2. *Electrical Conductivity and Temperature in the Earth.*

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Introduction.

In his former paper, the writer[1] attempted to determine the temperature-distribution within the earth from the distribution of the electrical conductivity and the theory of ionic crystals. The conductivity-distribution was obtained by the writer[2] from the standpoint of electromagnetic induction theory which has been developed by S. Chapman[3),4], A. T. Price[4),5], B. N. Lahiri, K. Terada[6] and the writer. Since the main part of the electric conduction in rocks at high temperature is due to the motion of ions as experimentally proved by T. Nagata[7] and H. P. Coster[8], the influence of pressure on the conductivity can be approximately estimated from the viewpoint of theoretical physics of solids. The writer's preliminary work was made under the assumptions that the compressibility does not depend on pressure and that the temperature in the earth increases proportionally to depth as was critically reviewed by B. Gutenberg[9]. These assumptions will be eliminated in the present paper. The pressure-effect on the activation energy of the motion of ions will be also examined more accurately. In short, the purpose of this paper is to revise the treatment in the previous study as accurately as possible.

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1) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **27** (1949), 17.

2) T. RIKITAKE, *Trans. Oslo Meeting, I. U. G. G. A. T. M. E.*, (1950), 435. *Bull. Earthq. Res. Inst.*, **28** (1950), 45, **29** (1951) 219 and 263.

3) S. CHAPMAN, *Phil. Trans. Roy. Soc. London A*, **218** (1919), 1.

4) S. CHAPMAN and A. T. PRICE, *Phil. Trans. Roy. Soc. London A*, **229** (1930), 427.

5) B. N. LAHIRI and A. T. PRICE, *Phil. Trans. Roy. Soc. London A*, **237** (1939), 509.

6) K. TERADA, *Geophys. Mag.*, **13** (1939), 63 and **16** (1948), 5.

7) T. NAGATA, *Bull. Earthq. Res. Inst.*, **15** (1937), 663.

8) H. P. COSTER, *M. N. R. A. S. Geophys. Suppl.*, **5** (1949), 193.

9) B. GUTENBERG, *Appl. Mec. Rev.*, **4** (1951), Rev. 3427.

In Part I of this paper, the effect of pressure on the ionic conductivity is estimated in detail. Then, after taking into account the pressure-effect, the temperature-distribution in the earth's mantle that is compatible with the conductivity-distribution is obtained while the crystal structure is assumed to be the same throughout the mantle. The temperature-distribution thus determined has a discontinuous increase at the depth of 400 *km* corresponding to the discontinuity of the electrical conductivity. Since we have no evidence for a particular source of heat at this depth, the discontinuity might be due to the fact that we assume the same crystal structure in both up- and downside of this level. From this viewpoint, the temperature below this level is recalculated in Part II under the assumption that the substance takes another phase below the depth of 400 *km*, the activation energy for the motion of ions being taken so as to avoid the discontinuity of temperature. As believed by a number of geophysicists, the substance might have a different phase below the depth of about 400 *km* which is called the "20° discontinuity" by seismologists. Hence it will be logical to assume another constant for the ionic mobility below this level though it will be impossible to prove experimentally.

Part I. The mantle having only one phase.

1. Ionic conductivity in the earth's mantle.

In the earth's mantle, which is believed to be composed largely of olivine and pyroxene, the motion of ions would be the main cause of electric conduction as proved in case of igneous rocks at high temperature. As was also shown experimentally, the conductivity of rocks at low temperature is electronic as is the case in most semi-conductors. In order to study the earth's mantle, then, we will consider only the ionic conduction.

As has been suggested first by J. Frenkel[10], the ionic conductivity in crystals is closely connected with the lattice defect. Since a very large field is required to remove an ion from its normal site in crystals, we suspect that there already exists lattice defect in crystals. In that case, ions may move by jumping into vacant sites or may move through interstitial positions. Since it is beyond the scope of this paper to mention the theory of lattice defect in detail, only the results of the

10) J. FRENKEL, *Z. Physik*, **35** (1926), 652.

theory will be dealt with. The electrical conductivity σ is expressed by[11)]

$$\sigma=\sigma_0 e^{-\epsilon_0/kT}\,, \qquad \text{(1)}$$

where

$$\sigma_0=(Ne^2\nu r^2/kT)e^{-\frac{\alpha r_0}{k}\left(\frac{d\epsilon}{dV}\right)_0}, \qquad \text{(2)}$$

$$\epsilon=\frac{1}{2}W+U\,, \qquad \text{(3)}$$

and where T, W and U denote respectively absolute temperature, energy required to make a lattice defect, and activation energy. k, N, e, ν, r, α and V denote respectively Boltzamann's constant, number of ion in unit volume, charge of electron, vibration frequency of ion at lattice point, interionic distance, thermal expansion coefficient, and volume per pair of ions. The suffix 0 means the quantity at zero temperature.

Strictly speaking, different conductivities of the type shown in (1) are responsible for respective ions and the total conductivity should be given by summing up all these conductivities. But it is assumed here that only a type of conductivity is effective in the earth's interior as approximately proved by the experiments of rocks at high temperature.

2. The effect of pressure on ϵ_0 and σ_0.

In order to estimate the conductivity at any pressure, $(\partial\epsilon_0/\partial p)_T$ and $(\partial \log \sigma_0/\partial p)_T$ are calculated in the following.

The energy required to make a lattice defect in ionic crystals is given by

$$W=U^{+}+U^{-}-E\,, \qquad \text{(4)}$$

in which U^+, U^- and E denote respectively the energy required to take off a positive ion, that required to take off a negative one and that required to make a pair of ions. According to the classical theory of ionic crystals, E is given by the sum of electrostatic interaction energy, repulsive interaction energy, van der Waals interaction energy and zero point energy. On neglecting the last two, we have

$$E=Ar^{-1}-Br^{-n}\,, \qquad \text{(5)}$$

in which A and B are both constants determined later, the first and

11) N. F. MOTT and R. W. GURNEY, *Electronic processes in ionic crystals*. Oxford, (1940).

second terms corresponding to electrostatic and repulsive interaction respectively.

To get the activation energy U, we should take into account the polarization energy due to the electric field in ion vacancies. Hence we have for positive ion

$$U^{+}=E-\frac{e^{2}}{2q^{+}r}(1-1/\kappa), \quad \text{......} (6)$$

and for negative ion

$$U^{-}=E-\frac{e^{2}}{2q^{-}r}(1-1/\kappa), \quad \text{......} (7)$$

in which κ denotes the dielectric constant, while $q^{+}r$ or $q^{-}r$ denote the effective radii of the respective vacancies. According to a more rigorous estimate, it is known that $q^{+}\simeq 0.6$ and $q^{-}\simeq 0.9$.

Since positive ions seem likely to contribute in the main to the conductivity of rocks as experimentally shown by Coster[8], it would be allowed to take the next expression as ϵ;

$$\epsilon=\frac{3}{2}E-\left(\frac{3}{q^{+}}+\frac{1}{q^{-}}\right)\frac{e^{2}}{4r}(1-1/\kappa). \quad \text{......} (8)$$

Now we are in a position to determine A and B in (5) from the condition of equilibrium. At zero temperature, there exist thermodynamical relations

$$\left(\frac{\partial E}{\partial V}\right)_{V=V_0}=-P, \quad \left(\frac{\partial^{2}E}{\partial V^{2}}\right)_{V=V_0}=\frac{1}{\chi_0 V_0}, \quad \text{......} (9)$$

where P and χ denote respectively pressure and compressibility.

On introducing the relation

$$V=c^{3}r^{3} \quad \text{......} (10)$$

where c denotes a constant peculiar to the crystal structure, we get

$$A=\frac{9-3(n+3)\chi_0 P}{(n-1)\chi_0}cV_0^{1/3}, \quad B=\frac{9-12\chi_0 P}{n(n-1)\chi_0}cV_0^{\frac{n+3}{3}}. \quad \text{......} (11)$$

Putting (11) in (5), the internal energy of the crystal becomes as follows;

$$E=\frac{3V_0}{(n-1)\chi_0}\left\{(3-\overline{n+3}\chi_0 P)(V_0/P)^{1/3}-\frac{3-4\chi_0 P}{n}(V_0/V)^{n/3}\right\}. \quad \text{...} (12)$$

At zero temperature or $V=V_0$, we have

$$E_0 = \frac{9V_0}{n\chi_0}\left(1 - \frac{\chi_0 P}{3}\right), \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

and, with the aid of the relation $-\left(\frac{d^3E}{dV^3}\right)_{V=V_0} = \frac{1}{\chi_0 V_0^2}\left\{1 + \frac{d}{dP}\left(\frac{1}{\chi_0}\right)\right\}$, the differential equation

$$\frac{1}{\chi_0^2}\frac{d}{dP}\left(\frac{1}{\chi_0}\right) = K - M\chi_0 P, \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where

$$K = \frac{(n+3)^2 - 16}{3(n-1)}, \qquad M = \frac{4}{9}(n+3).$$

Meanwhile we find $\chi_0 P < 0.1$ at depths shallower than about 1000 *km* in the earth's mantle. Hence the second terms in the righthand-side of (13) and (14) can be neglected, the pressure-effect on the compressibility being then shown by

$$\frac{1}{\chi_0} = \frac{1}{\chi_{00}} + KP, \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

in which χ_{00} denotes the compressibility at zero temperature and pressure.

In the next place, we find

$$\frac{d\epsilon_0}{dP} = -\chi_0 V_0 \frac{d\epsilon_0}{dV_0}$$

or

$$\epsilon_0 = \epsilon_{00} e^{-\int_0^P \frac{\chi_0 \nu_0}{\epsilon_0}\frac{d\epsilon_0}{dV_0} dP} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The quantity $\frac{V_0}{\epsilon_0}\frac{d\epsilon_0}{Vd_0}$ or $\frac{d\log\epsilon_0}{d\log V_0}$ is supposed to be -2 in the preliminary study. In this article, however, it will be accurately obtained through (8) and (13), while χ_0 is also calculated by (15). The pressure-distribution obtained by K. E. Bullen[12] is adopted. As to the value of n which amounts to 6 for light material such as lithium fluoride and to 12 for crystals with only very heavy ions such as caesium iodide, W. H. Ramsey[13] proved that it takes a value between 6 and 9 in the earth's mantle from the studies on the relation between compressibility and pressure as also expected by the fact that the heaviest element which is abundant in the earth is iron. The writer adopted $n=8$

12) K. E. BULLEN, *Bull. Seism. Soc. Amer.*, **30** (1940), 235 and **32** (1942), 19.
13) W. H. RAMSEY, *M. N. R. A. S. Geophys. Suppl.*, **6** (1950), 42.

from this view, the influence of slight difference of n on K in (15) being fairly small.

ϵ_{00} is taken to be $2.3\,eV$ as obtained from the experiments of various rocks such as basalt, gabbro, andesite, granite and peridotite. χ_{00} is assumed to be $0.90\times10^{-12}\,cgs$ which corresponds to the reduced value to zero temperature and pressure of the substance in the upper part of the earth's mantle. The change in the volume is estimated from the relation

$$V_0 = V_{00}(\chi_0/\chi_{00})^{1/K}, \qquad \text{(17)}$$

which is readily obtained from the definition of compressibility. Taking into consideration the fact that the density amounts to $3.3\,gr./cm^3$, V_{00} is estimated at $7.7\times10^{-23}\,cm^3$ for olivine whose molecular weight amounts to about 150. We further assume $r_{00}=3\times10^{-8}\,cm$ and $n=5$ as obtained for most ionic crystals. Looking at the expression (16), we find that ϵ_0 is most seriously affected by the change in χ_0, while the change of the polarization energy or that in the second term of the righthand-side of (8), which includes rather ambiguous quantities as stated above, will have scarcely any great influence on ϵ_0, the effect of the polarization energy being only a few percent as revealed in the numerical calculation.

On executing the integral in (16) numerically, the value thus obtained are shown in Table I for various depths.

Next, we must investigate the effect of pressure on σ_0. On logarithmically differentiating, we get

$$\frac{d\log\sigma_0}{dP} = \frac{d\log N}{dP} + \frac{d\log\nu}{dP} + \frac{2}{3}\frac{d\log V_0}{dP} - \frac{d}{dP}\left\{\frac{\alpha V_0}{k}\left(\frac{d\epsilon}{dV}\right)_{V=V_0}\right\} \qquad (18)$$

Meanwhile, the frequency of small vibration of ion at its normal site may be given by

$$\nu = \frac{1}{r}\sqrt{\frac{E}{m}}, \qquad \text{(19)}$$

where m denotes mass of the ion, the relation being derived from the equation of small motion of ions under the action of periodic field.

Taking into account further the relation

$$\frac{\alpha V_0}{\chi_0} = -3k\frac{d\log\nu}{d\log V_0}, \qquad \text{(20)}$$

which is obtained from the equilibrium condition of the Einstein-model

Table I.

Depth (*km*)	ϵ_0 (*eV*)	σ_0 T (*emu. deg.*)
0	2.3	0.10
33	2.3	0.10
100	2.5	0.11
200	2.8	0.12
300	3.1	0.13
400	3.3	0.13
500	3.7	0.14
600	4.0	0.15
700	4.2	0.16
800	4.7	0.17
900	5.0	0.18
1000	5.3	0.19
1200	6.1	0.21
1400	6.7	0.23
1600	7.4	0.25

of solids, the righthand-side of (18) can be estimated. Thus we have

$$\frac{d\log\sigma_0}{dP}=\left(\frac{K}{2}+\frac{2}{3}P\right)\chi_0, \qquad \cdots\cdots(21)$$

and then

$$\sigma_0=\sigma_{00}e^{\int_0^P\left(\frac{d\log\sigma_0}{dP}\right)dP}. \qquad \cdots\cdots(22)$$

According to the experiments of igneous rocks, $\sigma_{00}kT$ is estimated at 10^{-4} *emu* at about 1000° K. Adopting then $k\sigma_{00}=0.1$, $\sigma_0 T$ is calculated for various depths as also shown in Table I.

3. The temperature-distribution that is compatible with the conductivity-distribution.

As studied in section 3, ϵ_0 and σ_0 for any pressure can be estimated from their values at zero pressure, so we may readily determine the temperature in the earth's mantle that is compatible with the conductivity-distribution obtained from the electromagnetic induction theory by the writer himself. The change in the conductivity with the increase of depth are reproduced in Fig. 1. The temperature fit for the conductivity at various depths are determined as shown in Table II and Fig. 2.

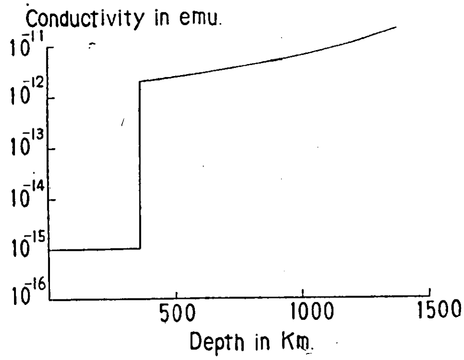

Fig. 1. The distribution of electrical conductivity in the earth as obtained from the electromagnetic induction theory.

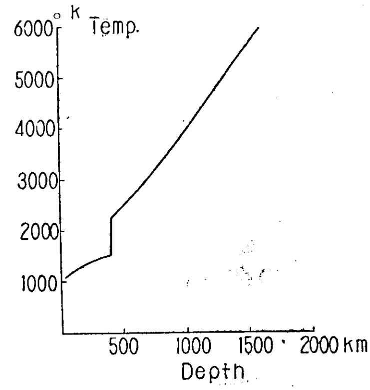

Fig. 2. The temperature in the earth's mantle having only one phase.

Table II.

Depth (km)	T ($°K$)
33	1080
100	1180
200	1370
300	1460
400	1540
400	2260
600	2750
800	3340
1000	3940
1200	4620
1400	5320
1600	6020

As also studied by the writer, the conductivity deeper than about 1500 km is ambiguous because the induced electric currents do not penetrate into such a great depth. Hence the temperature deeper than this level is also indeterminable.

Corresponding to the discontinuous increase in the conductivity at a depth of 400 km, there occurs a jump of the temperature. But it is doubtful that such a discontinuity of temperature really exists because we have no particular evidence for the source of heat at this depth. The discontinuity should be due to the fact that we assume only one phase of substance throughout the mantle.

Part II. The Mantle having another phase below the depth of 400 km.

1. The temperature-distribution which has no discontinuity and is also compatible with the conductivity-distribution.

The behavior of the bulk modulus and density seems anomalous from just below the depth of 400 km to 1000 km. Some geophysicists believed that this anomaly is due to a transition to a crystal structure which is more stable at high pressure. As was shown in Part I of this paper, there occurs a discontinuous increase in temperature at the depth of 400 *km* in case of the mantle having only one phase, the existence of such a discontinuity being implausible from physical consideration. Hence it will be natural to take into account the change in physical conditions in the deeper part of the mantle. For the sake of simplicity, the writer assumes here that the bulk modulus and density at zero temperature and pressure become respectively 2.1×10^{12} *cgs* and $4.2\, gr./cm^3$ in the region deeper than 400 *km* as obtained from extrapolation of Bullen's distribution, while the same values are adopted in the upper part of the mantle. The pressure-effect is estimated in a similar way with Part I. Though we have no experimental value for ϵ_0 and σ_0 in this region, ϵ_0 should be chosen so as to avoid the discontinuity of temperature at the depth of 400 *km*, while $k\sigma_0$ at zero temperature and pressure is assumed to amount to 0.1 as before. It it easily found out that the temperature becomes continuous by assuming $\epsilon_{00} = 1.8\, eV$ below the depth of 400 *km*. Adopting these values, the temperature-distribution in the deeper part of the mantle is determined as done in Part I. The results are shown in Table III and Fig. 3 together with the results of the upper region as determined in Part I.

Table III.

Depth (*km*)	T (°*K*)
33	1080
100	1180
200	1370
300	1460
400	1540
600	1730
800	1950
1000	2240
1200	2530
1400	2850
1600	3170

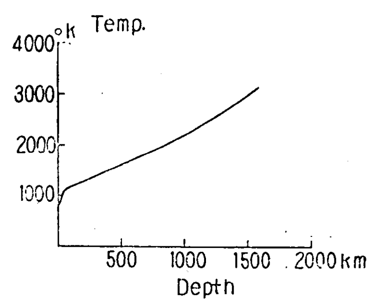

Fig. 3. The temperature in the earth's mantle having a different phase below the depth of 400 *km*.

2. Some considerations on the changes in density and compressibility in the earth's mantle.

With the aid of the temperature-distribution thus obtained, we shall examine here the distribution of density and compressibility in the earth's mantle.

The effect of pressure and temperature can be written as follows;

$$\frac{1}{\chi(T,P)}=\frac{1}{\chi_{00}}+P\left[\frac{\partial}{\partial P}\left(\frac{1}{\chi}\right)\right]_{00}+T\left[\frac{\partial}{\partial T}\left(\frac{1}{\chi}\right)\right]_{00}+\frac{1}{2}P^2\left[\frac{\partial^2}{\partial P^2}\left(\frac{1}{\chi}\right)\right]_{00}+\ldots\ldots,$$

$$V(T,P)=V_{00}+P\left(\frac{\partial V}{\partial P}\right)_{00}+T\left(\frac{\partial V}{\partial T}\right)_{00}+\frac{1}{2}P^2\left(\frac{\partial^2 V}{\partial P^2}\right)_{00}+\ldots\ldots.$$

Neglecting higher order terms we have

$$\frac{1}{x}=\frac{1}{\chi_{00}}+K\left(P-\frac{\alpha_{00}}{\chi_{00}}T\right)+\ldots\ldots, \qquad (23)$$

$$\frac{\rho_{00}}{\rho}=\frac{V}{V_{00}}=1-\chi_{00}P+\alpha_{00}T+\ldots\ldots. \qquad (24)$$

α_{00}, which is estimated from (20), amounts to 1.7×10^{-5} and 1.2×10^{-5} per *degree*, respectively for upper and lower side of the mantle, the values being reasonable judging from the experimental results of rocks. The results calculated from (23) and (24) are graphically shown in Figs. 4 and 5, where the density and bulk modulus due to Bullen's investigation are also shown. Though some discrepancies are found between

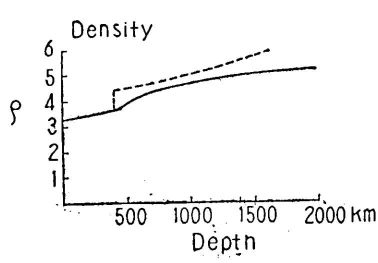

Fig. 4. Bullen's distribution of density (full line) and the calculated one (broken line).

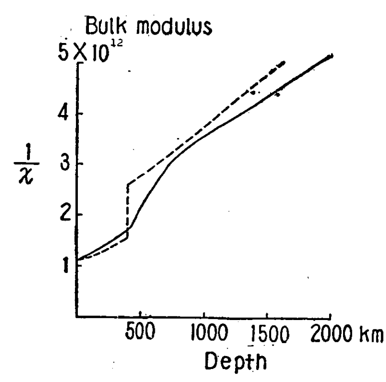

Fig. 5. Bullen's distribution of bulk modulus (full line) and the calculated one (broken line).

Bullen's and the calculated distribution, it might be said that the general aspects are roughly agreeing. Hence the temperature-distribution obtained in this paper seems to be approximately consistent with the distribution of density and bulk modulus in the earth's mantle.

Summary and Conclusion.

The theory of ionic crystals is applied to study the electrical conductivity in the earth's mantle in a manner improving that of the writer's previous study. The constants included are estimated as accurately as possible. The changes in physical conditions in up- and down-sides of the "20° discontinuity" is also taken into acconnt.

After taking into account the pressure-effect on the conductivity, it is found that the temperature likely amounts to 1000° K just below the earth's crust and then increases almost proportionally to depth, the rate of increase being estimated at 1.4° K per *km*. The distribution is roughly consistent with the distribution of density and bulk modulus in the earth's mantle which is determined by Bullen.

The earth's internal temperature has been hitherto studied from standpoints quite different from the one considered here, such as from the studies in the cooling of the earth and the pressure-effect on melting point. In order to compare the present result with those obtained before, the distribution obtained by H. Jeffreys[14], E. C. Bullard[15] and B. Gutenberg[16] are shown in Fig. 6 together with the writers'. However, all these distribution are by no means conclusive. Much more experiments for various quantities, especially under high pressure and temperature, are necessary before a more concrete knowledge on the earth's internal temperature is to be obtained. The writer is grateful to Dr. T. Nagata for his helpful advice.

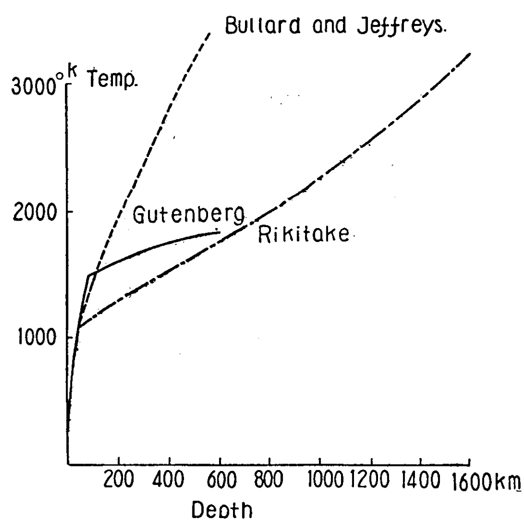

Fig. 6. The temperature-distribution within the earth obtained by Bullard, Jeffreys, Gutenberg and the writer.

14) H. JEFFREYS, *The Earth*. Cambridge (1924), p. 79.
15) E. C. BULLARD, *M. N. R. A. S. Geophys. Suppl.*, **4** (1939), 534.
16) B. GUTENBERG, *Physics of the Earth, VII. Internal Constitution of the Earth*. 1st ed., p. 153.

2. 地球内の電氣傳導度と溫度

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筆者はかつて地球内の電氣傳導度の分布より，イオン結晶の理論にもとずいて，地球内の溫度の分布を求めることを試みた．その際壓縮率が一定であること，溫度が深さとともに直線的に變化すること等の假定のもとに計算を行つた．本論文に於いては，それ等の假定を捨て，出來得る限りの正確な議論を行う．高溫に於ける岩石の實驗より，電氣傳導度は主としてイオンにより

$$\sigma = Ae^{-B/T}$$

(T は絕對溫度) が成立つことがしられているが，この式中の常數 A および B の壓力による變化は，イオン結晶中の格子缺陷理論を應用することにより，かなり正確に見つもることが出來る．したがつて K.E. Bullen によつて求められている地球内の壓力分布を使用し，A および B の壓力零の値より出發して，地球磁氣學的に求めた σ に對して T を決定出來る．地球の mantle が一定の結晶構造をもつているとすると岩石の實驗値より推定した溫度は $400\,km$ の深さに於て不連續をもつことになり，物理的に不合理であるから，この深さの下では物質は異つた相をとるとし，溫度の不連續がないという條件のもとに活性化エネルギーを見つもる．かくして決定した溫度分布は地殼の下で $1000°K$, $1600\,km$ の深さで $3000°K$ という値となり，その間は $1.4°K/km$ の增溫率でほとんど直線的に增加する．

このようにして求めた溫度の影響を壓力とともに考慮すると，密度および壓縮率の分布を求めることが出來るが，Bullen の得た分布と大體に於いて調和することがわかつた．