21. *Experimental Studies on the Mechanism of Generation of Elastic Waves III.*

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1. Introduction.

In the previous papers[1], we found peculiar disturbances on the surface of a semi-infinite elastic body; that is, when the surface is subjected to an impulsive vertical force, characteristic deformation occurs in the vicinity of the origin, and then propagates away forming finally the complete type of the free Rayleigh wave. It is considered to be one of the fundamental projects in the studies of elastic waves to explain how surface waves can be generated and propagated. Of course, efforts made by Rayleigh, Lamb, Sezawa, Nakano and others[2], have enriched our knowledge on the characters of surface waves, but these efforts usually made on the matter observed at distant points from the origin, and concerning the problem how the energy of shocks can be transformed to that of surface waves, we are not yet given enough explanation by physicists either theoretically or experimentally.

It is not easy to describe the processes of generation of surface waves in different case *en bloc*, but from the results of previous experiments, we may conceive in imagination three stages in the generation of surface waves, that is,

1) Initial deformation of the surface in the vicinity of the origin.
2) Propagation of the strain energy to more distant places. Initial deformation of the medium, whose characters are improper to free surface waves, changes its form to more proper one.
3) Deformation, which arranged the complete form of surface waves in this way, propagates with their own velocities.

The main object of the present paper is to make some considerations on the second stage, which seems to involve the most essential mechanism. As the first step, exciting the origin on the surface in different mode,

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1) K. KASAHARA, *Bull. Earthq. Res. Inst.*, **30** (1952), 259-268: *ibid*, **31** (1953), 71-79.

2) See for example, Y. SATÔ, *Kagaku*, **19** (1949), 311-319 (in Japanese).

formation of free surface waves are investigated experimentally; and then some mathematical considerations have been made on the strain energy accompanying the surface waves.

Throughout these studies, all the phenomena have been regarded as two-dimensional under the consideration that it will bring into the results no essential discrepancies.

2. Experiments.

A block of agar-agar is used as the elastic medium, whose dimensions and elastic characters are listed in Table I. To pick-up the elastic disturbances on the surface, a variable capacity type transducer is employed and an electromagnetic type oscillograph is used in recording. Details of these instruments were reported in the previous paper[3].

Table I.

Dimensions of the medium	length 70 cm, width 35 cm, depth 15 cm.	
Elastic constants of the medium	Young's modulus	9.11×10^4 c. g. s.
	Rigidity	3.24×10^4 c. g. s.
	Poisson's ratio	0.41
	Density	1.0

To cause elastic disturbances in the medium, a special wave-generater is constructed. As shown in Fig. 1 and 2, a pair of eccentric wheels W_1 and W_2 are mounted on the same shaft S, which is driven by a synchronous motor M transiently or continuously. To W_1 and W_2 are connected rods R_1 and R_2, as well as strings S_1 and S_2, respectively, which transmit the back and forth motions of W_1 and W_2. A vibrator made of an aluminum bar (20 cm long) lies beneath the surface of the medium, being supported in its neutral position with holders H and rubber packings P. When the elastic properties of S_1, S_2, and P are chosen properly the vibrator can displace in moderate amplitudes proportionally to the motion of R_1 and R_2. As can be seen in Fig. 1, and 3 the vibrator is excited with W_1 and W_2 in two directions (perpendicular to each other) at the same time, and these circumstances allow us to obtain different modes of excitations of the origin by changing the combination of eccentricities and phase angles of two wheels.

3) *loc cit.*

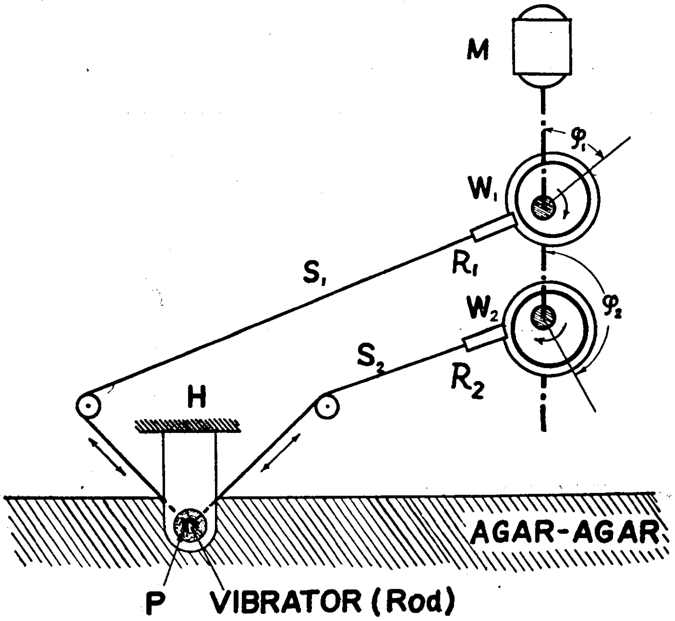

Fig. 1. H: holders: M: synchronous motor; P: rubber packing; R_1, R_2: connecting rods; S_1, S_2: strings; W_1, W_2: eccentric wheels; φ_1, φ_2; phase angles of W_1, W_2, respectively.

In the course of experiments, transient excitations are employed under the consideration that the continuous method will bring into the results undesirable effects of the walls of the medium. Experiments are made employing the phase angle $\varphi_1-\varphi_2$ as the indicator of the mode of excitation, while other conditions including amplitudes of excitation are kept unchanged. As is well known, the condition of particle motions which correspond to the excitation of stationary free surface waves is decided uniquely and we can imagine innumerable sets of conditions which do not satisfy the characters of free surface waves. The phase angle seems most convenient to indicate these differences, because a slight change in the phase angle between two components of excitation will cause easily a remarkable difference in the character

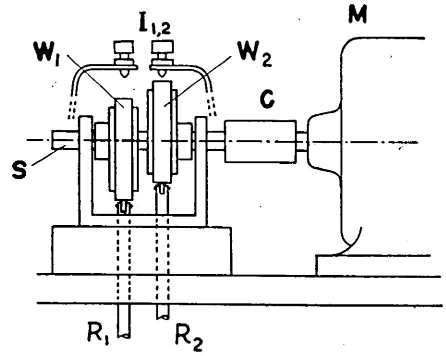

Fig. 2.

of origin. And therefore, we can see the outline of the phenomena in this way, without making other series of experiments changing other factors, such as amplitude ratios, decay of amplitudes with depth *etc.*

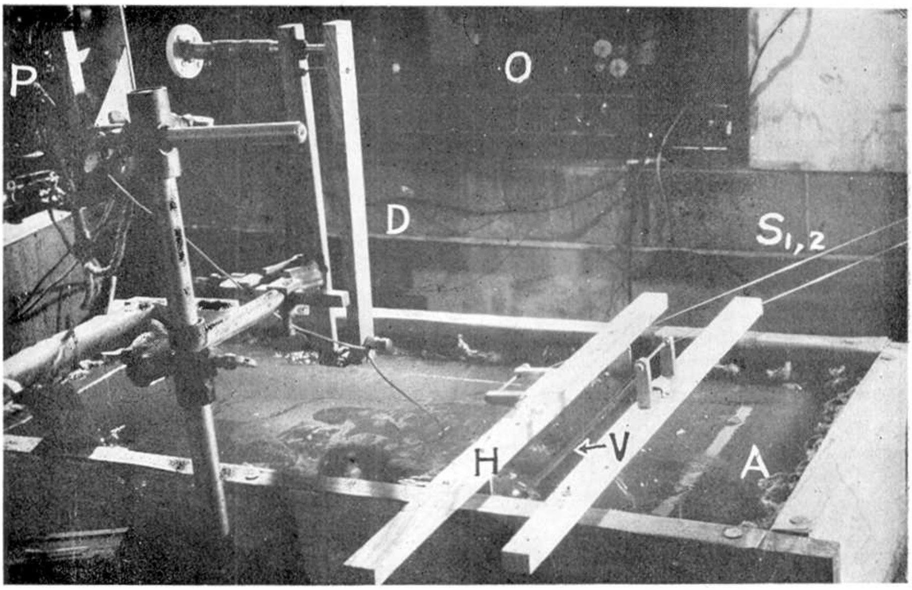

Fig. 3 A: agar-agar; D: detector; H: holder; O: oscillograph; P: Pierce's circuit; $S_{1,2}$: strings; V: vibrator.

Fig. 4 illustrates loci of particle motions observed at different points on the surface. Eccentric wheels are driven suddenly by three or four rotations, and taking the beginning of drive as the origin of time, we traced particle motions about x/V_R time after, where V_R and x represents the velocity of the surface wave and the distance of the point from the origin, respectively. The motion thus obtained coincides actually with the first main shock.

Four values of the phase angle are picked out, that is,

$$\text{a) } 0, \qquad \text{b) } \pi/2, \qquad \text{c) } \pi, \qquad \text{d) } 3\pi/2.$$

Actual motion of the origin corresponding to these phase angles are drawn in Fig. 4 in the column of origin. As can be seen in the figure, case b) is the most similar to the particle motion under the free surface wave, while case d) is the most different one among them. Cases a) and c) are between both extreme cases.

At those points distant enough from the origin, particles on the surface move according to the conditions of the free surface wave in any

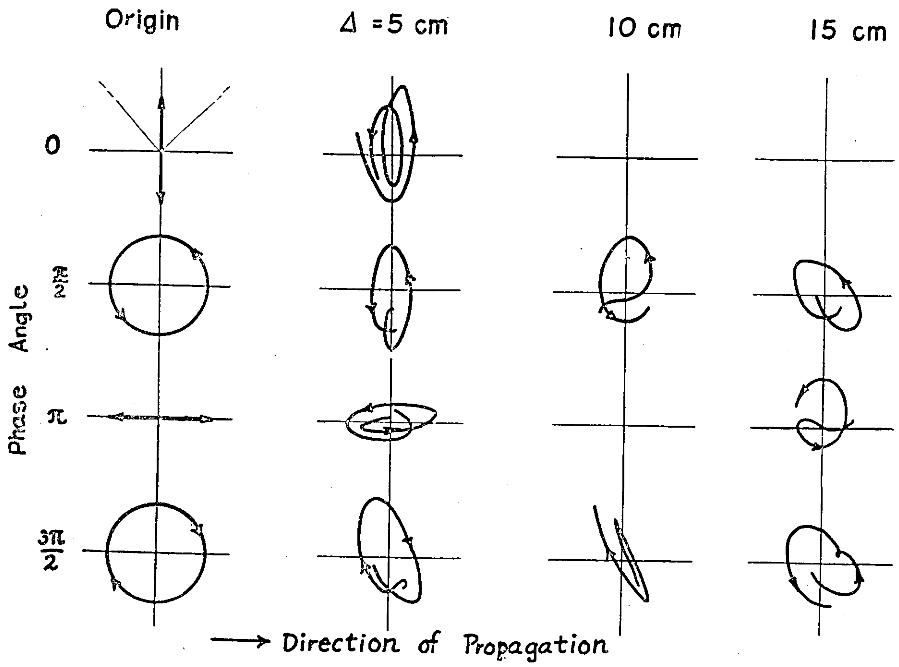

Fig. 4 Loci of particle motions, observed.

case of the phase angle. Loci at points more than 15 cm distant are considered to belong to this group. On the other hand, at nearer points about several centimetres distant, particle motions are affected severely by the condition of excitation, that is by the phase angle. When the origin is excited in the similar mode of the surface wave as in the case of b), particles on the surface show normal motions as stationary surface waves, while in the case of d), the abnormal motion of the origin affects the particle motions remarkably. In other two cases, the phenomena show intermediate characters.

Change of forms of loci in the case of d) is considered to contain valuable suggestions on the present problem, but further considerations will have to be given in the future, when the stress distribution in the medium becomes measurable with more improved instruments.

3. Mathematical Considerations.

Corresponding to the present experiment, simple mathematical considerations will be made in the following, that is, strain energies accompanying the free surface wave and other modes of surface deformations will be compared together. Needless to say, the Rayleigh wave is the unique surface wave which satisfies boundary conditions of the free surface of a semi-infinite elastic body, and therefore, we cannot expect in actual case another kind of surface wave to be travelling along the surface. However, as clarified in the experiment, complicated forms of deformations exist in the vicinity of the origin and so, the change of the form to that of the surface wave becomes essential. To find an answer to the question why a surface deformation of an abnormal mode is transformed to that corresponding to the Rayleigh wave, the above-stated comparison of strain energies seems worth trying.

We will consider a semi-infinite body of two dimensions. The x axis is taken parallel to the surface, while the positive z axis is directed into the medium. The origin of the axes lies on the surface. As to the expressions of the deformations we consider the following relations of displacements u and v, viz.,

$$\left.\begin{aligned} u&=K\{(2K^2-k^2)e^{-\alpha z}-2\alpha\beta e^{-\beta z}\}\ C_0 \sin(pt-Kx), \\ v&=-\alpha\{(2K^2-k^2)e^{-\alpha z}-2K^2e^{-\beta z}\}\ C_0 \sin(pt-Kx-\theta), \end{aligned}\right\} \quad (1)$$

where, $\alpha=\surd(K^2-h^2)$, $\beta=\surd(K^2-k^2)$,

$$h^2=\frac{p^2\rho}{\mu}, \qquad k^2=\frac{p^2\rho}{\lambda+2\mu},$$

and K is the real root of the equation, $(2q^2-k^2)^2-4q^2\alpha\beta=0$.

As (1) coincides with the expressions of the free surface wave in the case of $\theta=\frac{\pi}{2}$, (Fig. 5 and 6) the equations are adoptable for the present purpose, when we consider θ as a variable.

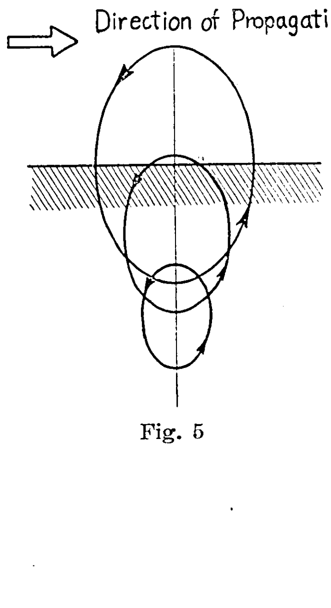

Fig. 5

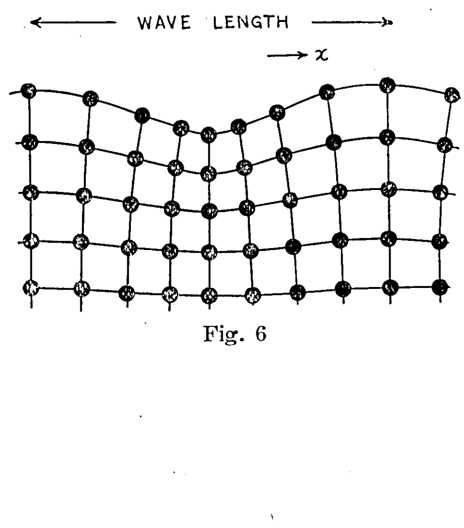

Fig. 6

The strain energy of a unit volume of a deformed elastic body is expressed, in general, as follows,

$$V_0=\frac{1}{2}\lambda e^2+\frac{1}{2}G\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\frac{1}{2}G\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2 \tag{2}$$

where λ, G, and e represents one of the Lamé's constants, rigidity, and volume dilatation, respectively. Corresponding to the deformation (1), each term of (2) becomes,

$$\left.\begin{aligned}
&\frac{1}{2}\lambda e^2=\frac{1}{2}\lambda\Big[A^2\cos^2(pt-Kx)+2AD\cos(pt-Kx)\\
&\qquad\times\sin(pt-Kx+\theta)+D^2\sin^2(pt-Kx+\theta)\Big],\\
&\frac{1}{2}G\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right\}=\frac{1}{2}G\Big[A^2\cos^2(pt-Kx)\\
&\qquad+D^2\sin^2(pt-Kx+\theta)\Big],\\
&\frac{1}{2}G\left\{\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right\}^2=\frac{1}{2}G\Big[C^2\sin^2(pt-Kx)+2BC\sin(pt-Kx)\\
&\qquad\times\cos(pt-Kx+\theta)+B^2\cos^2(pt-Kx+\theta)\Big],
\end{aligned}\right\} \tag{3}$$

where

$$\left.\begin{aligned}
&\mathrm{A}=-K^2C_0\{(2K^2-k^2)e^{-\alpha z}-2\alpha\beta e^{-\beta z}\},\\
&\mathrm{B}=-K\alpha C_0\{(2K^2-k^2)e^{-\alpha z}-2K^2e^{-\beta z}\},\\
&\mathrm{C}=-K\alpha C_0\{(2K^2-k^2)e^{-\alpha z}-2\beta^2e^{-\beta z}\},\\
&\mathrm{D}=-\alpha C_0\{\alpha(2K^2-k^2)e^{-\alpha z}-2\beta K^2e^{-\beta z}\}.
\end{aligned}\right\} \tag{4}$$

The total strain energy accompanying the deformation (1) at a given time can be obtained by integrating the sum of the righthand side of (3) from 0 to ∞ with regard to z and from $-\frac{pt}{K}$ to $\frac{2\pi-pt}{K}$ with regard to x, viz.,

$$\left.\begin{aligned}
&\int V_0\,dx\,dz=\frac{\pi}{2}\Big[(\lambda+2G)(A'^2+D'^2)+G(B'^2+C'^2)\\
&\qquad+2(\lambda\cdot A'D'-G\cdot B'C')\sin\theta\Big],
\end{aligned}\right\} \tag{5}$$

(per every unit wave-length).

where, for brevity,

$$\left.\begin{aligned} A' &= -\frac{K^2}{\alpha}C\left\{(2K^2-k^2)-2\alpha^2\right\}, \\ B' &= -\frac{KC}{\beta}\left\{(2K^2-k^2)\beta-2K^2\alpha\right\}, \\ C' &= -KC\{(2K^2-k^2)-2\alpha\beta\}, \\ D' &= -\alpha C\{(2K^2-k^2)-2K^2\}. \end{aligned}\right\} \quad (6)$$

As all quantities in (5) except θ can be regarded as constant in the present case, total strain energy depends only on the third term of the equation. Taking into consideration the relations $K>\alpha>\beta$ and $\lambda\geqq G$, we can see that $\lambda\cdot A'D'-G\cdot B'C'$ in the third term is always positive. (5) depends, therefore, only on the value of $\sin\theta$, and reaches to its minimum value when $\theta=\frac{\pi}{2}$ (Fig. 7). This case coincides with the Rayleigh wave,

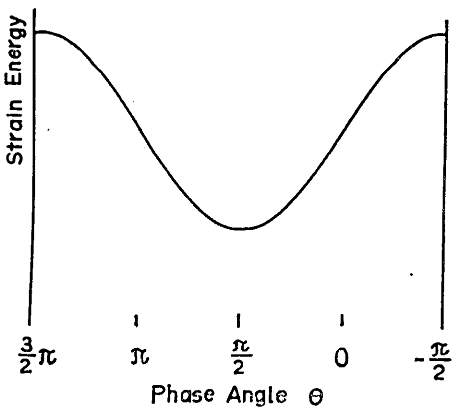

Fig. 7

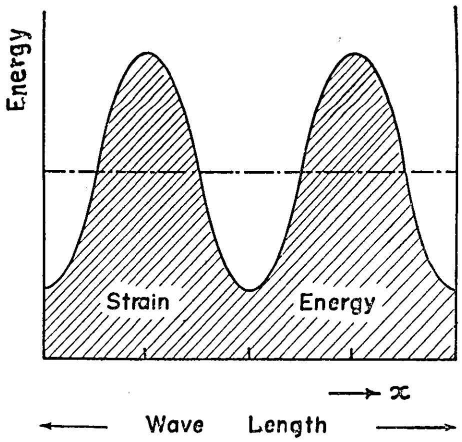

Fig. 8

leading to the conclusion: among the surface deformations of (1), one showing the phase angle equal to that of the Rayleigh wave is accompanied by the minimum strain energy, while the maximum corresponds to the phase angle of $\frac{3\pi}{2}$. Of course, this conclusion is to be expected from the standpoint that the Rayleigh wave is the unique solution that satisfies the condition of the surface free from stresses. However, the discussion here described is not without significance because it make us possible to treat the problem quantitatively.

The strain energy involved in the elementary column of the medium can also be calculated from (3). Fig. 8 is the numerical example for the medium deformed according to the conditions of the free surface wave.

4. Concluding Remarks.

In the present paper we have treated model experiments and some mathematical considerations on the problem how the complete type of the free surface wave can be produced from the initial deformation of the surface. The experiments, carried out by exciting the origin in two directions with constant amplitudes and variable phase angles, showed that even complicated modes of deformations are changeable to normal form of the surface wave and the more proper become the conditions of excitation, the more easily forms the surface wave.

Mathematical considerations made in correspondence to the experiments proved that the total strain energy accompanying the Rayleigh wave is less than that of any other deformation of which phase angle does not satisfy the given condition.

The results discussed in this paper is believed to be suggestive in the problem of generation of the surface wave. However, the present study is not yet complete enough to discuss the matter in more detail, and further discussions will have to be postponed.

Finally, the writer wishes to express his hearty thanks to Prof. Hagiwara, who has shown interest in this series of studies and encouraged him. The writer's thanks are also due to Mr. Saito for his kind assistance in the course of experiments. The present work has been supported by the Grant in Aid for the Miscelleraneous Scientific Researches of the Department of Education.

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21. 弾性波の発生機構に関する実験 (第3報)

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源点に起された表面変形が遠方へ伝播しながら次第に Rayleigh 波を形づくって行くことは，前からの実験で確かめられている．Rayleigh 波に対応しない変形状態が，正規の表面波に移って行く機構を研究する目的で簡単な実験と考察を試みた．

媒質中の源点を，互に直交する二方向の力で運動を起させる．その振巾を一定に保ったまゝ，両成分の位相関係を変化させ，源点から発射される弾性変形の伝播を考察した．位相角をどんな組合わせにしても十分遠方では完全な表面波が形成されるが位相角が正規 Rayleigh 波に近づく程，表面波の形成は源点近くから認められる．

之に対応する弾性論的考察として Rayleigh 波に相当する変位状況のうちで，水平，垂直両成分の位相差を変数として，全歪エネルギーを計算して見ると，Rayleigh 波の條件が充されるとき，最小値をとり，それと 180° 位相が異なるとき最大値をとる．
