8. *Study on Surface Waves VIII.*
Nomogram for the Phase Velocity of Love-Waves and Maximum Thickness of the Surface Layer.

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1. Introduction.

In the theoretical and observational study of seismic waves, we always pay attention to their velocity, especially when the waves have the property of dispersing. Consequently our study on Love-waves necessarily leads us to the examination of their velocity and we have published papers concerning this subject[1].

Hitherto, we chose some typical distribution of material and performed numerical calculations. Such a way of investigation, however, is not perfect for our practical purposes, for the variety of material constants and their distribution are not few, and we cannot calculate all the possible combinations of them. Thus it becomes desirable to make a nomogram by which we can obtain the velocity of Love-waves corresponding to any given state of media, or to get a material constant by means of the data of velocity and period[2].

2. Formula.

We adopted the following formula[3] which seems to be the most convenient for the present.

$$\omega_n = \sec\theta\cdot[\mathrm{Arctan}\,\{\chi\sec\theta\sqrt{(\sin^2\theta-\sin^2\theta_0)}\}+n\pi] \quad \ldots\ldots(2.1)^{4)}$$

where, ω_n: frequency of waves belonging to the n-th branch of Love-waves,

1) Y. SATÔ, "Study on Surface Waves," I, *Bull. Earthq. Res. Inst.*, **29** (1951), 1. III, *ibid.*, 435. IV, *ibid.*, 519. V, *ibid.*, **30** (1952), 1.

2) This nomogram will be of use in another case, too. Cf. K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **17** (1939), 685.

3) Y. SATÔ, "Study on Surface Waves, I. Velocity of Love-Waves," *Bull. Earthq. Res. Inst.*, **29** (1951), 1. Expression (4.2).

4) In this paper we assume n to be zero or one.

χ $=\mu'/\mu$ (μ' and μ the rigidity of the lower and upper medium respectively),

$\operatorname{cosec}\theta_0 = V_s'/V_s$ (V_s' and V_s the velocity of S-waves in the lower and upper medium respectively),

$\operatorname{cosec}\theta$: phase velocity of Love-waves measured by the unit V_s, and throughout this paper, the unit length is the thickness of the layer H, and that of time H/V_s, the physical meaning of which is quite evident.

By somewhat modifying the above expression we obtained the following form,

T(=period measured by the unit H/V_s)

$$=2\pi \div \left[\frac{\frac{V}{V_s}}{\sqrt{\frac{V^2}{V_s^2}-1}} \operatorname{Arctan} \left\{ \frac{\frac{V}{V_s}}{\sqrt{\frac{V^2}{V_s^2}-1}} \cdot \chi \sqrt{\frac{V_s^2}{V^2}-\frac{V_s^2}{V_s'^2}} \right\} + n\pi \frac{\frac{V}{V_s}}{\sqrt{\frac{V^2}{V_s^2}-1}} \right] \qquad \text{.................(2.2)}$$

and calculated the values of T corresponding to the given values of V/V_s.

3. Explanation of the figure and direction for use.

At first we will explain the nomogram for the computation of the fundamental branch.

In Fig. 1, [1] is the axis of V_s/V_s'. When some value of V_s/V_s' is given, we take a point A on this axis corresponding to this assigned value.

Next, α is the curve of $1\Big/\sqrt{\frac{V_s^2}{V^2}-\frac{V_s^2}{V_s'^2}}$. At first we chose some value of V/V_s and determine a point B on [2]-axis, and then we take a curve α which begins from the very point. The point C on [2]-axis

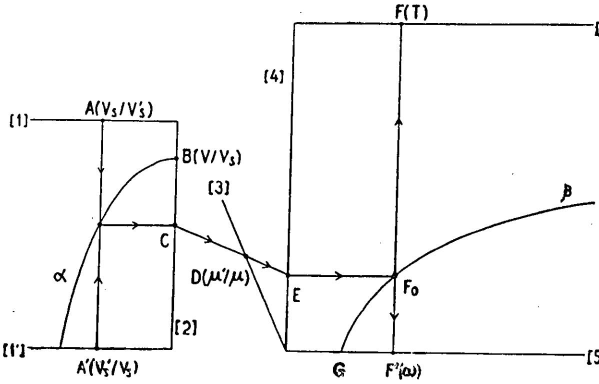

Fig. 1.

shows the value $1\Big/\sqrt{\frac{V_s^2}{V^2}-\frac{V_s^2}{V_s'^2}}$.

[3] is the axis of $\chi\equiv\mu'/\mu$. When the distribution of matter is determined, the value of χ is naturally given, therefore we can take a point D on [3]-axis.

Making a straight line passing the points C and D, we obtain a point E on [4]-axis, which is equal to [2]÷[3], or $1\Big/\chi\sqrt{\frac{V_s^2}{V^2}-\frac{V_s^2}{V_s'^2}}$,

Finally, the curve β gives

$$2\pi\div\left[\frac{\frac{V}{V_s}}{\sqrt{\frac{V^2}{V_s^2}-1}}\cdot\text{Arctan}\left\{\frac{\frac{V}{V_s}}{\sqrt{\frac{V^2}{V_s^2}-1}}\cdot\frac{1}{[4]}\right\}\right] \qquad \text{.........(3.1)}$$

or the value of the right hand member of the expression (2.2). (Point F on [5]-axis). V/V_s, the parameter of the curve β, must of course be same with that of α.

The value of $\omega=2\pi/T$ can be found on the lower side of the figure ([5′]-axis).

Axis [1′] gives V_s'/V_s, or the reciprocal of the [1]-axis.

The second nomogram which gives the first higher branch of waves shall be used without further explanation.

When we intend to get $\chi=\mu'/\mu$ by means of the data of the velocity and period, we can of course utilize this nomogram. However the procedure is very easy and it does not seem necessary to state the precise way of usage here.

4. Numerical example.

Let us give some numerical example in this section.

We assume

$$\begin{cases} \chi\equiv\mu'/\mu=5.0\,, \qquad \rho'/\rho=1.25\,, \\ \sin\theta_0\equiv V_s/V_s'=0.50\,, \end{cases} \qquad \text{........(4.1)}$$

and obtain the period corresponding to

$$V/V_s=\text{cosec}\,\theta=1.05,\ 1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5,\ 1.6,\ 1.7,\ 1.8,\ 1.9 \quad \text{...(4.2)}$$

The results obtained by strict calculation and those by the nomogram are given in the following table. The coincidence seems to be satisfactory.

Table I.

V/V_s	T (numerical calculation)	T (nomogram)	V/V_s	T (numerical calculation)	T (nomogram)
1.05	1.281	1.28	1.5	3.762	3.75
1.1	1.791	1.79	1.6	4.169	4.16
1.2	2.469	2.47	1.7	4.663	4.65
1.3	2.962	2.96	1.8	5.391	5.38
1.4	3.375	3.37	1.9	6.954	6.97

5. Maximum thickness of the surface layer.

If the velocity of S-waves and the rigidity of both media are given, and the period and the phase velocity of Love-waves are observed, we can determine the thickness of the surface layer from this nomogram easily. However, even if the complete data are not given, we can give a maximum value of thickness only from the data V_s, V and T'' (period of Love-waves in second).

A glance at the Plate I or the curve β in Fig. 1 shows that T has a minimum value corresponding to a given value of V/V_s (point G in Fig. 1). This is a somewhat interesting and peculiar phenomenon, for by means of these data only we can calculate the upper limit or the maximum value of H, the thickness of the layer. Whatever data we may add to those given above, the estimated thickness of the layer can not exceed this maximum value H_{max}.

Employing a set of numerical data, we will explain the procedure of getting this thickness.

If we give

$$V_s = 3.40 \text{ km/sec},$$

when the observed data of Love-waves are

$$V = 4.01 \text{ km/sec},$$

and $$T'' = 45 \text{ sec},$$

then we find from Plate I

$$T_{min} = 2.12, \qquad \text{..............(5.1)}$$

this same value is expressed by

$$T'' \Big/ \left(\frac{H_{max}}{V_s}\right) = 153/H_{max}. \qquad \text{..............(5.2)}$$

Therefore, combining (5.1) and (5.2), we obtain at once

$$H_{max}=72\,\text{km}\ .$$

In order to get the value of H_{max} alone, however, it is neither necessary nor convenient to use Plate I. We will here make another nomogram useful for computing the upper limit of thickness of the surface layer.

Getting back to the first

$$\omega_n=\sec\theta\cdot[\text{Arctan}\,\{\chi\sec\theta\sqrt{(\sin^2\theta-\sin^2\theta_0)}\}+n\pi]\ . \quad .....(5.3)$$

Therefore, the maximum value of ω_0 is

$$\omega_{max}=\frac{\pi}{2}\sec\theta=\frac{\pi}{2}\frac{V}{V_s}\Big/\sqrt{\frac{V^2}{V_s^2}-1} \quad ....(5.4)$$

and this expression is equal to $2\pi\Big/\left(\frac{T''}{H/V_s}\right)$; thus

$$H_{max}=L\Big/4\sqrt{\frac{V^2}{V_s^2}-1} \quad ....(5.5)$$

in which $L(=T''V)$ implies the wave-length of Love-waves.

We give the numerical relation V/V_s versus H_{max}/L in Table II and Fig. 2.

Table II.

V/V_s	H_{max}/L	V/V_s	H_{max}/L	V/V_s	H_{max}/L
1.05	0.7808	1.6	0.2002	4.5	0.0645
1.10	0.5455	1.8	0.1670	5.0	0.0510
1.15	0.4402	2.0	0.1444	5.5	0.0462
1.20	0.3770	2.2	0.1276	6.0	0.0423
1.25	0.3333	2.4	0.1146	6.5	0.0389
1.30	0.3010	2.6	0.1042	7.0	0.0361
1.35	0.2756	2.8	0.0956	7.5	0.0336
1.40	0.2552	3.0	0.0884	8.0	0.0315
1.45	0.2381	3.5	0.0745	9.0	0.0280
1.50	0.2236	4.0	0.0645	10.0	0.0251

Using the same data again, we will calculate H_{max}.

Since $V/V_s=1.18$, we get from Fig. 2

$$H_{max}/L=0.400\ ,$$

while

$$L=T''V=180\,\text{km}\ ,$$

therefore

$$H_{max}=72\,\text{km}.^{5)}$$

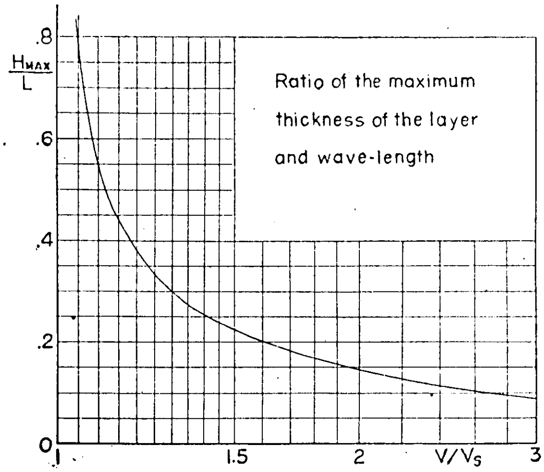

Fig. 2.

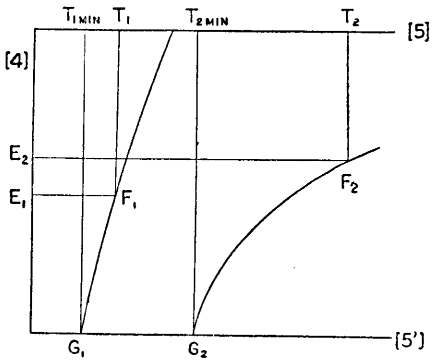

Fig. 3.

5.1 Precision of this method.

Although we can determine H_{max} , this method would be useless if this estimated value is much more larger than the real thickness of layer. We will examine, in this section, the condition wherein the estimated H_{max} does not differ so much from the real one and may be used effectively.

For the determination of this value we used the left terminal point G of the curve β in Fig. 1 or Fig. 3 and obtained T_{min}. However, the real thickness of the layer should be determined by means of T (cf. Fig. 2, [5]-axis). Therefore, if the difference $T\sim T_{min}$ is small, or $|\Delta T/\Delta[E]|_\beta$ is small, the estimation of T_{min} will be fairly accurate.

5) If we have the complete data with regard to the media, i.e. μ'/μ and ρ'/ρ, we can strictly determine the thickness of the layer. For example, if

$$\mu'/\mu=2.00, \qquad \rho'/\rho=1.22,$$

taking on [1]-axis $V_s/V_s'=0.782$ [2]-axis $V/V_s=1.18$ [3]-axis $\mu'/\mu=2.00$, and the curve with a parameter $V/V_s=1.18$, we can determine the points B_0, C, E, F_0 and finally F. In this case $F(=T)=3.72$.

Now, T implies $T=T''/\left(\frac{H}{V_s}\right)$,

therefore, introducing $T''=45$ sec. $V_s=3.40$ km/sec.
we obtain $H=41$ km.

On the other hand, the smaller the velocity of Love-waves is, the steeper the slope of the curve β will become. Thus we can employ slow, or short Love-waves more effectively than the fast, or long ones. (cf. Fig. 2.)

However, if we compare Plate I and II, we can notice by a glance that the curve β is much steeper in Plate II than in Plate I. Therefore, if we can observe Love-waves with a node in the layer, they will be of more use than those without a node.

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8. 表面波の研究 VIII

ラブ波の位相速度を求める図表と表面層の厚さの極大値

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1. ラブ波の伝播を研究する場合に，その速度は忘れる事のできない重要なものであるが，物質常数の分布に関するあらゆる組合せを与へて，これを計算し盡すといふ事は到底できない．

そこで，上下二つの物質内でのS波の速度比及び剛性率の比を与へた時に，ラブ波の速度と周期(もしくは周回振動数)との関係を与へる計算図表を作った．層内に一つも節のない場合を図版Iに，一つある場合を図版IIに示す．使用法は第3節と第1図から明らかであらう．

正しい計算値と比べた時，精度はこの種のものとしてほぼ滿足しうるものと思ふ．

2. ラブ波の周期，速度及び上下の媒質内でのS波の速さ，剛性率の比を与へれば，表面層の厚さはもとより決定する．しかし，図表をみて気づく事であるが，ラブ波の周期と速度，それに加へて表面層内でのS波の速さだけをしれば，層の厚さがとりうる最大値は他の物質常数の如何にかかはらず決定する．この事は興味ある事と思ひ，そのための一二の計算とグラフとを用意した．