22. *Examination of the Assumption of the Concentrated Mass in the Vibration of Framed Structure (Rahmen).*

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Since the exact solution for the vibration problem of framed structure (Rahmen) is a very complicated work, we often adopt a simplified method using various assumptions.

In this paper, we studied, for the purpose of deciding the limitation of approximation made under such an assumption, in what cases the value of the natural period of Rahmen obtained from the exact solution is in agreement with the value obtained under the assumption of concentrated mass.

The vibration problem of Rahmen is treated exactly if the equation of motion (1) regarding each member of the structure is solved by giving the boundary conditions met with each structure mode.

$$EI\frac{\partial^4 y}{\partial x^4}+\rho a\frac{\partial^2 y}{\partial t^2}=0. \qquad (1)$$

In equation (1), x is coordinate, y deflection, E Young's modulus and I, a and ρ represent moment of inertia of cross-section, sectional area and density respectively.

As to single-storied Rahmen of infinite span in the case of clamped conditions at the floors and the base, viz., the boundary conditions of usual Rahmen, frequency equation becomes as follow:

$$\begin{aligned}&2(\zeta^3\nu)^{1/4}\{(\cos\alpha\,\mathrm{sh}\alpha+\sin\alpha\,\mathrm{ch}\,\alpha)+\alpha\nu\phi(\cos\alpha\,\mathrm{ch}\,\alpha-1)\}\\&\quad\times\{\sin\beta(\mathrm{ch}\,\beta-1)+\mathrm{sh}\,\beta(1-\cos\beta)\}\\&\quad-\{(\cos\alpha\,\mathrm{ch}\,\alpha+1)+\alpha\nu\phi(\cos\alpha\,\mathrm{sh}\,\alpha-\sin\alpha\,\mathrm{ch}\,\alpha)\}(\cos\beta\,\mathrm{ch}\,\beta-1)=0^{1)}.\end{aligned} \qquad (2)$$

where α and β are the functions of frequency p;

$$\alpha^4=\frac{\rho_1 a_1 l_1^4}{E_1 I_1}p^2, \qquad \beta=\alpha\phi\left(\frac{\nu}{\zeta}\right)^{1/4}. \qquad (3)$$

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1) K. SEZAWA and K. KANAI, "Vibration of a Singled-storyed Framed Structure", *Bull. Earthq. Res. Inst.*, **10** (1932), p. 770, Eq. (26).

Also

$$\phi=\frac{l_2}{l_1}, \quad \zeta=\frac{E_2I_2}{E_1I_1}, \quad \nu=\frac{\rho_2a_2}{\rho_1a_1} \tag{4}$$

where suffix 1 indicates being a vertical member and suffix 2 a horizontal one.

In order to derive the frequency from equation (2), it is necessary to carry out a very complicated calculation using the trial and error method. Therefore, by assuming each member of Rahmen to be massless, the method of concentrating the whole mass on the panel points is often adopted. Then the equation of motion of each member can be written as follows:

$$\partial^4y/\partial x^4=0. \tag{5}$$

The frequency equation under the boundary clamped conditions at the floors and the base is

$$\frac{mp^2l_1^3}{E_1I_1}=\frac{3(\phi+12\zeta)}{\phi+3\zeta}.\text{[2]} \tag{6}$$

Suppose that the mass which acts as an inertia force concentrating at the panel point is the sum of the masses of horizontal members and the upper part of vertical ones. Then the concentrated mass, m, by using the notation in Fig. 1, is expressed as follows:

$$m=\rho_2a_2l_2+\rho_1a_1l_0, \tag{7}$$

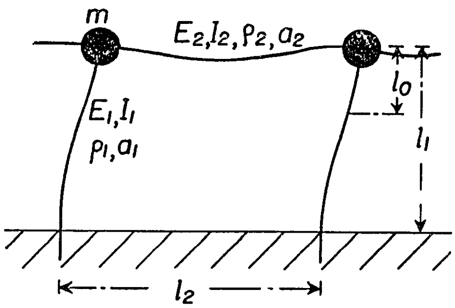

Fig. 1.

Introducing equation (7) into equation (6)

$$\frac{\rho_1a_1l_1^4}{E_1I_1}p^2(\equiv\alpha^4)=\frac{3(\phi+12\zeta)}{(\nu\phi+\kappa)(\phi+3\zeta)} \tag{6'}$$

where $\kappa=l_0/l_1$.

By applying various quantities of the material for horizontal and vertical members, the natural period of Rahmen will be exactly obtained from equation (2) in a form of α in equation (3). Then, introducing the value of α obtained in equation (2) into equation (6′) we can obtain the value of $\kappa(\equiv l_0/l_1)$. In other words we can tell how much mass of

2) K. SEZAWA and K. KANAI, "Theory of the Aseismic Properties of the Brace Struts .....", *Bull. Earthq. Res. Inst.*, **16** (1938), p. 710, Eq. (40′).

the upper part of vertical member must be added to the mass of horizontal member as a concentrated mass used in approximate calculation, in order to get the natural period equal to the one given by an exact calculation.

Under such considerations, the values of $\kappa(\equiv l_0/l_1)$ in regard to various cases are determined from equations (2) and (6′), and are shown in Tables I and II.

Table I. The values of $(\rho_1 a_1/E_1 I_1)^{1/2} l_1^2 p$.

$\frac{l_2}{l_1}$	$\frac{\rho_2 a_2}{\rho_1 a_1}$	1			2		5
	$\frac{E_2 I_2}{E_1 I_1}$	1	2	10	2	5	5
0.2		4.482	4.515	4.545	3.893	3.917	2.941
0.5		3.542	3.616	3.683	2.880	2.924	2.019
1		2.699	2.816	2.924	2.135	2.200	1.457
2		1.899	2.044	2.199	1.498	1.584	1.028
5		1.032	1.184	1.411	0.848	0.963	0.615

Table II. The values of l_0/l_1.

$\frac{l_2}{l_1}$	$\frac{\rho_2 a_2}{\rho_1 a_1}$	1			2		5
	$\frac{E_2 I_2}{E_1 I_1}$	1	2	10	2	5	5
0.2		0.37	0.37	0.38	0.37	0.37	0.38
0.5		0.35	0.37	0.38	0.36	0.37	0.38
1		0.34	0.35	0.37	0.36	0.37	0.38
2		0.33	0.35	0.37	0.36	0.37	0.36
5		0.99	0.65	0.37	0.99	0.51	0.82

If horizontal member is heavy or weak beyond a certain extent, the effect of its vertical motion increases as the span grows larger, and it makes the summary method unsuitable. And also this is the reason why the values in the bottom line in Table I are somewhat peculiar.

To summarize, if the span of Rahmen is within the double of the height, the sum of the mass of horizontal members and 33–38 percent of the mass of vertical members is to be thought of as a concentrated mass regardless of the material of horizontal and vertical members. Consequently this study permits us to express the practical equation which will determine the natural period regarding a one-storied framed structure as follows:

$$T=\frac{2\pi}{3}\sqrt{\frac{\frac{\rho_1 a_1 l_1^4}{E_1 I_1}\left(1.1+3\frac{\rho_2 a_2 l_2}{\rho_1 a_1 l_1}\right)\left(1+3\frac{E_2 I_2 l_1}{E_1 I_1 l_2}\right)}{1+12\frac{E_2 I_2 l_1}{E_1 I_1 l_2}}}. \qquad (8)$$

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22. 架構構造物の振動問題における集中荷重の仮定の吟味

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架構構造物の振動問題を正確に解くことは，非常に繁雑なため，しばしば種々の仮定のもとに，簡易化した解法がとられている．

この研究では，1 層無限張間の架構構造物の固有周期を，集中荷重の仮定で求めた値と，正確な解法で求めた値とが，どのような場合に，どの程度に合うかをしらべた．

その結果，かなり安心して使える実用計算式として (8) 式が得られた．
