# 25. *Analyses of Geomagnetic Field by Use of Hermite Functions.*

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(Read Sept. 16, 1952.—Received Sept. 20, 1952.)

## 1. Introduction.

It is of much importance and interest to interpret geomagnetic anomalies in both pure and applied geophysics. As has been summarized by E. H. Vestine and N. Davids[1], we have a good number of methods for analyzing and interpreting geomagnetic anomalies. Among these methods, the method of Fourier series developed by T. Nagata[2] seems to be one of the most convenient owing to the ease with which it can be applied in practice. The idea of the method by which the subterranean mass distribution may be obtained directly from the anomaly observed on the earth's surface was first adopted tentatively by C. Tsuboi and T. Fuchida[3] for finding out the density distribution concentrated on a subterranean plane directly from the gravity values observed on the earth's surface.

According to the T-N (Tsuboi-Nagata) method, we take a certain region in which the anomalies are predominant and then the anomalies are subjected to a Fourier analysis. Hence we are obliged to assume that the same anomalies are repeated periodically outside the said region, the distribution of anomaly then being different from the actual one. In so far as we discuss the anomaly and the corresponding subterranean structure within the limited region taken above, the result of the method seems to be approximately correct as already shown by Tsuboi and Nagata for various examples.

We, however, have sought after a more rigorous method by which we could treat not the limited are aalone but the whole area in question. Such a method of analysis would be particularly useful for studying an isolated anomaly and the corresponding underground structure. From

1) E. H. VESTINE and N. DAVIDS, *Terr. Mag.*, **50** (1945), 1.
2) T. NAGATA, *Bull. Earthq. Res. Inst.*, **16** (1938), 550.
3) C. TSUBOI and T. FUCHIDA, *Bull. Earthq. Res. Inst.*, **15** (1937), 636.

this viewpoint, the writer would like to propose here a method with the aid of Hermite functions. This method is to be applied in studying an anomaly together with the corresponding magnetization concentrated on a subterranean plane. The validity of the result will be under no spatial restrictions.

## 2. Theory for the two dimensional analysis.

It is well known that any function of $x$ defined in a region from $x=-\infty$ to $x=+\infty$ can be expressed with a series of Hermite polinominals multiplied by $e^{-\frac{x^2}{2}}$. We shall, in the first place, express the geomagnetic anomaly, for instance the anomaly in the vertical component $\Delta Z(x)$ which is observed on the earth's surface, with a series as

$$\Delta Z(x)=\sum_{n=0}^{\infty} K_n H_n(x) e^{-\frac{x^2}{2}}, \tag{1}$$

where $H_n(x)$ is a Hermite polinominal defined by

$$\left.\begin{aligned} H_{2m}(x)&=\sum_{s=0}^{m}(-1)^s\frac{(2m)!}{(2m-2s)!s!}(2x)^{2m-2s}, \\ H_{2m+1}(x)&=\sum_{s=0}^{m}(-1)^s\frac{(2m+1)!}{(2m-2s+1)!s!}(2x)^{2m-2s+1}, \end{aligned}\right\} \tag{2}$$

respectively for even and odd degrees. The first several polinominals become as follows;

$$\left.\begin{aligned} &H_0=1, \\ &H_1=2x, \\ &H_2=4x^2-2, \\ &H_3=8x^3-12x, \\ &H_4=16x^4-48x^2+12, \\ &H_5=32x^5-160x^3+120x, \\ &H_6=64x^6-480x^4+720x^2-120, \\ &H_7=128x^7-1344x^5+3360x^3-1680x, \\ &H_8=256x^8-3584x^6+13440x^4-13440x^2+1680, \\ &H_9=512x^9-9216x^7+48384x^5-80640x^3+30240x, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \tag{3}$$

If we define $\phi_n(x)$ by

$$\phi_n(x)=H_n(x)e^{-\frac{x^2}{2}}, \tag{4}$$

$\phi_n(x)$'s form a system of orthogonal functions in a region from $x=-\infty$ to $x=+\infty$ as easily proved. The values of $\phi_n(x)$ for various $x$ will be given in Table I.

We have from (1)

$$K_n=\frac{1}{2^n n!\sqrt{\pi}}\int_{-\infty}^{\infty}\Delta Z(\xi)\phi_n(\xi)d\xi, \tag{5}$$

because we know that

$$\int_{-\infty}^{\infty}\{\phi_n(\xi)\}^2 d\xi=\int_{-\infty}^{\infty}\{H_n(\xi)\}^2 e^{-\xi^2}d\xi=2^n n!\sqrt{\pi}. \tag{6}$$

Taking next the typical term $\phi_n(x)$ we shall apply a Fourier integral to it. If we assume that $\phi_n(x)$ can be expressed as

$$\phi_n(x)=\int_0^{\infty}A_n(u)\cos uxdu+\int_0^{\infty}B_n(u)\sin uxdu, \tag{7}$$

we readily obtain

$$\left.\begin{aligned} &A_{2m}(u)=\frac{2}{\pi}\int_0^{\infty}\phi_{2m}(\xi)\cos u\xi d\xi, \qquad B_{2m}(u)=0,\\ &A_{2m+1}(u)=0,\;\; B_{2m+1}(u)=\frac{2}{\pi}\int_0^{\infty}\phi_{2m+1}(\xi)\sin u\xi d\xi, \end{aligned}\right\} \tag{8}$$

by taking into consideration that $\phi_n(x)$ is an even or an odd function corresponding to even or odd $n$'s.

To calculate the integrals in (8), we should take into account (2), whence we get

$$A_{2m}(u)=\frac{2}{\pi}\sum_{s=0}^{m}(-1)^s\frac{2^{2m-2s}(2m)!}{(2m-2s)!s!}\int_0^{\infty}\xi^{2m-2s}e^{-\frac{\xi^2}{2}}\cos u\xi d\xi. \tag{9}$$

On the other hand, it is proved that

$$\left.\begin{aligned} &\int_0^{\infty}e^{-\frac{t^2}{2}}\cos ut\,dt=\sqrt{\frac{\pi}{2}}e^{-\frac{u^2}{2}},\\ &\int_0^{\infty}te^{-\frac{t^2}{2}}\sin ut\,dt=\sqrt{\frac{\pi}{2}}ue^{-\frac{u^2}{2}},\\ &\int_0^{\infty}t^2e^{-\frac{t^2}{2}}\cos ut\,dt=\sqrt{\frac{\pi}{2}}(1-u^2)^{-\frac{u^2}{2}}, \end{aligned}\right\} \tag{10}$$

## Table I. Table for Hermite Functions.

| $x$ | $\phi_0$ | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $\phi_5$ | $\phi_6$ | $\phi_7$ | $\phi_8$ | $\phi_9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.000 | 0 | -2.000 | 0 | 12.00 | 0 | -120.0 | 0 | 1680 | 0 |
| 0.2 | 0.9802 | 0.3921 | -1.830 | -2.290 | 9.906 | 22.28 | -90.14 | -303.4 | 1141 | 5311 |
| 0.4 | 0.9231 | 0.7385 | -1.255 | -3.958 | 4.366 | 35.16 | -15.53 | -434.3 | -196.5 | 6845 |
| 0.6 | 0.8353 | 1.002 | -0.4678 | -4.571 | -2.678 | 33.35 | 66.81 | -320.3 | -1320 | 3538 |
| 0.8 | 0.7262 | 1.162 | 0.4067 | -3.997 | -8.835 | 17.84 | 116.9 | -26.25 | -1680 | -2300 |
| 1.0 | 0.6065 | 1.213 | 1.213 | -2.426 | -12.13 | -4.852 | 111.6 | 281.4 | -1001 | -6502 |
| 1.2 | 0.4868 | 1.168 | 1.830 | -0.2804 | -11.66 | -25.73 | 54.80 | 440.3 | 289.4 | -6350 |
| 1.4 | 0.3753 | 1.051 | 2.192 | 1.934 | -7.748 | -37.13 | -26.60 | 371.1 | 1411 | -1982 |
| 1.6 | 0.2780 | 0.8896 | 2.291 | 3.772 | -1.674 | -35.53 | -91.40 | 116.1 | 1729 | 3398 |
| 1.8 | 0.1979 | 0.7124 | 2.167 | 4.959 | 4.837 | -22.26 | -128.5 | -195.5 | 1095 | 6872 |
| 2.0 | 0.1353 | 0.5412 | 1.894 | 5.412 | 10.23 | -2.165 | -111.5 | -420.0 | -123.9 | 6243 |
| 2.2 | 0.08892 | 0.3912 | 1.543 | 5.227 | 13.74 | 18.63 | -55.17 | -466.3 | -1290 | 1787 |
| 2.4 | 0.05614 | 0.2695 | 1.181 | 4.592 | 14.95 | 35.04 | 18.25 | -388.9 | -1837 | -5119 |
| 2.6 | 0.03405 | 0.1771 | 0.8526 | 3.725 | 14.26 | 44.33 | 88.26 | -73.02 | -1628 | -7332 |
| 2.8 | 0.01984 | 0.1111 | 0.5825 | 2.818 | 12.28 | 46.25 | 136.0 | 206.8 | -741.5 | -7462 |
| 3.0 | 0.01111 | 0.06666 | 0.3777 | 2.000 | 9.732 | 42.40 | 157.1 | 433.6 | 402.6 | -4516 |
| 3.2 | $0.0^{2}5864$ | 0.03753 | 0.2285 | 1.312 | 7.026 | 34.47 | 150.4 | 548.9 | 1405 | 21.19 |
| 3.4 | $0.0^{2}3091$ | 0.02102 | 0.1367 | 0.8458 | 4.931 | 26.76 | 132.6 | 580.6 | 2095 | 4952 |
| 3.6 | $0.0^{2}1535$ | 0.01105 | 0.07650 | 0.5066 | 3.189 | 18.91 | 104.3 | 523.9 | 2311 | 8254 |
| 3.8 | $0.0^{3}7319$ | $0.0^{2}5396$ | 0.03959 | 0.2793 | 1.885 | 12.09 | 76.00 | 410.1 | 2082 | 9354 |
| 4.0 | $0.0^{3}3355$ | $0.0^{2}2684$ | 0.02080 | 0.1557 | 1.121 | 7.719 | 50.55 | 312.1 | 1786 | 9303 |
| 4.2 | $0.0^{3}1478$ | $0.0^{2}1241$ | 0.01013 | 0.08013 | 0.6108 | 4.502 | 31.70 | 212.2 | 1367 | 7851 |
| 4.4 | $0.0^{4}6252$ | $0.0^{3}5502$ | $0.0^{2}4717$ | 0.03931 | 0.3170 | 3.665 | 18.65 | 134.4 | 921.3 | 5957 |
| 4.6 | $0.0^{4}2542$ | $0.0^{3}2339$ | $0.0^{2}2101$ | 0.01839 | 0.1563 | 1.802 | 10.33 | 79.55 | 587.2 | 4129 |
| 4.8 | $0.0^{5}9930$ | $0.0^{4}9532$ | $0.0^{3}8953$ | $0.0^{2}8213$ | 0.07338 | 0.8382 | 5.406 | 44.22 | 348.8 | 2641 |
| 5.0 | $0.0^{5}3725$ | $0.0^{4}3725$ | $0.0^{3}3651$ | $0.0^{2}3502$ | 0.03279 | 0.3747 | 2.674 | 23.14 | 193.9 | 1569 |
| 5.2 | $0.0^{5}1344$ | $0.0^{4}1398$ | $0.0^{3}1427$ | $0.0^{2}1428$ | 0.01532 | 0.1610 | 1.255 | 11.44 | 101.4 | 871.6 |
| 5.4 | $0.0^{6}4656$ | $0.0^{5}5028$ | $0.0^{4}5337$ | $0.0^{3}5563$ | $0.0^{2}6149$ | 0.06629 | 0.5586 | 5.343 | 49.94 | 453.8 |
| 5.6 | $0.0^{6}1550$ | $0.0^{5}1736$ | $0.0^{4}1913$ | $0.0^{3}2073$ | $0.0^{2}2361$ | 0.02632 | 0.2362 | 2.369 | 23.22 | 218.8 |
| 5.8 | $0.0^{7}4956$ | $0.0^{6}5749$ | $0.0^{5}6570$ | $0.0^{4}7391$ | $0.0^{3}8175$ | $0.0^{2}9938$ | 0.09503 | 0.9955 | 10.22 | 102.6 |
| 6.0 | $0.0^{7}1522$ | $0.0^{6}1826$ | $0.0^{7}2161$ | $0.0^{4}2520$ | $0.0^{3}2892$ | $0.0^{2}3591$ | 0.03636 | 0.3971 | 4.256 | 42.33 |
| 6.2 | $0.0^{8}4496$ | $0.0^{7}5576$ | $0.0^{6}6824$ | $0.0^{5}8238$ | $0.0^{4}9802$ | $0.0^{2}1246$ | 0.01328 | 0.1509 | 1.685 | 17.70 |
| 6.4 | $0.0^{8}1275$ | $0.0^{7}1633$ | $0.0^{6}2064$ | $0.0^{5}2577$ | $0.0^{4}3173$ | $0.0^{3}4125$ | $0.0^{2}4620$ | 0.05450 | 0.6330 | 6.987 |
| 6.6 | $0.0^{9}3476$ | $0.0^{8}4588$ | $0.0^{7}5987$ | $0.0^{6}7719$ | $0.0^{5}9827$ | $0.0^{3}1309$ | $0.0^{2}1533$ | 0.01875 | 0.2261 | 2.684 |
| 6.8 | $0.0^{10}9102$ | $0.0^{8}1238$ | $0.0^{7}1667$ | $0.0^{6}2215$ | $0.0^{5}2912$ | $0.0^{4}3975$ | $0.0^{3}4855$ | $0.0^{2}6149$ | 0.07683 | 0.9465 |
| 7.0 | $0.0^{10}2290$ | $0.0^{9}3206$ | $0.0^{8}4442$ | $0.0^{7}6091$ | $0.0^{6}8258$ | $0.0^{4}1108$ | $0.0^{3}1468$ | $0.0^{2}1923$ | 0.02486 | 0.3173 |
| 7.2 | $0.0^{11}5535$ | $0.0^{10}7970$ | $0.0^{8}1137$ | $0.0^{7}1605$ | $0.0^{6}2242$ | $0.0^{5}3101$ | $0.0^{4}4241$ | $0.0^{3}5735$ | $0.0^{2}7665$ | 0.1012 |
| 7.4 | $0.0^{11}1285$ | $0.0^{10}1902$ | $0.0^{9}2790$ | $0.0^{8}4053$ | $0.0^{7}5830$ | $0.0^{6}8306$ | $0.0^{4}1171$ | $0.0^{3}1633$ | $0.0^{2}2253$ | 0.03074 |
| 7.6 | $0.0^{12}2868$ | $0.0^{11}4359$ | $0.0^{10}6569$ | $0.0^{9}9810$ | $0.0^{7}1452$ | $0.0^{6}2128$ | $0.0^{5}3090$ | $0.0^{4}4441$ | $0.0^{3}6318$ | $0.0^{2}8892$ |
| 7.8 | $0.0^{13}6148$ | $0.0^{12}9592$ | $0.0^{10}1484$ | $0.0^{9}2277$ | $0.0^{8}3462$ | $0.0^{7}5220$ | $0.0^{6}7802$ | $0.0^{4}1154$ | $0.0^{3}1690$ | $0.0^{2}2452$ |
| 8.0 | $0.0^{13}1265$ | $0.0^{12}2024$ | $0.0^{11}3214$ | $0.0^{10}5061$ | $0.0^{9}7903$ | $0.0^{7}1224$ | $0.0^{6}1880$ | $0.0^{5}2862$ | $0.0^{4}4314$ | $0.0^{3}6445$ |

$$\left.\begin{aligned}
&\int_0^\infty t^3 e^{-\frac{t^2}{2}} \sin ut\,dt = \sqrt{\frac{\pi}{2}}(3u-u^3)e^{-\frac{u^2}{2}}, \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\int_0^\infty t^{2k} e^{-\frac{t^2}{2}} \cos ut\,dt = \frac{(-1)^k}{2^k}\sqrt{\frac{\pi}{2}}\,H_{2k}\left(\frac{u}{\sqrt{2}}\right)e^{-\frac{u^2}{2}}, \\
&\int_0^\infty t^{2k+1} e^{-\frac{t^2}{2}} \sin ut\,dt = \frac{(-1)^k}{2^{k+1/2}}\sqrt{\frac{\pi}{2}}\,H_{2k+1}\left(\frac{u}{\sqrt{2}}\right)e^{-\frac{u^2}{2}}.
\end{aligned}\right)$$

With the aid of (10), (9) becomes

$$A_{2m}(u) = \sqrt{\frac{2}{\pi}}\sum_{s=0}^{m}(-1)^m \frac{2^{m-s}(2m)!}{(2m-2s)!\,s!} H_{2m-2s}\left(\frac{u}{\sqrt{2}}\right)e^{-\frac{u^2}{2}}. \tag{11}$$

Meanwhile we have the relations

$$\left.\begin{aligned}
&\sum_{s=0}^{m}\frac{2^{m-s}(2m)!}{(2m-2s)!s!}H_{2m-2s}\left(\frac{u}{\sqrt{2}}\right) = H_{2m}(u), \\
&\sum_{s=0}^{m}\frac{2^{m-s+1/2}(2m)!}{(2m-2s+1)!s!}H_{2m-2s+1}\left(\frac{u}{\sqrt{2}}\right) = H_{2m+1}(u),
\end{aligned}\right\} \tag{12}$$

which are easily proved. Substituting (12) into (11), we get finally

$$A_{2m}(u) = (-1)^m\sqrt{\frac{2}{\pi}}\phi_{2m}(u). \tag{13}$$

And in a similar way, we have also

$$B_{2m+1}(u) = (-1)^m\sqrt{\frac{2}{\pi}}\phi_{2m+1}(u). \tag{14}$$

Assuming that the anomaly is caused by a magnetization concentrated on an underground plane at a depth of $D$ with the direction of magnetization inclining $\theta$ from the horizontal plane, Nagata[2] has shown that the Fourier coefficients of the magnetization are given as

$$\left.\begin{aligned}
a_n(u) &= \frac{e^{uD}}{2\pi u}\{A_n(u)\sin\theta - B_n(u)\cos\theta\}, \\
b_n(u) &= \frac{e^{uD}}{2\pi u}\{A_n(u)\cos\theta + B_n(u)\sin\theta\},
\end{aligned}\right\} \tag{15}$$

respectively for harmonics $\cos ux$ and $\sin ux$. Applying (15) to the present problem and summing up with respect to $n$ or $m$, the magnetization corresponding to the anomaly is formally expressed as

$$J(x)=\frac{1}{2^{1/2}\pi^{3/2}}\sum_{m=0}^{\infty}(-1)^m K_{2m}\int_0^{\infty}\frac{e^{uD}}{u}\phi_{2m}(u)\sin(ux+\theta)du$$

$$-\frac{1}{2^{1/2}\pi^{3/2}}\sum_{m=0}^{\infty}(-1)^m K_{2m+1}\int_0^{\infty}\frac{e^{uD}}{u}\phi_{2m+1}(u)\cos(ux+\theta)du. \qquad (16)$$

In an actual analysis, $K_n$ would be easily obtained by numerical integrations in (5) because, in so far as we treat an isolated anomaly, $\Delta Z$ becomes fairly small for large values of $|x|$. And moreover since the integrals in (16) contain the factor $e^{-\frac{u^2}{2}}$ which becomes fairly small for large $u$, the integrals can be calculated by means of the same method for an assumed value of $D$.

The Hermite function method stated here will be applied not only to an anomaly in vertical geomagnetic force but also to that in horizontal force as well.

We should pay attention, however, to the fact that the integral $\int_0^{\infty}\frac{e^{uD}}{u}\phi_{2m}(u)\cos ux\,du$ that is included in (16) does not generally converge because the integrand reaches infinity at $u=0$. In that case, the difficulty can be avoided by analyzing magnetic potential which may be obtained by

$$\Delta W(x)=\frac{2}{\pi}\int_{-\infty}^{\infty}\Delta Z(x')\log|x-x'|\,dx', \qquad (17)$$

where the integral can be calculated numerically. After that the magnetic potential is expanded into a series of Hermite functions as before. If we denote the coefficient by $K'_n$, the corresponding magnetization is expressed as

$$J(x)=-\frac{1}{2^{1/2}\pi^{3/2}}\sum_{m=0}^{\infty}K_2{}'_m\int_0^{\infty}e^{uD}\phi_{2m}(u)\sin(ux+\theta)du \qquad (18)$$

$$+\frac{1}{2^{1/2}\pi^{3/2}}\sum_{m=0}^{\infty}K'_{2m+1}\int_0^{\infty}e^{uD}\phi_{2m+1}(u)\cos(ux+\theta)du,$$

where the integrals included are all convergent.

## 3. A special case.

In order to examine the relation between the present method and the T-N method, we shall study a special and simple case in which the anomaly is given as

$$\Delta W = \cos \alpha x \qquad \text{for} \quad -\infty < x < \infty, \tag{19}$$

and the direction of magnetization is taken downwards or $\theta=\pi/2$ for the sake of simplicity.

In that case, we have

$$K'_{2m} = \frac{1}{2^{2m-1}(2m)!\sqrt{\pi}} \int_0^\infty \phi_{2m}(t) \cos \alpha t dt, \; K_{2m+1}=0 \tag{20}$$

from (5) and

$$J(x) = \frac{1}{2^{1/2}\pi^{3/2}} \sum_{m=0}^{\infty} (-1)^m K'_{2m} \int_0^\infty e^{uD} \phi_{2m}(u) \cos uxdu. \tag{21}$$

As proved in the last section, we get

$$\int_0^\infty \phi_{2m}(t) \cos \alpha t dt = (-1)^m \sqrt{\frac{\pi}{2}} \phi_{2m}(\alpha), \tag{22}$$

and hence

$$K'_{2m} = \frac{(-1)^m}{2^{2m-1/2}(2m)!} \phi_{2m}(\alpha). \tag{23}$$

Substituting (23) into (21), we obtain

$$J(x) = \frac{-1}{2\pi^{3/2}} \sum_{m=0}^{\infty} \frac{\phi_{2m}(\alpha)}{2^{2m-1}(2m)!} \int_0^\infty e^{uD} \phi_{2m}(u) \cos uxdu. \tag{24}$$

Since the right-hand side of (24) is identical with the expansion of $-\dfrac{e^{\alpha D}\cos \alpha x}{2\pi}$ in a series of Hermite functions whose variable is $\alpha$, it follows that

$$J(x) = -\frac{e^{\alpha D}\cos \alpha x}{2\pi}, \tag{25}$$

which agrees exactly with the result obtained by the T-N method. Thus it is made clear that if the harmonics such as $\cos \alpha x$ and $\sin \alpha x$

are defined throughout the whole space both mothods give the same result.

## 4. A typical example for an isolated anomaly.

Now we are going to study an example in order to compare the writer's method to T-N one in case of an isolated anomaly. We shall consider an anomaly in the vertical geomagnetic component which is given as

$$\Delta Z=\begin{cases} 0 & \text{for} \quad x<-\pi, \\ \sin x & \text{for} \quad -\pi<x<\pi, \\ 0 & \text{for} \quad \pi<x. \end{cases} \tag{26}$$

According to the T-N method of Fourier series, we obviously find only one harmonic $\sin x$ by taking the region from $x=-\pi$ to $x=\pi$. And so the magnetization concentrated on an underground plane at a depth $D$ with an assumed downward direction is immediately obtained as

$$J=\frac{e^{D}\sin x}{2\pi} \tag{27}$$

On the other hand, $\Delta Z$ given by (26) is expressed fairly well with five odd terms of Hermite functions. That is

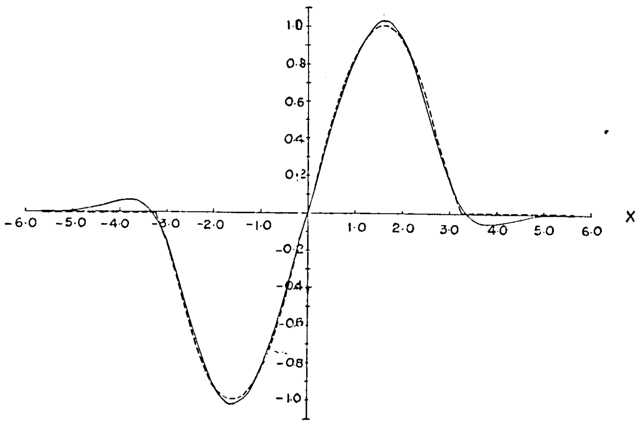


Fig. 1. The approximation of the curves defined by 0 for $x<-\pi$, $\sin x$ for $-\pi<x<\pi$ and 0 for $\pi<x$ (broken line) with five Hermite functions (full line).

$$\Delta Z = \sum_{m=0}^{4} K_{2m+1}\phi_{2m+1}(x), \tag{28}$$

where

$$K_1 = 0.8601, \quad K_3 = 0.0760, \quad K_5 = -0.000154,$$
$$K_7 = -0.000140, \quad K_9 = -0.00000278. \tag{29}$$

The approximations of this series seems to be good as shown in Fig. 1, the error being within a few per cent. With the aid of the values of $K_1$, $K_3$, ...., we can calculate the magnetization by (16), for example when $D$ is assumed as 0 and 1. The results are shown in Figs. 2 and 3 respectively together with the calculated results obtained by the T-N method.

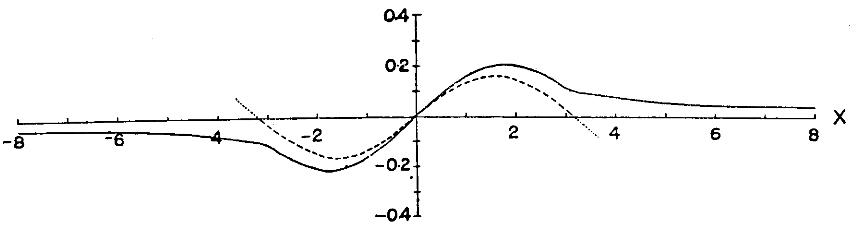


Fig. 2. The magnetization obtained by T-N method (broken line) and by the Hermite function method (full line) for $D=0$.

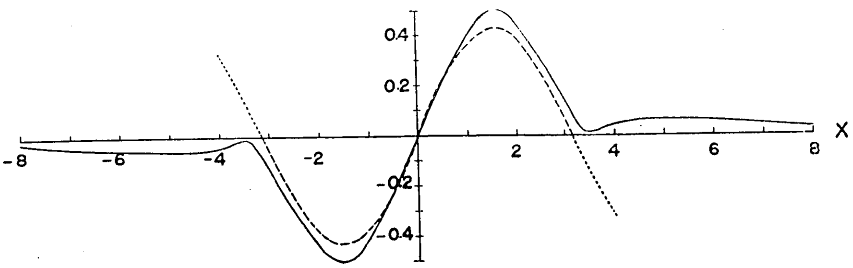


Fig. 3. The magnetization obtained by T-N method (broken line) and by the Hermite function method (full line) for $D=1$.

As clearly shown in these figures, the discrepancies between the results obtained by the T-N method and those by the Hermite function method become fairly large at the ends of the anomaly distribution. However, we find no serious difference in the central part of the distribution as may be naturally expected. Hence the T-N method may be applied if one does not wish to keep rigorousness near the ends of the anomaly distribution.

## 5. Separation of geomagnetic field into external and internal origin parts.

When the $X$- and $Z$- components of a geomagnetic field are given over a plane surface $z=0$, it is obvious that the field can be separated into two parts originating from above and below $z=0$ respectively. Such a separation has already been discussed by Vestine[1] by use of Fourier series and surface integrals. Meanwhile the Fourier series method has been applied to some actual problems concerning the field under the auroral zone by A. G. McNish[4] and Nagata[5], while the writer[6],[7] applied the same method to the SF-variation (a geomagnetic variation associated with the solar-flare effect) and to the anomalous distribution of geomagnetic disturbance of short period in Japan.

According to the Hermite function method developed in this paper, it is also possible to separate the geomagnetic field into the external and internal origin parts. Suppose the field on $z=0$ is given as

$$X_0=\sum_n K_n\phi_n(x),\quad Z_0=\sum_n L_n\phi_n(x), \tag{30}$$

these components can be written in the next forms;

$$\left.\begin{aligned} X_0&=\sqrt{\frac{2}{\pi}}\sum_{m=0}^{\infty}(-1)^m\left\{K_{2m}\int_0^\infty \phi_{2m}(u)\cos ux\, dn+K_{2m+1}\int_0^\infty \phi_{2m+1}(u)\sin ux\, du\right\},\\ Z_0&=\sqrt{\frac{2}{\pi}}\sum_{m=0}^{\infty}(-1)_m\left\{L_{2m}\int_0^\infty \phi_{2m}(u)\cos ux\, du+L_{2m+1}\int_0^\infty \phi_{2m+1}(u)\sin ux\, du\right\},\end{aligned}\right\} \tag{31}$$

using the relations (13) and (14).

On the other hand we assume the magnetic potential $W$ which satisfies $\nabla^2 W=0$. It is given as

$$W=\int_0^\infty [\{e_c(u)e^{-uz}+i_c(u)e^{uz}\}\cos ux+\{e_s(u)e^{-uz}+i_s(u)e^{uz}\}\sin ux]\, du. \tag{32}$$

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4) A. G. McNISH, *Terr. Mag.*, **43** (1938), 67.
5) T. NAGATA, *Rep. Ionos. Res. Japan*, **4** (1950), 87.
6) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 219.
7) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **30** (1952), 207.

Then we get $X_0$ and $Z_0$ from (32) as follows;

$$\left.\begin{aligned} X_0 &= \int_0^\infty [\{e_c(u)+i_c(u)\}u\sin ux - \{e_s(u)+i_s(u)\}u\cos ux]\,du, \\ Z_0 &= \int_0^\infty [\{e_c(u)-i_c(u)\}u\cos ux + \{e_s(u)-i_s(u)\}u\sin ux]\,du. \end{aligned}\right\} \quad (33)$$

Comparing (31) with (33), we have the next simultaneous equations

$$\left.\begin{aligned} e_c(u)+i_c(u) &= \sqrt{\frac{2}{\pi}}\sum_m(-1)^m K_{2m+1}\frac{\phi_{2m+1}(u)}{u}, \\ e_c(u)-i_c(u) &= \sqrt{\frac{2}{\pi}}\sum_m(-1)^m L_{2m}\frac{\phi_{2m}(u)}{u}, \\ -e_s(u)-i_s(u) &= \sqrt{\frac{2}{\pi}}\sum_m(-1)^m K_{2m}\frac{\phi_{2m}(u)}{u}, \\ e_s(u)-i_s(u) &= \sqrt{\frac{2}{\pi}}\sum_m(-1)^m L_{2m+1}\frac{\phi_{2m+1}(u)}{u}. \end{aligned}\right\} \quad (34)$$

from which we can obtain $e_c$, $i_c$, $e_s$ and $i_s$. With these coefficients we have the external and internal parts of $X$ on $z=0$ as written below after some calculations, the suffices $e$ and $i$ denoting the parts caused by the origins in $z>0$ and $z<0$ respectively;

$$\left.\begin{aligned} X_e &= \frac{1}{2}(X_0+X'), \\ X_i &= \frac{1}{2}(X_0-X'), \end{aligned}\right\} \quad (35)$$

where

$$X' = \sqrt{\frac{2}{\pi}}\sum_{m=0}^{\infty}(-1)^m\left\{L_{2m}\int_0^\infty \phi_{2m}(u)\sin ux\,du - L_{2m+1}\int_0^\infty \phi_{2m+1}(u)\cos ux\,du\right\}. \quad (36)$$

And we have also

$$\left.\begin{aligned} Z_e &= \frac{1}{2}(Z_0-Z'), \\ Z_i &= \frac{1}{2}(Z_0+Z'), \end{aligned}\right\} \quad (37)$$

where

$$Z'=\sqrt{\frac{2}{\pi}}\sum_{m=0}^{\infty}(-1)^m\left\{K_{2m}\int_0^{\infty}\phi_{2m}(u)\sin ux\,du-K_{2m+1}\int_0^{\infty}\phi_{2m+1}(u)\cos uxdu\right\}. \quad (38)$$

## 6. Conclusions.

By using Hermite functions, the writer proposed a rigorous method of obtaining subterranean magnetization by which the geomagnetic anomaly on the earth's surface is caused. As shown in the example, the present method is more accurate than the one based on Fourier series which has been developed by Tsuboi and Nagata. Though the Hermite function method seems to be rather complicated, the writer is now arranging the tables of integrals such as $\int_0^{\infty}e^{uD}\phi_n(x)\frac{\cos}{\sin}ux\,du$ for various $D$ and $x$, which will be useful for actual analyses. The theory of separation of geomagnetic field into external and internal origin parts is also discussed in this paper.

In conclusion the writer is grateful to Miss Y. Hishiyama and Miss Y. Naito who assisted the writer in the course of the numerical work. A part of this study was conducted with the financial aid of research grant from the Department of Education and the writer's sincere thanks are also due to the Department.

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## 25. エルミット函数による地磁気の解析

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エルミット函数の直交性を利用して，地表で観測された磁場分布より地下の1平面上の帯磁分布を直接求める方法を考案した．この方法によれば，孤立した地磁気異常を解析する場合に正確な結果を与える．フーリエ級数による坪井―永田の方法とくらべる時，異常の中央部附近ではほとんど一致するが，末端ではいちじるしい喰違いが見られ，厳密を要する場合はエルミット函数法によるべきであると考えられる．地表の磁場を外部および内部に原因を有する部分に分ける方法も述べてある．

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