23. *Elastic and Viscous Properties of Volcanic Rocks at Elevated Temperature. Part 1.*

By Shūzō SAKUMA,

Earthquake Research Institute.

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1. Introduction.

In recent years, remarkable developments have been achieved in the field of elasticity and plasticity of rocks[1]. The most eminent and leading among them were the works done by D. Griggs and F. Birch. Especially, their studies on the physical behaviours of the materials which compose the earth's crust were full of instructions for solving difficult geological and geophysical problems. Their researches, however, were concentrated mainly on crystalline rocks, such as granite and marble which are the commonly accepted components of the crust in its deeper part. The earth's surface is composed not only of them but also of glassy rocks such as basalt and obsidian. On the other hand, the recent progress in the physics of volcanoes, partly advanced in Japan, calls for a more detailed knowledge on the physical properties of volcanic rocks and the parent magma. In this connection, the experimental studies by K. Kani, K. Iida and M. Ide and others are also to be remembered[2]. The present writer also planned to investigate the elasticity and viscosity of glassy rocks at elevated temperature and to throw some light on the difficult problem concerning the relation of the structure and history of rocks to their mechanical properties.

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[1] F. ADAMS and E. COKER, *Am. J. Sci.*, **29** (1910), 489.
L. ADAMS and R. GIBSON, *Proc. Nat. Akad. Sci.*, **15** (1929), 713.
F. BIRCH and D. BANCROFT, *J. Geol.*, **46** (1938), 59; **48** (1940), 752; *Am. J. Sci.*, **240** (1942), 457.
F. BIRCH, *Bull. Geol. Soc. Amer.*, **54** (1943), 263.
D. GRIGGS and others, *J. Geol.*, **44** (1936), 541; **47** (1939), 225; *Bull. Geol. Soc. Amer.*, **62** (1951) 853, 1385.

[2] K. KANI, *Proc. Imp. Acad. Japan* **10** (1934), 29 and 79; **11** (1935), 334, 383.
K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939), 59, 79.
M. IDE, *J. Geol.*, **15** (1937), 689.

2. Method and Instrument.

The ordinary bending or sagging method of measuring elasticity and slow viscous flow was adopted. Instead of measuring the subsidence of the centre of the beam, the change in curvature of the beam caused by the applied load was observed by the double reflection through the two rectangular prisms which were set face to face on the ends of the beam of the specimen. The knife-edges for supporting the specimen and applying the load, the two prisms and the framework for suspending the load were all made of fused silica, because the fused silica not only remains rigid and free from oxidation at high temperatures but is also easy of shaping. (Fig. 1 & 10).

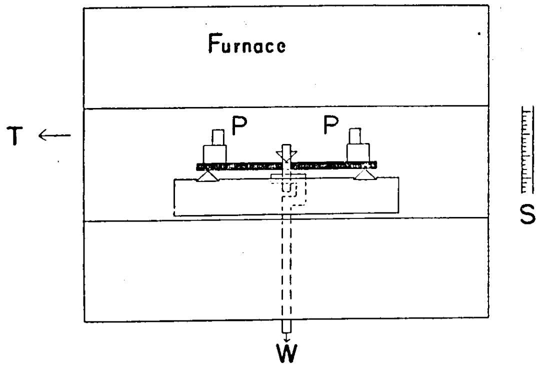

Fig. 1. Apparatus, P: prisms, T: telescope, S: scale, W: weight.

The two prisms, the specimen, the knife-edges and the suspender were mounted on a block of hard porcelain, which was reported to remain rigid up to 1300°C. The whole contrivance was inserted horizontally into a doubly wound Nichrome electric furnace. The incident and reflected light was allowed to enter and go out through the holes on the lids at both sides of the furnace. The load to bend the specimen with was suspended in the air by a suspender through a tiny hole in the furnace, right beneath the centre of the specimen. Though such holes disturbed the uniformity of temperature in both radial and axial direction in the furnace, the actual difference in temperature was found to be less than ±5°C within the space which was to be occupied by the specimen itself.

The temperature of the specimen was measured with the aid of a Pt-PtRh thermo-couple which was inserted in the furnace close to the specimen. The temperature was raised and lowered at the rate of about 200°C in one hour and the temperature was kept constant from ten minutes before and during a series of measurement. The load was applied and removed, and thus the difference in the image of the scale was read with the telescope. The flow of the specimen under load at constant temperatures was also observed.

In computing the material constants from the observed results, the writer adopted a formula which is availed of sometimes in the field of rheology. The time-derivatives of stress-components could be reckoned at zero in our experiment. Accordingly, the stress-strain equations became as follows, where z-axis is taken upward and x-axis perpendicular to the axis of the bending moment.

$$P_{yy}=P_{zz}=P_{zx}=P_{xy}=P_{yz}=0$$

$$\frac{P_{xx}}{\tau}=E(\dot{e}_{xx}+\tau'\ddot{e}_{xx})\,.$$

Put $P_{xx}=E\varphi z\,,$

$$e_{xx}=\frac{\varphi z}{\tau}t-\frac{\varphi z\tau'}{\tau}+C+C'e^{-\frac{t}{\tau'}}\,.$$

If perfectly elastic, ($\tau\rightarrow\infty$ and $\tau'\rightarrow 0$)

$$e_{xx}=\varphi z.$$

$$\therefore\quad e_{xx}=\varphi z\left\{\frac{t}{\tau}+\left(1-\frac{\tau'}{\tau}\right)\right\}+C'e^{-\frac{t}{\tau'}}\,.$$

Put $C'=\varphi' z$, we obtain

$$\begin{cases} e_{xx}=z\phi(t) \\ \phi(t)=\varphi\left\{\frac{t}{\tau}+\left(1-\frac{\tau'}{\tau}\right)\right\}+\varphi' e^{\frac{t}{\tau'}}\,. \end{cases}$$

The quantity of subsidence ζ is given by ordinary integrations,

$$\zeta=K\left\{\left(1-\frac{\tau'}{\tau}\right)+\frac{t}{\tau}\right\}+Ge^{-\frac{t}{\tau'}}\,,$$

where G and K are constants, and

$$\begin{cases} K=\frac{5wl^4g}{384EI}+\frac{Wl^3g}{48EI}\,, \\ I=\frac{1}{12}bd^3\,. \end{cases}$$

where

b and d : breadth and thickness of specimen,
w : weight of the specimen per unit length,
W : applied load,
l : distance between the supporting knife-edges.

ζ is connected to the observed scale-reading X through the equation

$$\zeta=\frac{l}{6(2D+a+2A)}X,$$

where

D : distance between the scale and a prism,
a : distance between the prisms,
A : length of the sides of prisms.

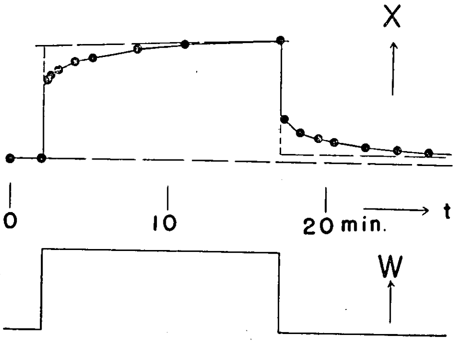

Fig. 2. An example of observations under constant load.

The material constants used here are those which were defined by M. Toda[3]. E is Young's modulus, and τ, τ' are the relaxation times for shear and voluminal change respectively. The coefficient of viscosity (η) can also be derived from the following formula

$$\eta=\frac{\tau E}{3}.$$

Fig. 2 illustrates an actually observed curve.

At first, a copper-plate was tested in the furnace up to 900°C and the result was satisfactory. The method proved to be also adequate for the purpose of measuring elasticity and viscosity of metals and alloys near their melting points, where the vibration method fails because of high attenuation.

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[3] M. TODA, 'Structure of liquids' (1947), Tokyo. (in Japanese)

3. Specimen.

The present specimens were cut out of a piece from the 1950 lava-flow of Volcano Oo-sima and were ground smoothly to the size of ca. $1.0 \times 0.3 \times 8.0\ \mathrm{cm}^3$ (Fig. 10–A). The chemical and petrographical studies were already carried out by H. Tsuya and R. Morimoto[4] of our Institute. According to their studies, the 1950 lava of Oo-sima is identified as an augite-hypersthene-bytownite-basalt (SiO_2 52.25 in wt %). The ground-mass, which occupies about 90% in volume, is composed of bytownite, pigeonite, iron ores, cristobalite and glass.

4. Results of Experiment.

Young's moduli of specimens, even when taken from one piece of lava, did not give the same value, but differed in accordance with their porosity (Fig. 3).

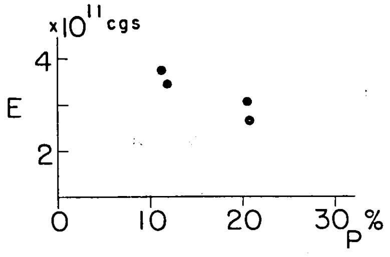

Fig. 3. Effect of porosity (P) on Young's modulus at room temperature.

It should be stated here that the present specimens followed Hooke's law fairly well even at elevated temperatures.

Young's modulus of the rock increased surprisingly with the rise in temperature until it began to show a decrease at nearly 700°C–800°C, where the viscous flow under the applied load also began to appear. The specimen behaved as a visco-elastic body at higher temperature than that up to 1100°C. We can determine the quantities of E, τ and τ' from the observed deformation and the rate of their change with time.

This tendency of elasticity to vary with the rise in temperature is similar to the results concerning amorphous silica[5]. However, this

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[4] H. Tsuya and R. Morimoto, *Bull. Earthq. Res. Inst.*, **29** (1951), 563.

[5] R. Sosman, 'The Properties of Silica' (1927), New York.
K. Iida, *Bull. Earthq. Res. Inst.*, **13** (1935), 665.

tendency differs from that of common materials, such as metals and most of natural rocks⁶⁾.

At the cooling stage, more extraordinary and characteristic phenomena took place. The values of Young's modulus were estimated at higher values than those in the heating process. The modulus attained to its maximum at about 650°C and then decreased.

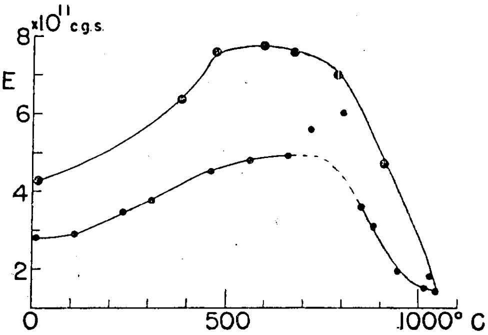

Fig. 4. Variation of Young's modulus with temperature.
Specimen No. 4.
larger circle: cooling stage.

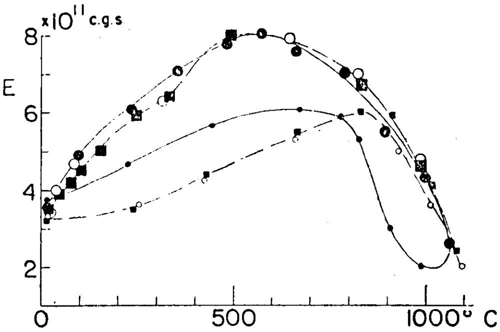

Fig. 5. Variation of Young's modulus with temperature.
Specimen No. 1.
larger marks: cooling stage,
closed circle : 1st run,
square : 2nd run,
open circle : 3rd run.

If the same specimen was heated and cooled again, the modulus showed less gradient of increase in the heating process, and the elasticity was observed to decrease at a higher temperature than before. In the

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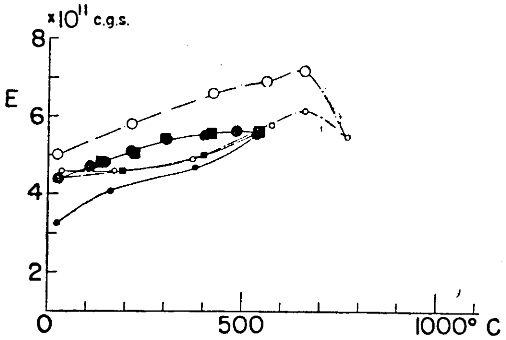
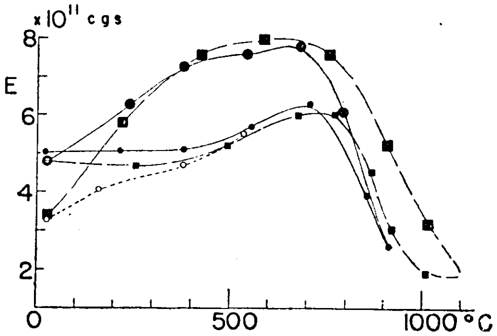
6) F. BIRCH, *loc. cit.*, p. 269.

cooling process, the modulus changed with temperature along the same curve of the first run of cooling. The *E-T* relation at the third run was almost the same as that of the above described second run. (Fig. 5)



6-a. 1st~3rd run.
larger marks : cooling stage,
closed circle : 1st run up to 540°C,
square : 2nd run 〃 ,
open circle : 3rd run up to 770°C,



6-b. 4th~5th run.
larger marks : cooling stage,
closed circle : 4th run up to 910°C,
square : 5th run up to 1100°C,
open circle : 1st run (for reference).

Fig. 6a, b. Variation of Young's modulus with temperature, for successively elevated highest temeratnres (T_M). Specimen No. 2.

In addition, the 'hysteresis' phenomena are concerned with the highest temperature (T_M) which the specimen has experienced in the laboratory. When we stopped heating and began cooling at a definite temperature, the above-described transition was repeated. However, when the turning temperature (T_M) was once raised, the matter became somewhat different. A series of experiments was carried out for a single specimen in this connection, as will be seen in Fig. 6. It is also to be remarked that, though the heating curves differ in general between the first and the second run, the values of E always agree at the highest temperature (T_M).

The difference due to the highest temperature (T_M) was also reflected in the value of Young's modulus at the room temperature at the end of cooling stage. (Fig. 7)

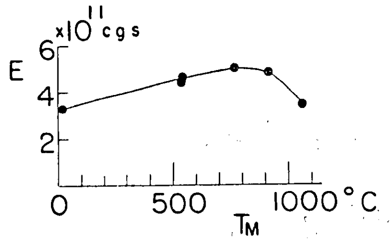


Fig. 7. Young's modulus at the room temperature after cooling from the highest temperature (T_M). (Specimen No. 2)

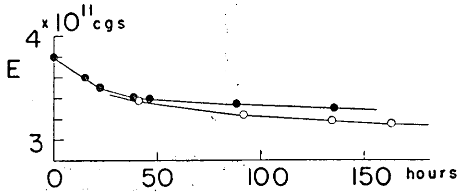
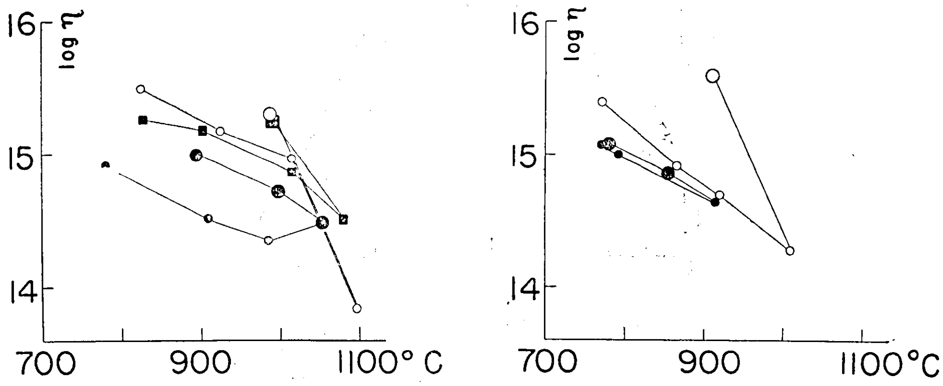


Fig. 8. Variation of Young's modulus at room temperature with time elapsed after cooling from above 1000°C.
closed circle : specimen No. 1,
open circle : specimen No. 2.



9-a. Specimen No. 1.
closed circle : 1st run,
square : 2nd run,
open circle : 3rd run.

9-b. Specimen No. 2.
closed circle : 3rd and 4th runs,
open circle : 5th run.

Fig. 9. Variation of viscosity coefficient (η) with temperature.
larger marks : cooling stage.

The value of E at the end of cooling process, however, varies slightly in course of time as will be seen in Fig. 8. This and other facts mentioned above lead us to assume the existence of some kind of change of internal structure of the specimen of rather slow velocity, such as crystallization or decomposition.

The relaxation time, or viscosity, was determined from the terminal velocity of deformation under a constant load at a constant temperature. The accuracy of this quantity was not so satisfactory because of the high viscosity and slow velocity of deformation in the present range of temperature. The value of viscosity of the order of 10^{14} poise at 1050°C far exceeds the viscosity cofficients which were obtained both by T. Minakami[7] in the field in its natural molten state and by K. Kani[8] experimentally at higher temperatures. However, by the experiences of those chemists who have remolten and analysed the lava of Oo-sima, and the results of experiments on electrical conductivity of the lava by T. Nagata[9], the writer is convinced that a discontinuity of viscosity does exist between 1100°C and 1150°C, at least in the remelting treatment. One of the causes of this high viscosity can be ascrided to the possible escape of volatile constituents and to the possible changes in crystalline structure. In fact, even in our experiments, the values of relaxation time (τ) increased not only in the cooling stages when compared with those in the preceding heating stages, but also increased with the repetition of experiments.

Further investigations on the mode of causation of elasticity, viscosity and their variations are now under way. This report is a mere approach to the problem and accordingly the writer refrains here from proposing any definite interpretation of the observed phenomena.

5. Acknowledgement.

The present writer wishes to express thanks to Dr. H. Tsuya, the director of our Institute, for the consent given to the writer regarding the present research. To Drs. T. Minakami, T. Nagata and R. Morimoto his cordial thanks are also due for their kind advices given in the course of the experiments.

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[7] T. MINAKAMI, *Bull. Earthq. Res. Inst.*, **29** (1951), 437.

[8] K. KANI, *loc. cit.*, p. 269.

[9] T. NAGATA, *Bull. Earthq. Res. Inst.*, **15** (1937), 663.

23. 高溫に於ける火山岩の粘性及び彈性 第1報

地震研究所 佐久間修三

火山物理學の最近の進歩に伴つて，火山岩の粘性，彈性に關する實驗的知識の必要が痛感されるので，高溫域に於けるこれらの性質を，まづ靜的に，撓みの方法によつて測定した．

1950年に噴出した大島熔岩のヤング率は，溫度上昇に伴つて增加し，700°C～800°Cに至つて急に減少し始める．粘性流動が觀測されるのも700°C以上の高溫域に於てであり，粘性は1050°Cで約10^{14} poiseであつた．ヤング率は冷却の過程では加熱の過程に於けるよりも一般に大きい値を示し，又この傾向は，試料が嘗て實驗室内で處理された最高の溫度にも關係するやうである．

これらの，異常な性質の原因については，火山岩中の揮發性成分の逸出，或は微細な結晶の析出などの可能性が考えられるが，尙今後の檢討を要する．

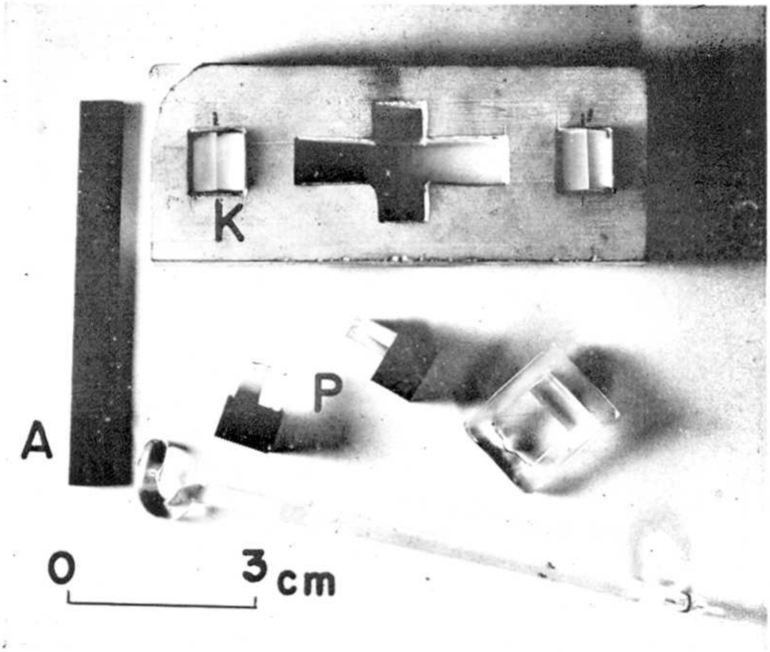

Fig. 10. A specimen and a part of the instrument.
A: specimen, K: knife-edge, P: prism.