17. *On the Electrical Conductivity in the Earth's Core.*

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1. Introduction.

The origin of the earth's main magnetic field which has long been a mystery in geophysics seems to be successfully explained by the coupling of magnetic field and fluid motion in the earth's core, the theory being developed by W. M. Elsasser[1] and E. C. Bullard[2]. Since the possibility of existence of a self-exciting dynamo was recently proved by H. Takeuchi and Y. Shimazu[3], the Elsasser-Bullards' theory is thought to be the most reliable one. According to the theory, the electrical conductivity in the core (σ_c) is assumed to amount to the order of 10^{-6} *emu*. Though the said order of σ_c is adopted from a physical consideration of metallic iron under high pressure and temperature, it would be of importance to criticize the value more precisely from the standpoints of both physics and geophysics as briefly stated by the writer in his former paper[4].

Geophysically speaking, the electrical state of the earth's interior can be inferred from the studies of electromagnetic induction into the earth by transient geomagnetic variations, and the theory has been developed by S. Chapman[5], A. T. Price[6], B. N. Lahiri[7] and the writer[8]. From these studies, we have an approximate knowledge about the electrical conductivity down to a depth of about 1500 *km*. The writer[9] also studied the length of the period of geomagnetic variation required to infer the electrical conductivity in the core. The conclusion was as follows: We should analyse geomagnetic variations having periods

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1) W. M. ELSASSER, *Phys. Rev.*, **69** (1946), 106, **70** (1946), 202.
2) E. C. BULLARD, *Proc. Roy. Soc. London A*, **197** (1949), 433, **199** (1949), 413.
3) H. TAKEUCHI and Y. SHIMAZU, *Journ. Phys. Earth.*, **1** (1952), 1.
4) T. RIKITAKE, *Geofisica pura e applicata*, **22** (1952), 37.
5) S. CHAPMAN, *Phil. Trans. Roy. Soc. London A*, **218** (1919), 1.
6) S. CHAPMAN and A. T. PRICE, *Phil. Trans. Roy. Soc. London A*, **229** (1930), 427.
7) B. N. LAHIRI and A. T. PRICE, *Phil. Trans. Roy. Soc. London A*, **227** (1939), 509.
8) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 45, 219, 263, **29** (1951), 61.
9) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **29** (1951), 539.

longer than 10 *years* in order to determine σ_c, but an analysis of such a long period variation would be almost impossible judging from the accuracy of geomagnetic observation to-day.

Though we failed to tackle the problem by geophysical method, some researchers studied the problem from physical standpoint. Elsasser made an estimate of σ_c as the electrical conductivity of metallic iron under high pressure and temperature. He assumes the relation

$$\sigma = c\theta^2/T,$$

which is valid for ordinary iron at high temperature, where σ, θ and T denote respectively the electrical conductivity, Debye temperature and absolute temperature. In the earth's core, the factor c is assumed to remain constant and T is taken to be 9000°. It becomes then

$$\sigma_c = 1.3 \times 10^{-5}\, emu.$$

Meanwhile, Bullard[2] has obtained $\sigma_c = 1 \times 10^{-6}\, emu$ by using empirically determined pressure and temperature variations.

According to Elsasser[11], we have powerful evidences for iron-nickel core as revealed by the relation between density and pressure in the earth. He shows that the density-pressure curve determined by K. E. Bullen[12] seems to continue to those of silicate group at low pressure and to those of substance having Z (atomic number)$\doteqdot$30 at high pressure, the former being based on laboratory experiments and the latter on a quantum-theoretical estimate for Thomas-Fermi-Dirac model.

A direct estimate[13] of σ_c based on Thomas-Fermi-Dirac model which is valid under enormous pressure also leads to the conclusion: $\sigma_c \simeq 10^{-4}$ *emu*.

As stated above, we may expect that σ_c amounts to the order of $10^{-5}\, emu$ or more from physical viewpoint. But it should be borne in mind that these estimates still include some unknown factors.

The writer, intending to obtain the possible range of σ_c, will make an estimate of σ_c from some geophysical standpoints which are quite different from those mentioned before.

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10) W. M. ELSASSER, *Rev. Mod. Phys.*, **22** (1950), 1.
11) W. M. ELSASSER, *Science*, **113** (1951), 105.
12) K. E. BULLEN, *Introduction to the theory of seismology*. Cambridge (1947).
13) H. MIKI, *Kagaku*, **21** (1951), 468.

2. Possibility of the existence of magneto-hydrodynamic waves in the earth's core.

As has been suggested first by H. Alfvén[14], there is a type of wave-motion, called magneto-hydrodynamic waves, in an electrically conducting fluid placed in a magnetic field. The magneto-hydrodynamic waves play an important rôle in solar physics such as the origin of sunspots, the heating of solar corona, and the generation of cosmic radiation. The existence of the magneto-hydrodynamic waves is experimentally observed[15] as the coupling between a magnetic field and sound waves in mercury.

In the earth's core which is thought to be more or less electrically conducting and to have a magnetic field, a kind of magneto-hydrodynamic waves would be excited if the boundary of the core is stimulated by seismic waves which come from the earth's mantle. The magneto-hydrodynamic waves thus produced, which are essentially transverse waves, would be propagated along the magnetic lines of force as fully studied by Alfvén[16] and would appear again at the surface of the core if the waves had sufficient intensity. The velocity of the waves which is proportional to the intensity of the magnetic field would be very small in case of non-extraordinary magnetic field. Thus we could observe seismic waves which pass through the core as the magneto-hydrodynamic waves, the damping effect in the core being assumed to be small. We should further pay attention to the fact that the seismic waves reflected at the surface of the core, such as ScS waves, would be much affected if the magneto-hydrodynamic waves of appreciable intensity are generated in the core.

According to the results of seismic observations, we cannot find phases of seismic waves which come through the core as the magneto-hydrodynamic waves and ScS waves, one of the most distinct phases of the waves reflected at the surface of the core, do not seem to be affected by the reaction of the hypothetical magneto-hydrodynamic waves. Hence the electrical property and magnetic field in the core should be within limits in which no magneto-hydrodynamic waves of appreciable intensity would be generated.

The writer would here like to determine the possible range of σ_c

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14) H. ALFVÉN, *Ark. f. Mat. Astr. o. Fys.*, **29B** (1942), No. 2.

15) S. LUNDQUIST, *Phys. Rev.* **76** (1949), 1805.

16) H. ALFVÉN, *Cosmical Electrodynamics* (1950), Oxford.

which does not contradict with the seismic observation mentioned above.

Combining the fundamental equations of electromagnetism with those of hydrodynamics, we have the next equations in the electrically conducting fluid.

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho}\,\mathrm{grad}\,p - \lambda[\vec{B}\,\mathrm{rot}\,\vec{B}], \tag{1}$$

$$\frac{\partial \vec{B}}{\partial t} = \mathrm{rot}\,[\vec{v}\,\vec{B}] + \nu_m \nabla^2 \vec{B}, \tag{2}$$

where $\vec{v}$, $\vec{B}$, p and ρ denote respectively velocity, magnetic induction, pressure and density and λ and ν_m are defined by

$$\lambda = \frac{1}{4\pi\mu\rho}, \quad \nu_m = \frac{1}{4\pi\mu\sigma}, \tag{3}$$

where μ and σ denote magnetic permeability and electrical conductivity, ν_m being called magnetic viscosity. In the above expressions, we have neglected the displacement current, the inertia term of fluid-motion and fluidal viscosity.

Now let us consider plane waves propagated along z-direction, while the initial magnetic field is also parallel to the z-direction. We have, then,

$$\vec{B} = \vec{B}_0 + \vec{b}$$

in which $\vec{B}_0$ and $\vec{b}$ correspond respectively to the initial magnetic field and the field due to the electric current. In such a simple case, the equations are reduced to

$$v_x = \mathrm{const.}, \tag{4}$$

$$\partial v_y/\partial t = \lambda B_0 \partial b_y/\partial z \tag{5}$$

$$\partial v_z/\partial t = -\rho^{-1}\partial p/\partial z - (\lambda/z)\partial(b_y^2)/\partial z, \tag{6}$$

$$\partial b_y/\partial t = -B_0 \partial v_y/\partial z - \partial(v_z b_y)/\partial z + \nu_m \partial^2 b_y/\partial z^2. \tag{7}$$

Neglecting, for the first time, the second term of the righthand-side of (7) and combining it with (5), we get a wave equation of the type

$$\partial^2 b_y/\partial t^2 = \lambda B_0^2 \partial^2 b_y/\partial z^2 + \nu_m \partial^3 b_y/\partial^2 z \partial t, \tag{8}$$

from which we get a typical solution

$$b_y = A e^{i\alpha\left(t + \frac{z}{\sqrt{\lambda B_0^2 + i\nu_m \alpha}}\right)}, \tag{9}$$

the solution being correct for $|B_0| \gg |b|$.

Putting (9) into (5) and integrating with respect to t, we obtain the velocity

$$v_y = \frac{A\lambda B_0}{\sqrt{\lambda B_0^2 + i\nu_m\alpha}} e^{i\alpha\left(t + \frac{z}{\sqrt{\lambda B_0^2 + i\nu_m\alpha}}\right)}, \tag{10}$$

and the displacement

$$u_y = \frac{A\lambda B_0}{i\alpha\sqrt{\lambda B_0^2 + i\nu_m\alpha}} e^{i\alpha\left(t + \frac{z}{\sqrt{\lambda B_0^2 + i\nu_m\alpha}}\right)}, \tag{11}$$

while v_z denotes the longitudinal wave of usual sense slightly modified by the magneto-hydrodynamic waves. v_z is not important in the following discussions.

Let us next consider a transverse wave expressed by

$$u_y = a e^{i\alpha\left(t + \frac{z}{c}\right)} \tag{12}$$

which comes from the mantle with null incident angle to the surface of the core, c denoting the velocity of the wave. We denote the reflected wave which is also a transverse one by

$$u_y = d e^{i\alpha\left(t - \frac{z}{c}\right)}. \tag{13}$$

From the continuity of tangential displacement and stress at the boundary, we have

$$\left.\begin{aligned} a + d &= \frac{A\lambda B_0}{i\alpha\sqrt{\lambda B^2_0 + i\nu_m\alpha}}, \\ a - d &= \frac{A\lambda B_0\rho}{i\alpha c\rho_c}, \end{aligned}\right\} \tag{14}$$

from which we get

$$\frac{d}{a} = \frac{1-x}{1+x} \tag{15}$$

where
$$x = \frac{\rho}{\rho_c}\frac{\sqrt{\lambda B_0^2 + i\nu_m\alpha}}{c}, \tag{16}$$

and where ρ_c denotes the density in the core.

Taking $\rho_c = 9.4$, $\rho = 5.7$ and $c = 7.3$ *km/sec*, *mod.* $\frac{d}{a}$ can be calculated for various combinations of σ and B_0, where μ is assumed as unity. As the period of ScS waves is usually observed to be several seconds, $\alpha=\frac{2\pi}{5}$ is adopted here. *mod.* $\frac{d}{a}$ (the ratio of the amplitude of the reflected wave to that of incident one) thus calculated is shown in Fig. 1 in which we see that *mod.* $\frac{d}{a}$ is scarcely affected by B_0 except the case of very large intensity such as $10^5\, emu<B_0$. *mod.* $\frac{d}{a}$ becomes small when σ_c approaches $10^{-12}\, emu$ where the phase change becomes also remarkable. Taking into consideration the fact that we have no reliable evidence for $d<a$ and phase difference as shown, for example, by H. Honda[17] we could tentatively adopt $B_0<10^4\, emu$ and $\sigma_c>10^{-8}\, emu$ from the consideration stated above.

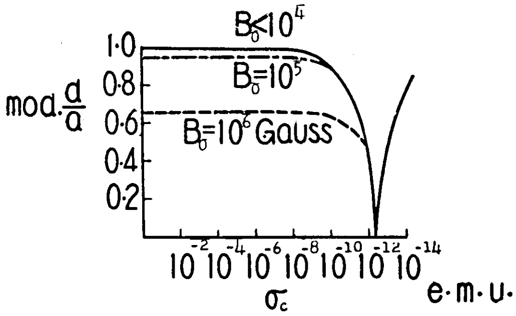

Fig. 1. The relation between the ratio of the amplitude of the reflected waves to that of the incident ones and the electrical conductivity in the core.

From (11), we can estimate the distance z_0 in which the amplitude of the magneto-hydrodynamic waves reduced to $1/e$. z_0 is tabulated in Table I in *cm* unit.

In a favorable case of distribution of magnetic lines of force and of high conductivity, we might observe the waves which come through the core. We tentatively assume here that the wave might be observed for $z_0>100\, km$ which corresponds to $\sigma_c>10^2$ for $B_0=10^2$, $\sigma_c>1$ for $B_0=10^3$, and $\sigma_c>10^{-3}$ for $B_0=10^4$ all in electromagnetic unit. However, we have

Table I

z_0 in *cm* unit.

$\sigma_c(emu)$ \ $B_0(emu)$	10	10^2	10^3	10^4
10^{-8}	3.6×10^3	3.6×10^3	3.6×10^3	3.8×10^3
10^{-6}	3.6×10^2	3.6×10^2	3.8×10^2	1.2×10^4
10^{-4}	3.6×10	3.8×10	1.2×10^3	1.2×10^6
10^{-2}	3.8	1.2×10^2	1.2×10^5	1.2×10^8
1	1.2×10	1.2×10^4	1.2×10^7	1.2×10^{10}
10^2	1.2×10^3	1.2×10^6	1.2×10^9	1.2×10^{12}

17) H. HONDA, *Geophys. Mag.*, **8** (1934), 161.

no evidence in seismology for such a wave. From this seismological evidence, we presume $\sigma_c<10^3$, $\sigma_c<1$, and $\sigma_c<10^{-3}$ respectively for $B_0=10^2$, $B_0=10^3$ and $B_0=10^4$. Since the magnetic field is not well known in the core, we cannot make any accurate estimate for σ_c, but if we take the view of Elsasser-Bullards' theory in which B_0 amounts to scores of *gauss*, we get $\sigma_c<10^3$ *emu*.

Combining the simple consideration on ScS waves with those of the damping of magneto-hydrodynamic waves in the core, we arrive at a conclusion $10^3>\sigma_c>10^{-8}$ in electromagnetic unit. Though the conclusion is rather incomplete, it is impossible to determine the possible range of σ_c accurately by the method studied above.

3. Some considerations on the shielding and cancelling effect of geomagnetic secular variation.

In his previous paper[18], the writer studied the electromagnetic shielding within the earth and its relation to the geomagnetic secular variation. Though only the case for the first degree component of spherical harmonic was discussed in that paper, it was made clear that the time-dependent part of geomagnetic secular variation due to electric currents flowing on the surface of the core would be fairly shielded by the mantle and cancelled by the electric currents induced in the conducting core.

In so far as we take a view that the geomagnetic secular variation is caused by the time-varying electric currents flowing near the surface of the core, the fact that we observe secular variation notwithstanding the above-mentioned shielding and cancelling effect would give some clue to the electrical conductivity in the core.

As has been pointed out by H. G. Macht[19] and as will be also shown in the following section, the second degree potential constituent would be more important for the actual secular variations than the first degree one. The writer will calculate here the shielding and cancelling effect for the second degree potential constituent together with the first degree one. As already shown in the writer's previous paper, the coefficient i_n of magnetic potential due to a time-dependent current-sheet on the surface of the core is given by

18) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **29** (1951), 263,
19) H. G. MACHT, Personal communication.

$$-(4\pi/a)K_n^m=q_1{}^{-n-1}\frac{2n+1}{n}z_1\left\{\frac{z_2}{2\nu}\Big(I_{\nu+1}(z_2)K_{\nu+1}(z_1)-K_{\nu+1}(z_2)I_{\nu+1}(z_1)\Big)\right.$$
$$\left.+\frac{F_{n-1}(q_2)}{F_n(q_2)}\Big(I_\nu(z_2)K_{\nu+1}(z_1)+K_\nu(z_2)I_{\nu+1}(z_1)\Big)\right\}i_n\,, \qquad (17)$$

the current-function being defined by

$$J_n^m=K_n^m S_n^m. \qquad (18)$$

z_1 and z_2 respectively denote $z_{\rho=q_1}$ and $z_{\rho=q_2}$, q_1 and q_2 mean the values of ρ at the upper and lower boundaries of the conducting mantle respectively, ρ denoting the ratio of the radial distance r to the radius of the earth a, z and ν are defined by

$$\rho^{1-l/2}=(l-2)z/(2\zeta),\quad \nu=(2n+1)/(l-2),\quad \zeta^2=4\pi a^2\sigma_0 p \qquad (19)$$

where the conductivity in the mantle is assumed to be expressed by $\sigma=\sigma_0\rho^{-l}$ as determined by the writer. The operator p should be replaced by $i\alpha$ in case of periodic variation having a period $2\pi/\alpha$.

If we neglect the shielding and cancelling effect wholly the coefficient $i_{n,0}$ of the magnetic potential at the earth's surface is given by

$$-(4\pi/a)K_n^m=q_2{}^{-n-1}\frac{2n+1}{n}i_{n,0}.$$

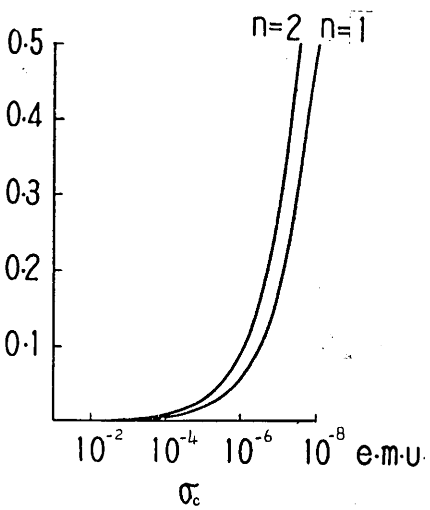

Fig. 2. The shielding and cancelling effect (*mod.* $i_2/i_{2,0}$ and *mod.* $i_1/i_{1,0}$) for various conductivity in the core.

By taking *mod.* $i_n/i_{n,0}$, we can estimate the shielding and cancelling effect as in the writer's previous paper. By taking into consideration

Table II

σ_c (*emu*)	*mod.* $i_1/i_{1,0}$	*mod.* $i_2/i_{2,0}$
10^{-8}	0.477	0.737
10^{-7}	0.166	0.265
10^{-6}	0.0536	0.0880
10^{-5}	0.0172	0.0287
10^{-4}	0.00543	0.00900
10^{-3}	0.00172	0.00287
10^{-2}	0.000543	0.000900

the fact that the secular variations of regional character have apparent periods or duration-time of the order of 100 *years*, *mod.* $i_2/i_{2,0}$ and *mod.*

$i_1/i_{1,0}$ are calculated for variation having 100-*year* period as shown in Table II and Fig. 2 in which σ_c is taken as abscissa. The functions K_ν, I_ν and F_n, which are essentially Bessel functions, are numerically calculated with the aid of ascending and descending power series.

As may be clearly seen in the figure, the shielding and cancelling effect become considerable as σ_c takes a high value. Hence the amplitude of the time-dependent electric currents, which correspond to the primary origin of the secular variation flowing near the surface of the core, would become enormously large in case of high conductivity.

But it seems unlikely that the current density is much larger than that required to produce the main dipole field. If we assume that the main dipole field is produced by the current flowing on the surface of the core, the current density amounts to 0.5 *emu/cm*. As for the secular variation, it amounts to as much as 100γ/*year* in South Africa. Adopting this value as a maximum of changing rate, the amplitude of the secular variation amounts to 1.6×10^{-2} *emu* for 100 *year* period variation. The amplitude of the electric currents on the surface of the core or K_n^m in (18) can be obtained by taking into account the shielding and cancelling effect. The current density for the second degree constituent thus calculated is tabulated in Table III.

Table III

σ_c	Amplitude of current density
10^{-8} *emu*	0.047 *emu*
10^{-7}	0.14
10^{-6}	0.41
10^{-5}	1.2
10^{-4}	4.0
10^{-3}	12
10^{-2}	40

Comparing the amplitude of the current density with that of the main magnetic field, we arrive at a conclusion $\sigma_c<10^{-5}$ *emu*. Though it is a matter of common sense to assume that the current density required to produce the secular variation should not be much larger than that required to produce the main field, we may tentatively presume that σ_c is not much larger than 10^{-5} *emu* from the geophysical consideration discussed in this section.

4. Mathematical studies on the magnetic dipole that corresponds to the regional secular variation.

F. J. Lowes and S. K. Runcorn[20] showed that the geomagnetic secular variation for the epoch 1922.5 can be expressed fairly well by

20) F. J. LOWES and S. L. RUNCORN, *Phil. Trans. Roy. Soc. London A*, **243** (1951). 525.

the field of 13 magnetic dipoles situated at depths of $(0.6\pm0.1)a$, the directions of these dipoles being all radial. Their conclusions seem to be interesting and suggestive together with similar conclusion reached by Elsasser[21] and A. G. McNish[22] concerning the non-dipole part of the main field. However, the said dipoles are under the influence of the shielding and cancelling effect in the earth, the effect being of some importance in estimating the strength and position of the primary origin of the dipole. From this viewpoint a mathematical study on the time-dependent magnetic dipole will be carried out here in detail.

Let us consider a magnetic dipole situated at the surface of the core, the direction of which is vertical to the surface and coincides with $\theta=0$ axis. If the distance between the dipole and the earth's centre is denoted by d, the magnetic potential due to this dipole can be written as

$$W_d=M\frac{r\cos\theta-d}{R^3}, \quad (R^2=r^2+d^2-2rd\cos\theta) \tag{21}$$

where M denotes the magnetic moment of the dipole. The right-hand-side of (21) is readily expanded into a series of zonal harmonics such as

$$W_d=\begin{cases} M\dfrac{1}{r}\sum\limits_{n=0}^{\infty} n\dfrac{d^{n-1}}{r^n}P_n(\cos\theta) & \text{for} \quad r>d, \\ -M\dfrac{1}{d^2}\sum\limits_{n=0}^{\infty}(n+1)\dfrac{r^n}{d^n}P_n(\cos\theta) & \text{for} \quad r>d, \end{cases} \tag{22}$$

whence we can replace the dipole by a magnetized shell or electric current sheet whose potential are expressed by (22).

Taking an earth-model, as shown in Fig. 3 in which the distribution of the electrical conductivity ($\sigma'=10^{-15}\,emu$, $\sigma_0=1.0\times10^{-12}\,emu$, $l=11$, $q_1=0.94$, $q_2=0.545$) is the same with the one adopted in the writer's preliminary study on the electromagnetic shielding in the earth, the typical term of the magnetic potential can be expressed as

$$W_n^m=ai_n(t)\rho^{-n-1}S_n^m, \tag{23}$$

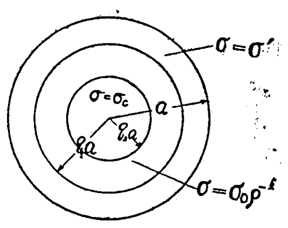

Fig. 3.

where the external origin part is not taken into account since the secular variation originates only inside the earth. The magnetic permeability is assumed as unity everywhere. S_n^m denotes

21) W. M. ELSASSER, *Phys. Rev.*, **60** (1941), 876.
22) A. G. MCNISH, *Trans. A.G.U. 21st Annual Meeting*, **II** (1940), 287.

spherical surface harmonic. The components of the magnetic field become

$$\left.\begin{aligned} H_r &= (n+1)\rho^{-n-2}\dot{i}_n S_n^m, \\ H_\theta &= -\rho^{-n-2}\dot{i}_n \partial S_n^m/\partial\theta, \\ H_\phi &= -\rho^{-n-2}\dot{i}_n \partial S_n^m/(\sin\theta\partial\phi). \end{aligned}\right\} \tag{24}$$

We have in the conducting medium vector potential $\overrightarrow{A}$ which satisfies the relation

$$\nabla^2\overrightarrow{A} = 4\pi\sigma(\rho)\partial\overrightarrow{A}/\partial t \qquad 25)$$

for slow variations. The typical term of the solutions of (25) becomes

$$\overrightarrow{A}_n^m = af_n(t, \rho)\,[r \operatorname{grad} S_n^m], \tag{26}$$

where f_n satisfies the differential equation

$$\partial(\rho^2\partial f_n/\partial\rho)/\partial\rho = \{n(n+1) + 4\pi a^2\sigma\rho^2\partial/\partial t\}f_n. \tag{27}$$

The components of the magnetic field are given as

$$\left.\begin{aligned} H_r &= -n(n+1)\rho^{-1}f_n S_n^m \\ H_\theta &= -\rho^{-1}\frac{\partial}{\partial\rho}(\rho f_n)\partial S_n^m/\partial\theta, \\ H_\phi &= -\rho^{-1}\frac{\partial}{\partial\rho}(\rho f_n)\partial S_n^m/(\sin\theta\partial\phi). \end{aligned}\right\} \tag{28}$$

The function f_n becomes respectively for the regions 2 and 3 as follows:

$$f_{n,2} = \rho^{-1/2}\{C_2K_\nu(z) + D_2I_\nu(z)\} \qquad \text{for region 2,} \tag{29}$$

where C_2 and D_2 are both functions of time, K_ν and I_ν denote modified Bessel functions, while ν and z are defined by (19), and

$$f_{n,3} = C_3\rho^n F_n(\rho) \qquad \text{for region 3,} \tag{30}$$

where C_3 also denotes a function of time. F_n was fully studied in the theory of electromagnetic induction within the earth as given in the well-known book "*Geomagnetism*"[23]. In the above treatment, $\partial/\partial t$ is replaced by p as is usual in operational calculus.

Now we get the next relation from the continuity at the boundary of the core or at $\rho = q_2$

$$\{\partial(\rho f_{n,2})/\partial\rho\}_{\rho=q_2} - \{\partial(\rho f_{n,3})/\partial\rho\}_{\rho=q_2} = Ma^{-3}q_2^{-2}(2n+1), \tag{31}$$

$$(f_{n,2})_{\rho=q_2} = (f_{n,3})_{\rho=q_2}. \tag{32}$$

23) S. CHAPMAN and J. BARTELS, *Geomagnetism* (1940), p. 738.

We have also

$$n(f_{n,2})_{\rho=q_1}=-i_n\,q_1^{-n-1} \tag{33}$$

$$\{\partial(\rho f_{n,2})/\partial\rho\}_{\rho=q_1}=i_n q_1^{-n-1} \tag{34}$$

from the boundary conditions at $\rho=q_1$.

Then, solving the simultaneous equations (31), (32), (33), and (34), we get, with the aid of recurrence formula, the relation

$$Ma^{-3}q_2^{-2}=q_1^{-n-1}z_1\left[\frac{z_2}{2\nu}\Big(I_{\nu+1}(z_2)K_{\nu+1}(z_1)-K_{\nu+1}(z_2)I_{\nu+1}(z_1)\Big)\right.$$
$$\left.+\frac{F_{n-1}(q_2)}{F_n(q_2)}\Big(I_\nu(z_2)\,K_{\nu+1}(z_1)+K_\nu(z_2)\,I_{\nu+1}(z_1)\Big)\right]i_n \tag{35}$$

where z_1 and z_2 denote respectively $z_{\rho=q_1}$ and $z_{\rho=q_2}$ as before. In the case of periodic variation having a period $2\pi/\alpha$, the operator p is replaced by $i\alpha$ in the above expressions. When $|z|\ll 1$, that is, for variations having period longer than about 10 *years*, we obtain i_n approximately as follows;

$$i_n=\frac{Ma^{-3}\,q_2^n}{\dfrac{F_{n-1}(q_2)}{F_n(q_2)}+\dfrac{i}{\nu(\nu+1)}\left\{\left(\dfrac{x_2}{2}\right)^2+\left(\dfrac{q_2}{q_1}\right)^{2n+1}\left(\dfrac{F_{n-1}(q_2)}{F_n(q_2)}-1\right)\left(\dfrac{x_1}{2}\right)^2\right\}}, \quad (z=x\sqrt{i}) \tag{36}$$

Table IV

(Unit: $Ma^{-3}q_2^n$)

n	$i_{n,0}$	*mod.* i_n	Phase dif.	n	$i_{n,0}$	*mod.* i_n	Phase dif.
1	0.162	0.0^2867	44.°5	16	0.0^3288	0.0^3141	35.°2
2	0.176	0.0155	43.7	17	0.0^3168	0.0^4865	34.7
3	0.144	0.0176	42.9	18	0.0^4965	0.0^4518	34.3
4	0.105	0.0162	42.2	19	0.0^4553	0.0^4308	33.8
5	0.0715	0.0133	41.8	20	0.0^4319	0.0^4185	33.4
6	0.0468	0.0102	41.1	21	0.0^4182	0.0^4109	33.0
7	0.0298	0.0^2742	40.3	22	0.0^4103	0.0^5638	32.6
8	0.0186	0.0^2516	39.8	23	0.0^5588	0.0^5377	32.2
9	0.0113	0.0^2347	39.2	24	0.0^5334	0.0^5220	31.7
10	0.0^2687	0.0^2230	38.5	25	0.0^5189	0.0^5128	31.3
11	0.0^2413	0.0^2150	38.0	26	0.0^5108	0.0^6752	30.9
12	0.0^2245	0.0^3957	37.4	27	0.0^6608	0.0^6437	30.5
13	0.0^2144	0.0^3598	36.9	28	0.0^6344	0.0^6254	30.2
14	0.0^3850	0.0^3374	36.4	29	0.0^6194	0.0^6146	29.9
15	0.0^3495	0.0^3230	35.9	30	0.0^6109	0.0^7837	29.6

while the coefficient $i_{n,0}$ for the earth, in which we neglect all the shielding and cancelling effect, is readily obtained from (22) or from the limiting case of (36) for $\sigma_c \to 0$. It becomes

$$i_{n,0}=Ma^{-3}q_2^n n. \tag{37}$$

By adopting $\sigma_c=10^{-6}\,emu$, the writer calculated here $i_{n,0}$ and i_n for 100-*year* period variation. The amplitude and phase difference of i_n are shown together with $i_{n,0}$ in Table IV. Owing to the slowness of convergence of the series, high degree terms such as $n=30$ are needed in order to calculate the magnetic field at the earth's surface. The behaviors of the magnetic field thus calculated are shown in Figs. 4 and 5

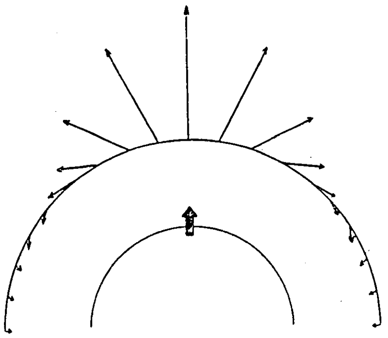

Fig. 4. The distribution of the magnetic force in a meridional section of the non-conducting earth. The primary magnetic dipole is shown with a thick arrow.

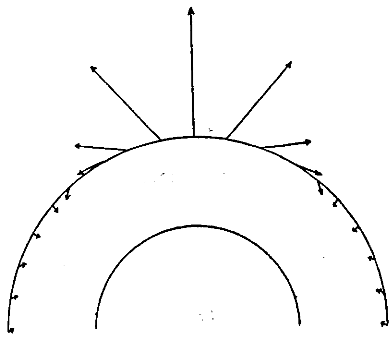

Fig. 5. The magnetic force for conducting earth. The magnitude is multiplied 5.86 times.

respectively for non-conducting and conducting earth where the phase differences for different harmonics are neglected because of the small differences for main harmonics. In the figures, we must pay attention to the fact that the field for conducting earth is multiplied 5.86 times in order to make the field at the pole agree with that of the non-conducting earth.

Looking at Fig. 5, we find that the distribution of the magnetic field on the earth's surface is fairly modified in case of conducting earth though the general character of it remains unchanged. If we approximate this distribution with a dipole, the position of it will become slightly shallower than that of non-conducting earth. Taking into account the accuracy of determination for actual secular variation, however, we may obtain a dipole whose position falls within a range as determined by Lowes and Runcorn if the secular variation is produced by a time-dependent dipole situated near the surface of the core. Hence, the assumption that σ_c takes a value of order of 10^{-6} *emu* seems to be consistent with the analysis of the actual secular variation.

An important fact, which is deduced from the calculus for $\sigma_c=10^{-6}$ *emu*, is that the changing rate of the magnetic moment of the primary dipole becomes several times larger than the one directly obtained from the analysis of the actual secular variation. As was pointed out by Lowes and Runcorn, a surface field of $100\gamma/year$ implies a dipole at depth $0.6a$ of strength 2.78×10^{22} *emu/year*. By taking into account the shielding and cancelling, the strength of the primary dipole will amount to 1.5×10^{23} *emu/year*. Even when we assume that this value corresponds to the rapidest part of periodic change having 100-*year* period, the amplitude of the periodic change in the strength is estimated as much as 2.4×10^{24} *emu*. When we compare this with the moment of the main magnetic dipole which amounts to 8.1×10^{25} *emu*, the values for the secular variation seems acceptable. As already treated in the last section, if σ_c is larger than 10^{-5} *emu*, the original strength of the dipole for secular variation will become much larger than that for the main field, such a great perturbation being implausible in the core.

5. Summary and conclusion.

In order to investigate the electrical conductivity in the earth's core, the writer studied the possibility of the magneto-hydrodynamic waves in the core and the shielding and cancelling effect in the earth together with the behavior of time-dependent dipole situated at the surface of the core. From the observational results in seismology that ScS waves are not likely affected by the magneto-hydrodynamic waves and we find no waves which come through the core as the magneto-hydrodynamic waves, it may be roughly concluded that $10^3>\sigma_c>10^{-8}$ *emu* while the intensity of magnetic field should be less than 10^4 *emu*.

After taking into account the shielding and cancelling effect for geomagnetic secular variation which is produced by the time-dependent electric currents flowing near the surface of the core, $\sigma_c>10^{-5}$ *emu* is tentatively concluded by adopting the criterion that the electric current density for the primary origin of secular variation will not be much larger than the one for the main dipole field, the study being done for a variation having a 100-*year* period. A similar conclusion is obtained from the study concerning the modification of the magnetic field of a time-dependent dipole situated at the surface of the core.

Although the estimate of σ_c mentioned in this paper is crude, it is of interest that the possible range of σ_c, say $10^{-5} > \sigma_c > 10^{-8}$ *emu*, fairly agrees with those obtained before from a quite different standpoint such as physics of metallic iron at high temperature and pressure. The writer is of opinion that Elsasser-Bullards' consideration that σ_c amounts to the order of 10^{-6} *emu* is fairly right judging from both physical and geophysical viewpoints.

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17. 地球核の電氣傳導度

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從來地球核の電氣傳導度は物性論的考察より 10^{-5}～10^{-6} *emu* 程度と考えられ，Elsasser-Bullard の地磁氣理論にもこの値が採用されている．筆者は核内に於ける Magneto-hydrodynamic wave (電磁流體波) の存在の可能性を吟味し，地磁氣永年變化が核表面に於ける變化電流によると考える時にその電流密度が地球主磁場のそれを越えないという常識的見解より，核の電氣傳導度は 10^{-5} *emu* より小さく 10^{-8} *emu* より大きいことを推論した．核表面に磁氣双極子を考えた時のシールディングによる磁場の分布形式の變化もあわせてしらべてある．
