28. *Energy Law of Earthquake Occurrence in the Vicinity of Tokyo.*

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In the former paper on Japanese earthquakes, the writer obtained a conspicuous energy law on the occurrence of larger earthquakes. An extension of the law so as to include smaller earthquakes will be made in this paper. Investigations related to the present problem have hitherto been made by senior authorities such as Prof. Terada, Prof. Ishimoto, Prof. Matuzawa and Prof. Gutenberg as well as Dr. Wadati and Dr. Iida. Further studies including some theoretical improvements and the actual examinations which support the present theory have here been made by the writer.

The energy of an earthquake sent out from the hypocentre is transmitted as elastic bodily and surface waves, diminishing their amplitudes with the hypocentral distance owing to the spreading of wave-fronts, reflection, refraction and absorption. The amplitude observed at a single station therefore depends firstly upon the magnitude of the energy sent out from the origin, and secondly upon the focal distance and thirdly upon the structure of the medium through which it is propagated. But, for simplicities' sake, the last item is here neglected, and the effects of reflection and refraction are not taken into consideration.

The amplitude is thus assumed to vary only with the focal distance. The effect of absorption is assumed to be proportional to e^{-kr}. Thus we put

$$A=A_0R(r)e^{-kr} \qquad (1)$$

where A_0 is the amplitude at distance $r=r_0$, so that

$$R(r_0)e^{-kr_0}=1$$

Let us now consider the frequency of the maximum amplitude of earthquakes observed at a station. If we denote by FdA_0 the frequency of earthquakes having amplitude A_0 at r_0 and occurring in unit volume

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per unit time, then F is generally a function of position (x, y, z), time, and amplitude A_0. But as we are here dealing with the statistical mean value, we will assume that F is independent of time, whereupon

$$F=\varphi(x, y, z)f(A_0) \tag{2}$$

Then the frequency $\delta n(A)dA$ of A between A and $A+dA$ observed at a station at $(0, 0, 0)$ is

$$\delta n(A)dA=\varphi(x, y, z)f(AR^{-1}e^{kr})d(AR^{-1}e^{kr})dx\,dy\,dz. \tag{3}$$

As already stated, we are dealing with the mean value of seismic activities, so we will assume φ to be constant in the domain $-\infty\leqq x\leqq\infty$, $-\infty\leqq y\leqq\infty$ and $z_1\leqq z\leqq z_2$ and to be 0 outside the domain. *This is approximately true* in the region concerned. By integrating (3) with respect to polar coordinates we obtain the frequency of earthquakes occurring at a distance between r and $r+dr$ from the station and having amplitude between A and $A+dA$ at the station. When r is sufficintly large it results in

$$\delta n_r(A)drdA=2\pi\varphi hrR^{-1}e^{kr}dr f(AR^{-1}e^{kr})dA, \tag{4}$$

where h denotes (z_2-z_1).

Next comes the determination of φ, h, k and the functional form of f. Empirically we know already that $f(\infty)=0$, so we will introduce Laurent's series

$$f=\sum_{i=1}^{\infty} a_i/A_0^i$$

into (4). Then integrating with respect to A within the variable interval $A_I=A_1\kappa^{I-1}$ and $A_{I+1}=A_1\kappa^I$, and denoting the result by $n_r(I)dr$, we have

$$n_r(I)=2\pi\varphi hrR^{-1}e^{kr}\left[a_1Re^{-kr}\log\kappa+\sum_{i=2}^{\infty}a_iR^ie^{-ikr}\frac{1}{(i-1)A_I^{i-1}}\{1-\kappa^{1-i}\}\right]. \tag{6}$$

We know also from actual observations that $\lim_{I=\infty} n_r(I)=0$, and thus $a_1=0$. For convenience' sake we will integrate this equation from r to $r+\rho$, for finite interval ρ, then

$$N(I, r, \rho)=2\pi\varphi h\sum_{i=2}^{\infty}\frac{a_i(1-\kappa^{1-i})}{(i-1)A_I^{i-1}}\int_r^{r+\rho}rR^{i-1}e^{-(i-1)kr}dr. \tag{7}$$

Observations at Tokyo show that $N(I, r, \rho)$ is invariably proportional to $e^{-\lambda r}$. This leads to the conclusion that rR^{i-1} is *constant* for only one admissible integer i greater than 1.

In addition to this, the observations at Tokyo show that

$$N(I, r, \rho) \infty 1/A_I \tag{8}$$

indicating that $i=2$, so that

$$R = r_0 e^{kr_0}/r, \tag{9}$$

and

$$f = a_2/A_0^2. \tag{10}$$

Thus we have

$$F(A_0) = \varphi a_2/A_0^2. \tag{11}$$

Since the energy of seismic waves is proportional to the square of amplitude, the energy of all earthquakes with amplitude between A_0 and A_0+dA_0 is proportional to $F(A_0)A_0^2 dA_0 = \varphi a_2 dA_0$ which is a constant, that is to say, the law of equipartition of energy holds true here.

Now we have

$$N(I, r, \rho) = 2\pi\varphi h a_2 r_0 (1-\kappa^{-1})(1-e^{-k\rho})(kA_I)^{-1} e^{-k(r-r_0)}. \tag{12}$$

Assuming that r′ is the minimum distance for the equation (12) to hold, we will sum up all the earthquakes with I and occurring at all distances. The result is equivalent to the limit of (12) when $\rho \to \infty$, at r'.

$$N(I) = N(I, r', \rho\to\infty) = 2\pi\varphi h a_2 r_0 (1-\kappa^{-1})(\kappa A_I)^{-1} e^{-k(r'-r_0)}. \tag{13}$$

Similarly if we take the limit of (12) for $I=1$, when $\kappa\to\infty$, the result is equivalent to the sum $N'(r, \rho)$ of all the earthquakes occurring at distances between r and $r+\rho$.

$$N'(r, \rho) = 2\pi\varphi h a_2 r_0 (1-e^{-k\rho})(kA_1)^{-1} e^{-k(r-r_0)}. \tag{14}$$

From (13) or (14) we obtain the total number $\overline{N}$ of earthquakes observed at a station

$$\overline{N} = 2\pi\varphi h a_2 r_0 (kA_1)^{-1} e^{-k(r'-r_0)}. \tag{15}$$

Introducting $\overline{N}$ into (12), (13) and (14) we have

$$N(I, r, \rho) = \overline{N}(\kappa-1)(1-e^{-k\rho})\kappa^{-I} e^{-k(r-r')}, \tag{16}$$

$$N(I) = \overline{N}(\kappa-1)\kappa^{-I}, \tag{17}$$

$$N'(r, \rho) = \overline{N}(1-e^{-k\rho})e^{-k(r-r')} = \overline{N}(1-e^{-k\rho})e^{-nk\rho} \tag{18}$$

provided $r = r' + n\rho$.

Now let us calculate the energy emitted from a unit volume in the layer considered. As we have seen, the number of earthquakes occurring in the same volume in unit time is $F(A_0)dA_0$. And the energy emitted in time t (duration of maximum amplitude) from the hypocentre of an earthquake having amplitude A_0 at r_0 is $4\pi r_0^2 vt\frac{1}{2}\sigma(2\pi A_0 e^{kr_0}/T)^2$. The total energy of all the earthquakes having amplitudes between A

and A_0+dA_0 from unit volume per unit time is thus obtained from the product of these two values. If we multiply this by the thickness of the layer, h, we have a measure of seismicity:

$$S=(2\pi/T)^2 A_1 \bar{N} k r_0 e^{k(r'+r_0)} \sigma v t d A_0, \tag{19}$$

which is a constant independent of A_0 and r_0, since $A_0 r_0 e^{kr_0}$ is invariant owing to (1) and (9). From a practical point of view it will be convenient to use the energy per 100 km. square and per 100 years. If we integrate (19) with respect to A_0 from its minimum to maximum value, we may have a better measure of seismicity, but still simpler and more direct is the number of earthquakes of magnitude M_k per unit area in unit time, that is

$$N(M_k)=\int_{A_{0I}}^{\kappa A_{0I}} hF(A_0)dA_0=\varphi h a_2(\kappa-1)A_{0I}^{-1}\kappa^{-1}. \tag{20}$$

Let us now compare the above theory with actual observations, and determine the constants $\bar{N}$, φa_2, κ, and k. As already pointed out by the late Prof. Ishimoto, Weber-Fechner's law holds true in the Japanese intensity scale and earthquake acceleration. The writer's later determination with ampler materials of the relation is

$$\bar{\alpha}=0\cdot 422(3.266)^I \doteqdot 0.45\times 10^{0.5I} \tag{21}$$

for the mean acceleration corresponding to I, while the lower limit $\alpha_l=0.25\times 10^{0.5I}$. And since the periods of earthquake motion manifesting maximum acceleration at Hongō (Tokyo) is invariably 0.3 sec., the acceleration and amplitude there may be assumed to be proportional. The writer therefore took the statistics of the earthquake observations at Tokyo in the 30 years from 1911 to 1940 with intensity I, and the result is shown in Table 1.

From this table we see that r' is the distance where the duration of preliminary tremor $(S-P)$ is ca. 5 sec. and $\bar{N}=1847$. From the values of $N(I)$ and $N'(r, \rho)$ in the last row and column of the table, we have by the method of least squares

$$\bar{N}=1785,\ \kappa=3.602;\ \text{and}\ \bar{N}=1730,\ kv=0.1467,$$

where v is the virtual velocity of $(S-P)$ wave for which the value satisfying $12\,v=100\,\text{km}$ is adopted here.

Using the observed $\bar{N}=1847$, κ and kv determined above, we have

$$N(I, r, \rho)=2499\times 10^{-0.556I}e^{-0.734n},$$

in 30 years, and the values calculated by this formula are tabulated in Table 1. Comparing the calculated values with those observed we find a fairly good conformity.

Table 1. Statistics of earthquakes with intensity I and duration of preliminary tremor $(S-P)$ observed at Hongō, Tokyo during 1911–1940. The number without bracket is the observed frequency, $N(I, r, \rho)$, while those in bracket are the values calculated from the above theory.

$(S-P)$ \ I	n	1	2	3	4	5	6	$N'(r, \rho)$
sec.		[29] (0)	[3] (0)	[1] (0)	[0] (0)	[0] (0)	[0] (0)	[33] (0)
5	0							
		810 (694)	208 (193)	55 (54)	9 (15)	5 (4.1)	1 (1.1)	1088 (960)
10	1							
		303 (333)	62 (92)	27 (27)	12 (7.1)	3 (2.0)	0 (0.5)	407 (461)
15	2							
		136 (160)	34 (44)	8 (12)	4 (3.4)	0 (0.9)	0 (0.3)	182 (221)
20	3							
		71 (77)	13 (21)	2 (5.9)	0 (1.6)	0 (0.5)	0 (0.1)	86 (106)
25	4							
		34 (37)	8 (10)	3 (2.8)	1 (0.8)	0 (0.2)	0	46 (51)
30	5							
		22 (18)	3 (4.9)	2 (1.4)	0 (0.4)	0 (0.1)	0	27 (24)
35	6							
		4 (8.5)	5 (2.4)	1 (0.7)	1 (0.2)	0	0	11 (12)
$N(I)$		1380 (1334)	333 (370)	98 (103)	27 (29)	8 (7.9)	1 (2.2)	1847

The seismicity S calculated by (19) thus becomes

$$S=0.9\times 10^{25} t d A_0 \text{ ergs}/(100\text{ km})^2 \ 100 \text{ years},$$

and from (21)

$$N(M_k)=1.5\times 10^{3-0.556M_k}/(100\text{ km})^2 \ 100 \text{ years},$$

while the number of all earthquakes occurring at any distance and manifesting intensity I at Tokyo in 100 years is, by (17)

$$N(I)=1.6\times 10^{4-0.556I}$$

The index 0.556 may be approximately replaced for the present by $\frac{1}{2}$ so long as the present statistical accuracy is concerned. Then a notably simple law $N(M_k)=C\times 10^{-0.5M_k}$ will be found to exist in the occurrence of earthquakes in the vicinity of Tokyo, and this law is by no means a peculiarity conbined to this region, since, it is nearly the same as the law for Japanese earthquakes already mentioned.

28.　東京附近に起る地震のエネルギー法則

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前論文に於て日本に起る比較的大きな地震の頻度が $10^{-0.5M_k}$ に比例して，M_k が大きくなると小くなる事を示したが，本論文に於てはもつと小さい地震の発生頻度にも同様な關係のあることを東京に於ける三十年間の地震の震度別並びに距離別回数の統計とその理論的考察から確めた．

東京に於て距離 r と $r+\rho$ の間の所で起り，震度が I である地震の回数 $N(I, r, \rho)$ は

$$N(I, r, \rho)=N(\kappa-1)(1-\mathrm{e}^{-k\rho})\ \kappa^{-I}e^{-\kappa(r-r')}$$

なる理論的関係でよく表わされる．ここに κ は震度階級の差に対する加速度の倍數で k は地殼表層に於ける吸收係數であり r' は上の公式が成り立つ最小距離である．　この式は地下或る深さにある一定の厚さの層の中だけに地震が振幅の二乗に逆比例した頻度で起る事を意味し，実際の観測からは $\kappa=10^{0.556}$ で前論文に得た日本に起る大きな地震の発生法則の常数 $10^{0.537}$ と極めて近く，実際の震度と加速度の関係を示す式の常數 $10^{0.514}$ とかなり近い事は偶然とは思われない．これ等は何れも現在の観測精度から $10^{0.5}$ と見てよかろう．

之が正しいとすれば，日本に起る地震は少くとも有感地震以上ではその発生頻度は，今地震動の周期を別問題にすれば，最大振幅の２乗に逆比例する．換言すればエネルギーの等配則が成り立つと云う事が出来る．

因みに上の結果から最大動は吸收作用で 100km の間で 0.17 に減じ，從つてエネルギーは約 1/30 に減ずる事になり，地震のエネルギーを論ずる時決して無視出來ない．之の點は將來他の方面から大いに確めなければならない問題であろう．
