27. *On the Energy Law of Occurrence of Japanese Earthquakes.*

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As far back as in 1862 R. Mallet estimated the *vis viva* of the Neapolitan earthquake of 1857, and T. C. Mendenhall proposed a method of estimating the energy of earthquakes, and applied it to the Charleston earthquake of 1886, although his assumption was in no wise perfect. O. Enya later discussed the same problem from a broader point of view. H. F. Reid calculated the energy released from the fault in the California earthquake of 1906, and B. Galitzin estimated the energy of the Pamir earthquake from the seismogram observed at a single station Pulkovo. Navarro-Neumann also determined the energy of a number of earthquakes by his own and Galitzin's formula. Recently in Japan, K. Wadati inferred relative magnitudes (not energy) of earthquakes by means of intensity-distance ($I-\Delta$) curve. More recently C. F. Richter and B. Gutenberg made a thorough investigation on the problem based on the observed maximun amplitude by means of the Wood-Anderson torsion seismometer. The writer also made a magnitude scale and calculated the energy for each magnitude, and determined the magnitudes of Japanese earthquakes from 1885 to 1943. The résumé of the study is given here.

The writer first determined the $I-\Delta$ and $A-\Delta$ curves for normal earthquakes using the data of some hundred earthquakes in Japan. The result is

$$e^{I}=\left(\frac{100}{\Delta}\right)^{2}e^{M_k-0.00183(\Delta-100)}\,, \qquad (1)$$

$$A=\left(\frac{100}{\Delta}\right)^{1/2}e^{m-0.00305(\Delta-100)} \qquad \text{for } 100\leqq\Delta\leqq700 \qquad (2)$$

$$=\left(\frac{100}{\Delta}\right)^{2}e^{m+1.21-0.00041(\Delta-600)} \qquad \text{for } 700\leqq\Delta\leqq2000 \qquad (2')$$

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where A is the maximum amplitude in μ at epicentral distance Δ measured in km, while M_k and m are I and $\log_e A$ at $\Delta=100$ km respectively. Between M_k and m the following relation was found to exist

$$m=3.4+1.2M_k \quad \text{or} \quad \log_{10} A_{100} \doteqdot 1.5+0.5M_k. \tag{3}$$

By means of these formula we can infer the magnitude M_k of any earthquake from the $I-\Delta$ or $A-\Delta$ curve observed.

The energy of earthquake as sent out from the origin may be calculated by the formula

$$4\pi r^2 vt \tfrac{1}{2}\sigma(2\pi A/T)^2 \tag{4}$$

where v and σ are the velocity in and the density of the earth's crust and A, T and t are the amplitude, period and duration of earthquake motion at distance r respectively. Since energy is proportinal to the square of amplitude, the energy is concentrated to the maximum phase so that we may use for A the maximum amplitude and its equivalent duration for t.

From the observations at Tokyo the writer obtained the relation

$$\overline{\alpha}=(2\pi/T)^2 A=0.412(3.266)^I \doteqdot 0.45\times 10^{0.5I} \text{ gals} \tag{5}$$

for the mean acceleration $\overline{\alpha}$ of earthquakes of intensity I in Japanese scale. Introducing this into the equation (4) we have

$$\overline{\frac{dE}{dt}}=3\times 10^{17+M_k} \qquad \text{ergs/sec.} \tag{6}$$

since M_k is the intensity at $r \doteqdot \Delta=100$ km.

In the above calculation we have assumed A to be the mean value of A on the sphere $r=r_0$. From observations in Japan we see that earthquakes are in no wise of uniform amplitude in all azimuths. The writer has proved the existence of such earthquakes of which the displacement may be expressed as

$$\begin{aligned} u \doteqdot \mathbf{i}_1 \frac{\mathfrak{A}}{r}\cos(pt-hr)P_n^m(\cos\theta)\sin m\varphi \\ +\mathbf{i}_2\frac{\mathfrak{B}}{r}\cos(pt-kr)\frac{dP_n^m(\cos\theta)}{d\theta}\sin m\varphi \\ +\mathbf{i}_3\frac{m\mathfrak{B}}{r}\cos(pt-kr)\frac{P_n^m(\cos\theta)}{\sin\theta}\cos m\varphi, \end{aligned} \tag{7}$$

where $n=2$ and $m=0$, 1, or 2. In these cases the time mean of the total energy flux calculated as usual is

$$\frac{\overline{dE}}{dt}=\frac{\pi(n+m)!}{(2n+1)(n-m)!}\sigma p^2\{\mathfrak{A}^2V+n(n+1)\mathfrak{B}^2\mathfrak{B}\}=4\pi r^2\tfrac{1}{2}\sigma p^2\overline{\mathfrak{B}}^2\mathfrak{B}C \quad (8)$$

in which V and $\mathfrak{B}$ are the velocities of P- and S-waves respectively, and $\overline{\mathfrak{B}}$ is the mean amplitude at $r=r_0$ of S-wave for all azimuths. The constant C is about 1.5 for $n=2$, $m=0$; or 2.2 for $n=2$ and $n=1$ or 2, provided that Poisson's ratio of the crust is $\frac{1}{4}$ and $\mathfrak{A}/\mathfrak{B}=\frac{1}{2}\sim\frac{1}{4}$ as is usually the case with deep earthquakes hitherto observed in this country.

Thus we find that the usual formula (4) should be multiplied by ca. 2 if we consider the mechanism of occurrence and use the mean amplitude, an effect rather unimportant compared with the case when the amplitude observed at a single station (7) is used, since the latter is subjected to a wide variation owing to instruments and underground-conditions. It is also to be remarked that the amplitude observed on the earth's surface is about twice that of the incident wave owing to the effect of reflection, so that the energy calculated by (8) from observed maximum amplitude should be divided by 4. We may therefore roughly substitute 1 for the coefficient 3 in the right hand side of the equation (6). It must also be added that the writer's later investigation showed that the attenuation of amplitude even to such small distance as $r=100$ km. is not negligible. If this result of the Tokyo observation is to be adopted, the energy passing the sphere $r=100$ km. is about one thirtieth of the energy emitted from the hypocentre. This is a grave ploblem for future investigation.

The $I-\Delta$ or $A-\Delta$ curve expressed in (1) or (2) was applied to the observations of all normal earthquakes which took place from 1885 to 1943, and their magnitudes were determined. The magnitudes of the earthquakes in recent 40 years of which the epicentres were more accurately determined are represented on the map in Fig. 1 with the magnitudes of the circles denoting the meizoseismic area (in which the demolition of houses is to be seen, i. e. $I=6$).

The statistics of the number N of earthquakes of magnitude M_k in the same 40 years from 1904 to 1943 shows

M_k	4	5	6	7	8
N	275	84	21	1	2

which is expressed by the least square solution

$$N=273\times10^{-0.537(M_k-4)}. \quad (9)$$

The statistics of smaller earthquakes seems imperfect from two reasons, that is (1) the records of smaller earthquakes in this country were not kept regularly, and (2), as is evident from Fig. 1, most of the earthquakes occur under the ocean bed off the Pacific coast of NE Japan and smaller earthquakes were liable to be omitted from previous observations. The energy law of smaller earthquakes is therefore examined from a different viewpoint by making use of observations at Tokyo the result of which is reported in next paper. The result also proves the energy law

$$N \propto 10^{-0.556 M_K}. \qquad (10)$$

The difference of the index 0.556 from the above 0.537 is not significant from the statistical point of view. So it may be concluded that the number of Japanese earthquakes diminishes inversely to power equal to half the magnitude of 10, that is

$$N \propto 10^{-0.5 M_K} \qquad (11)$$

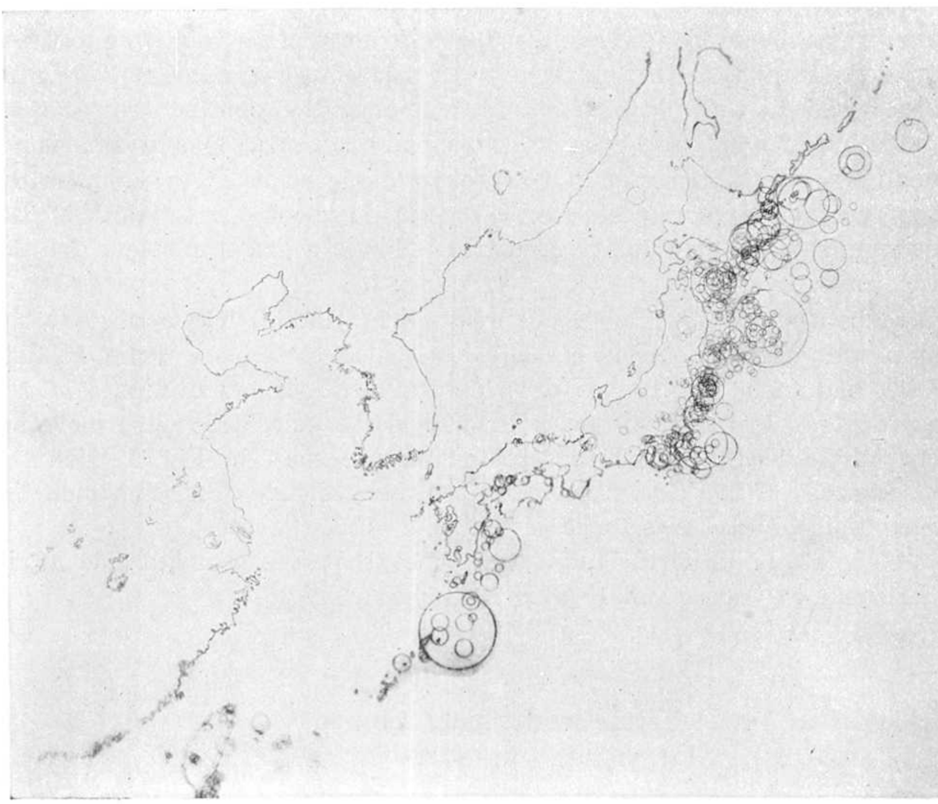

Fig. 1. Map of the meizoseismic Area of Large Earthquakes in Japan during 1904—1943

27. 日本に起る地震のエネルギー法則

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地震のエネルギーは R. Mallet が 1857 年のナポリ地震について計算したのが始りで, T. C. Mendenhall, Galitzin, Navarro-Neuman, Jeffreys 等の計算があり, 最近では C.F. Richter, B. Guteberg 等が特殊の地震計により観測した最大振幅からその地震のエネルギーを計算する方式を確立した. 筆者も日本に於ける地震観測結果を用いて同様な事を行つた.

それは標準の $I(\triangle)$ 及び $A(\triangle)$ 曲線 (1) 及び (2) を定めそれを用いて震央距離 $\triangle=100$km の所に於ける震度 $I\equiv M_k$ 及び, 最大振幅 A_{100} (單位 μ) を用いるもので

$$A_{100} \doteqdot 10^{1.5+0.5M_k}$$

の關係があり, 最大動の継続時間をtとすればエネルギー E は

$$E \doteqdot t \cdot 10^{17+M_k} \quad \text{エルグ}$$

である.

この様にして筆者は日本古來の地震の大きさ M_k を定めたが, それ等の頻度を統計して見ると 1911～1940 の間の比較的大きな地震については

$$N(M_k)=273\times 10^{-0.537\ (M_k-4)}$$

の關係が得られた.

統計の精度から見て

$$N(M_k) \propto 10^{-0.5M_k}$$

と見て差支へないであろう. 即ち地震の頻度はその大きさ M_k が二階級昇るごとに10分の一に減少することになる.