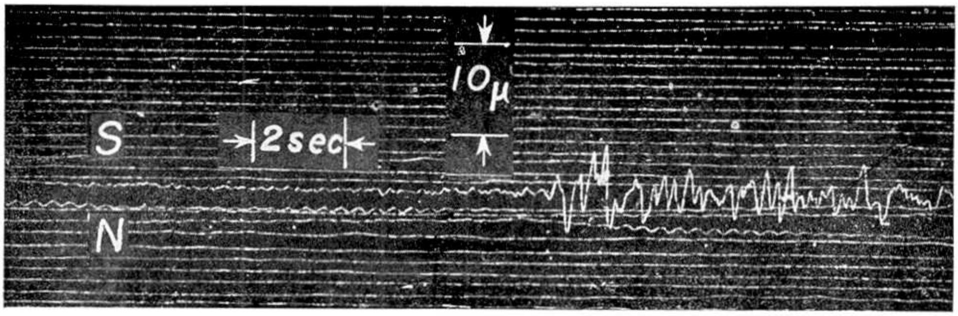

Fig. 4-a. Earthquake No. 409. 300 m. depth.

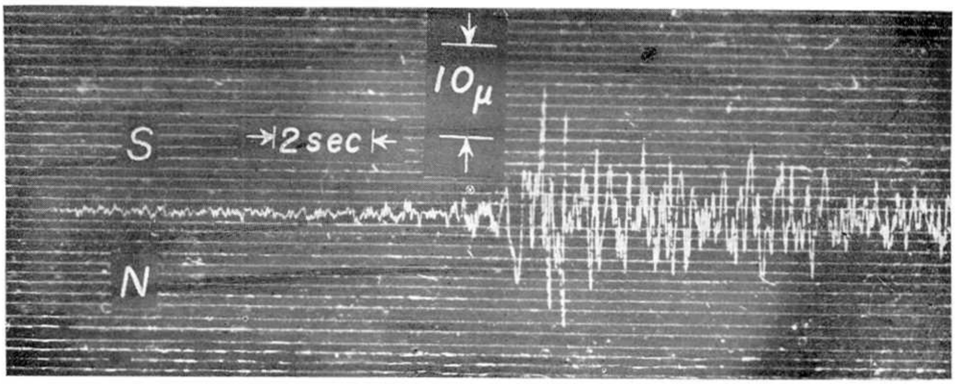

Fig. 4-b. Earthquake No. 409. Mine.

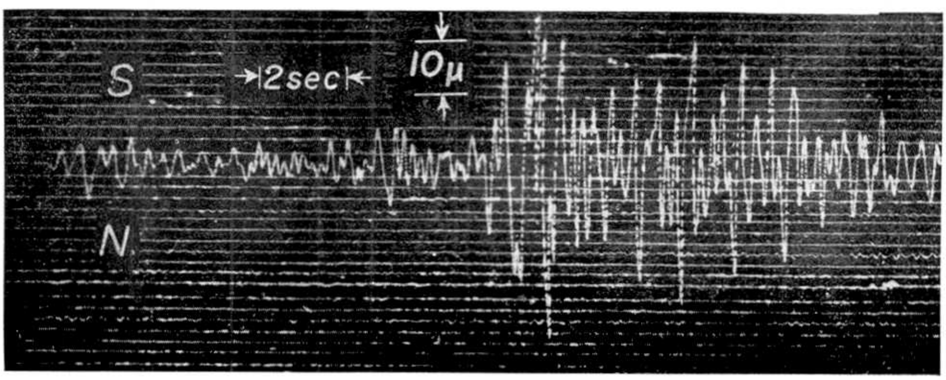

Fig. 4-c. Earthquake No. 409. School.

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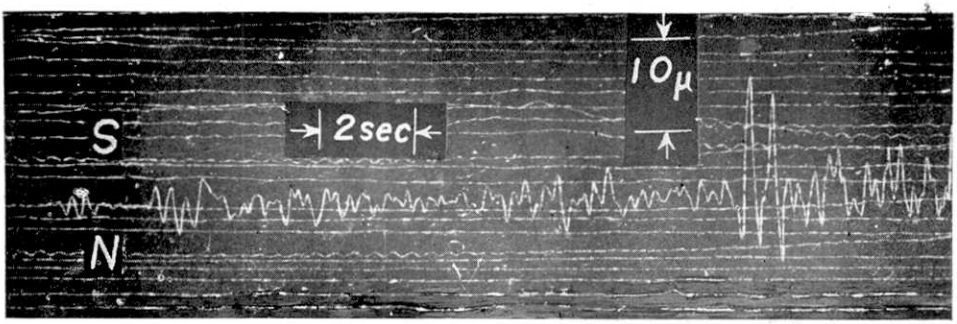

Fig. 5-a. Earthquake No. 446. 300 m depth.

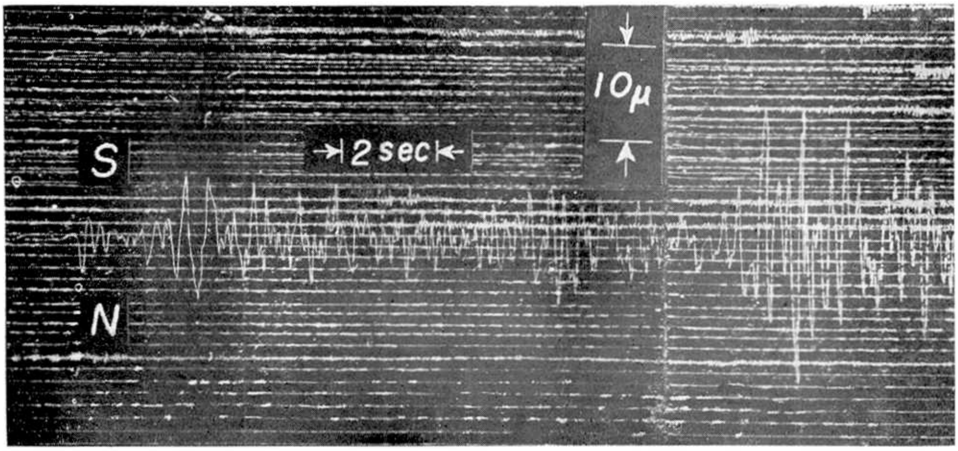

Fig. 5-b. Earthquake No. 446. Mine.

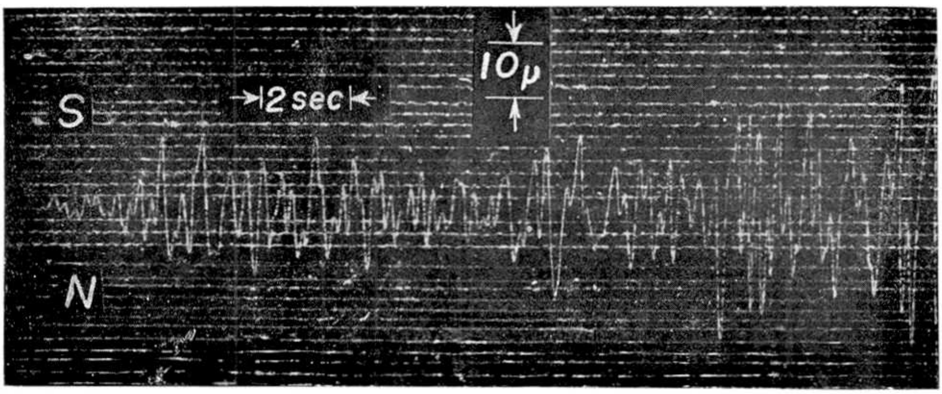

Fig. 5-c. Earthquake No. 446. School.

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