# 4. *Relation between the Nature of Surface Layer and the Amplitudes of Earthquake Motions.*

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1. The present paper is the continuation of my research work[1] on the effect of solid viscosity of surface layer on the earthquake motion. In the previous paper, I investigated mathematically the case of a layer, and carried out the numerical calculations for some special cases. It was found that the diagramatical features of the results of the previous investigation closely resemble the features of the spectrogramic diagrams of earthquake motions of the ground which were obtained by the accelerometer at several places in Tokyo and Yokohama.

It is a well-known fact that the earthquake damage done to Japanese style wooden buildings at the past earthquakes were closely related with the thickness of alluvium. Recently, it seems that the problem of the relation between the earthquake damage to Japanese style wooden buildings and the thickness, the velocity of seismic waves, the damping coefficient and etc. of superficial soil layer recieved the attention of many seismologists.

In connection with these problems, in the first part of this paper, the oscillations of doubly stratified visco-elastic layers excited by seismic waves will be dealt with mathematically. Next, the numerical calculation of the problem of a stratified visco-elastic layer excited by seismic waves will be carried out again more elaborately because of its increasing importance in connection with the problem of earthquake-proof constructions.

2. In the present chapter, we will investigate the case in which primary distortional waves propagated vertically upwards through the doubly stratified visco-elastic layers residing on the semi-infinite visco-elastic body.

Let, $\rho$, $\mu$, $v$, $\xi$ and $H$ be the density, elastic constant, velocity, coefficient of solid viscosity and thickness of layer, and $u$ the displace-

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1) K. KANAI, "The Effect of Solid Viscosity of Surface Layer on the Earthquake Movements", *Bull. Earthq. Res. Inst.*, **28** (1950), 31.

ment. Suffix 1, 2, 3 represent the first layer, the second layer and the bottom medium respectively. Then the equation of the motion of each medium may be written as follows:

$$\rho_n \frac{\partial^2 u_n}{\partial t^2} = \left(\mu_n + \xi_n \frac{\partial}{\partial t}\right) \frac{\partial^2 u_n}{\partial z^2} \cdot [n=1, 2, 3] \quad (1)$$

If the incident waves in the lowest medium be of the type

$$u_0 = a\, e^{i(pt+f_3 z)}, \quad \ldots\ldots\ldots\ldots\ldots (2)$$

the resulting displacements of the lowest medium, the second layer and the first layer are expressed by

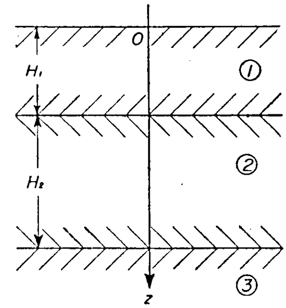

Fig. 1.

$$\left.\begin{aligned} u_3 &= a\, e^{i(pt+f_3 z)} + A\, e^{i(pt-f_3 z)}, \\ u_2 &= B\, e^{i(pt+f_2 z)} + C\, e^{i(pt-f_2 z)}, \\ u_1 &= D\, e^{i(pt+f_1 z)} + E\, e^{i(pt-f_1 z)}, \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (3)$$

where $A$, $B$, $C$, $D$, $E$ are arbitrary constants determined by boundary conditions and $f_1$, $f_2$, $f_3$ have the meaning such that

$$f_1^2 = \frac{\rho_1 p^2}{\mu_1 + i\xi_1 p}, \quad f_2^2 = \frac{\rho_2 p^2}{\mu_2 + i\xi_2 p}, \quad f_3^2 = \frac{\rho_3 p^2}{\mu_3 + i\xi_3 p}, \quad \ldots (4)$$

in which $2\pi/p$ is the period of waves.

The boundary conditions at the plane, $z=0$, $z=H_1$, $z=H_1+H_2$ are as follows:

$$z=0; \qquad \left(\mu_1 + \xi_1 \frac{\partial}{\partial t}\right) \frac{\partial u_1}{\partial z} = 0, \quad \ldots\ldots\ldots\ldots (5)$$

$$z=H_1; \qquad u_1 = u_2, \quad \ldots\ldots\ldots\ldots (6)$$

$$\left(\mu_1 + \xi_1 \frac{\partial}{\partial t}\right) \frac{\partial u_1}{\partial z} = \left(\mu_2 + \xi_2 \frac{\partial}{\partial t}\right) \frac{\partial u_2}{\partial z}, \quad \ldots\ldots\ldots (7)$$

$$z=H_1+H_2; \qquad u_2 = u_3, \quad \ldots\ldots\ldots\ldots (8)$$

$$\left(\mu_2 + \xi_2 \frac{\partial}{\partial t}\right) \frac{\partial u_2}{\partial z} = \left(\mu_3 + \xi_3 \frac{\partial}{\partial t}\right) \frac{\partial u_3}{\partial z}. \quad \ldots\ldots\ldots (9)$$

Introducing (3) in (5)-(9) by means of (4), we find finally

$$\frac{|u_{1_{z=0}}|}{2|u_0|} = \frac{2}{\sqrt{\Phi_1^2 + \Phi_2^2}}, \quad \ldots\ldots\ldots\ldots (10)$$

where

$$
\begin{aligned}
\left.\begin{matrix}\Phi_1\\ \Phi_2\end{matrix}\right\} &= \{1+\alpha\cos(N_2-N_1)\}\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}(P_2+P_1)\left.\begin{matrix}\cosh\\ \sinh\end{matrix}\right\}(Q_2+Q_1)\\
&\pm\}\alpha\sin(N_2-N_1)\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}(P_2+P_1)\left.\begin{matrix}\sinh\\ \cosh\end{matrix}\right\}(Q_2+Q_1)\\
&+\{\beta\cos(N_3-N_1)+\gamma\cos(N_3-N_2)\}\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}(P_2+P_1)\left.\begin{matrix}\sinh\\ \cosh\end{matrix}\right\}(Q_2+Q_1)\\
&\pm\{\beta\sin(N_3-N_1)+\gamma\sin(N_3-N_2)\}\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}(P_2+P_1)\left.\begin{matrix}\cosh\\ \sinh\end{matrix}\right\}(Q_2+Q_1)\\
&+\{1-\alpha\cos(N_2-N_1)\}\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}(P_2-P_1)\left.\begin{matrix}\cosh\\ \sinh\end{matrix}\right\}(Q_2-Q_1)\\
&\mp\alpha\sin(N_2-N_1)\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}(P_2-P_1)\left.\begin{matrix}\sinh\\ \cosh\end{matrix}\right\}(Q_2-Q_1)\\
&-\{\beta\cos(N_3-N_1)-\gamma\cos(N_3-N_2)\}\left.\begin{matrix}\cos\\ \sin\end{matrix}\right\}(P_2-P_1)\left.\begin{matrix}\sinh\\ \cosh\end{matrix}\right\}(Q_2-Q_1)\\
&\mp\}\{\beta\sin(N_3-N_1)-\gamma\sin(N_3-N_2)\}\left.\begin{matrix}\sin\\ \cos\end{matrix}\right\}(P_2-P_1)\left.\begin{matrix}\sinh\\ \cosh\end{matrix}\right\}(Q_2-Q_1), \qquad (11)
\end{aligned}
$$

$$
\left.\begin{aligned}
&\alpha=\frac{\rho_1 v_1 M_1}{\rho_2 v_2 M_2}, \quad \beta=\frac{\rho_1 v_1 M_1}{\rho_3 v_3 M_3}, \quad \gamma=\frac{\rho_2 v_2 M_2}{\rho_3 v_3 M_3},\\
&M_n=\left\{1+\left(\frac{\xi_n p}{\mu_n}\right)^2\right\}^{1/4}, \quad N_n=\frac{1}{2}\tan^{-1}\frac{\xi_n p}{\mu_n}, \quad [n=1,2,3]\\
&P_n=\frac{pH_n}{v_n}\cdot\frac{\cos N_n}{M_n}, \quad Q_n=\frac{pH_n}{v_n}\cdot\frac{\sin N_n}{M_n}. \quad [n=1,2]
\end{aligned}\right\}\ldots\ldots(12)
$$

The numerical calculations are now being worked out.

In the case of dilatational waves, it is necessary to replace $\mu$ by $\lambda+2\mu$ and $\xi$ by $\eta+2\xi$.

3. In this chapter, we will examine the special case of the previous chapter in which $H_1=0$, that is, $P_1=Q_1=0$, in (10)-(12). It means the case of a stratified visco-elastic layer. Then from (10)-(12) we find

$$
\frac{|u_{2_{Z=0}}|}{2|u_0|}=\frac{1}{\sqrt{\Phi_1'^2+\Phi_2'^2}}, \quad\ldots\ldots\ldots\ldots\ldots\ldots(10')
$$

$$
\left.\begin{aligned}
\Phi_1' &= \cos P_2\cosh Q_2+\gamma\,\{\cos(N_3-N_2)\cos P_2\sinh Q_2\\
&\qquad\qquad +\sin(N_3-N_2)\sin P_2\cosh Q_2\},\\
\Phi_2' &= \sin P_2\sinh Q_2+\gamma\,\{\cos(N_3-N_2)\sin P_2\cosh Q_2\\
&\qquad\qquad -\sin(N_3-N_2)\cos P_2\sinh Q_2\},
\end{aligned}\right\}\ldots\ldots(11')
$$

$$\left.\begin{aligned} \gamma = \frac{\rho_2 v_2 M_2}{\rho_3 v_3 M_3}, \quad N_n = \frac{1}{2}\tan^{-1}\frac{\xi_n p}{\mu_n} \quad [n=2,\,3], \\ P_2 = \frac{pH_2}{v_2}\frac{\cos N_2}{M_2}, \quad Q_2 = \frac{pH_2}{v_2}\frac{\sin N_2}{M_2}\,. \end{aligned}\right\} \quad \cdots\cdots\cdots(12')$$

If $\xi_3 \to 0$, that is to say, $N_3 \to 0$ in equation (11′), these are coincident with the result obtained previously. The form of equation (11′) is somewhat simplified for carrying out numerical calculations easily compared with the previous one[2].

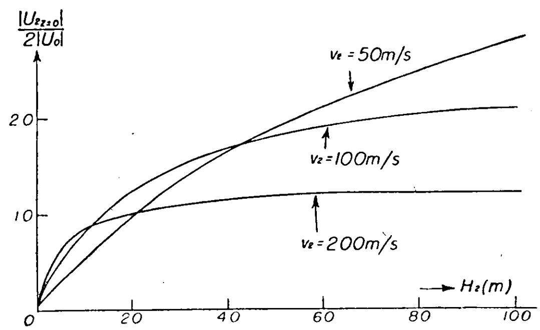


Fig. 2. The case of $\xi_2=10^6$ C.G.S., $\rho_2=1.5$, $\rho_3=2.25$, $v_3=1.7$ km/sec, $p=2\pi/T=2\pi/(4H_2/v_2)$.

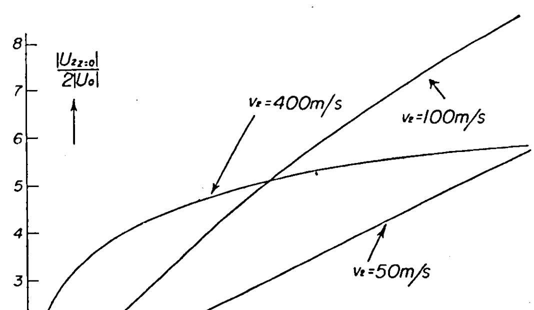


Fig. 3. The case of $\xi_2=10^7$ C.G.S., $\rho_2=1.5$, $\rho_3=2.25$, $v_3=1.7$ km/sec, $p=2\pi/T=2\pi/(4H_2/v_2)$.

Using (10′)-(12′), we calculated the ratio of the displacements at the free surface of a layer to twice the displacements of incident waves for six cases, namely $v_2=50$ m/sec, 100 m/sec, 200 m/sec when $\xi_2=10^6$

2) *loc. cit.*, 1), equation (14).

C.G.S. and $v_2=50$ m/sec, 100 m/sec, 400 m/sec when $\xi_2=10^7$ C.G.S., besides under the conditions of $\xi_3 \to 0$, $\rho_2=1.5$, $\rho_3=2.25$, $v_3=1.7$ km/sec and $p=2\pi/T_0=2\pi/(4H_2/v_2)$ for different values of the thickness of layer. Here, twice the displacements of incident waves means the displacements at the free surface when bed rocks are disclosed, and $T_0(=4H_2/v_2)$ is the period of synchronous vibration of a layer. The results are shown in Figs. 2 and 3.

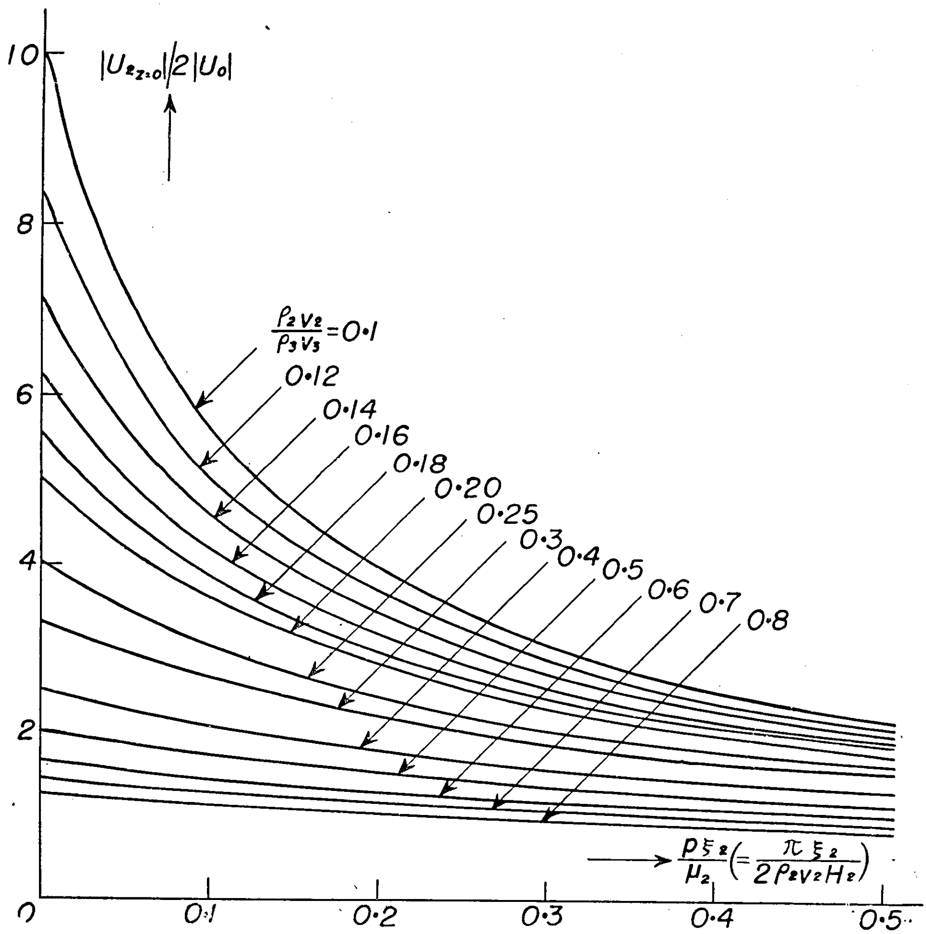


Fig. 4. Relation among the amplitude at the surface, impedance ratio of two medium, coefficient of viscosity and thickness of surface layer.

From these figures, it will be seen that, when the thickness of weak layer is fairly large, as the velocity of weak layer decreases, the amplitudes of the earthquake motions at free surface increases. While, when the thickness of weak layer is small, this relation is not so simple. In all cases, as the thickness of weak layer increases, the amplitudes mentioned above increases.

The diagrammatical feature of the case of small velocity of layer and large solid viscosity as when $\xi_2=10^7$ and $v_2=50$ m/s, 100 m/s resemble the relation[3] between the damage ratio of Japanese-style wooden buildings in the former Tokyo city at the time of the Kwanto earthquake of 1923 and the thickness of alluvium. On the other hand, the curve of the case of large velocity of layer at large viscosity and of small viscosity resemble the relation between the damage ratio of Japanese-style wooden buildings in Yokohama city at the time of the Kwanto earthquake of 1923 and the thickness of alluvium[4].

Table I.

| $\frac{\rho_2 v_2}{\rho_3 v_3}$ \ $\frac{\pi\xi_2}{2\rho_2 v_2 H_2}$ | 0 | 0.01 | 0.05 | 0.1 | 0.3 | 0.5 |
|---|---|---|---|---|---|---|
| 0.1 | 10.00 | 9.27 | 7.18 | 5.60 | 3.03 | 2.14 |
| 0.12 | 8.34 | 7.82 | 6.28 | 5.04 | 2.85 | 2.05 |
| 0.14 | 7.15 | 6.76 | 5.58 | 4.57 | 2.70 | 1.96 |
| 0.16 | 6.25 | 5.96 | 5.01 | 4.19 | 2.56 | 1.89 |
| 0.18 | 5.55 | — | 4.55 | 3.86 | 2.43 | 1.82 |
| 0.20 | 5.00 | — | 4.18 | 3.58 | 2.31 | 1.75 |
| 0.25 | 4.00 | — | 3.46 | 3.04 | 2.07 | 1.61 |
| 0.30 | 3.33 | — | 2.95 | 2.64 | 1.87 | 1.48 |
| 0.40 | 2.50 | — | 2.28 | 2.09 | 1.57 | 1.28 |
| 0.50 | 2.00 | — | 1.85 | 1.72 | 1.35 | 1.12 |
| 0.60 | 1.67 | — | 1.56 | 1.47 | 1.18 | 1.00 |
| 0.70 | 1.43 | — | 1.35 | 1.28 | 1.05 | 0.89 |
| 0.80 | 1.25 | — | 1.19 | 1.14 | 0.95 | 0.82 |

3) H. KAWASUMI, *Journal Archtect. Inst. Japan*, Vol. **66**, No. 773 (1951), 12, (in Japanese).

4) S. OMOTE, "Earthquake Damage in Yokohama City Due to the Great Kwanto Earthquake of September 1, 1923", *Bull Earthq. Res. Inst.*, **27** (1949), 57.

Since it seems that very few observations have been made in regard to the velocity of transmission of distortional waves through the ground and the coefficient of solid viscosity, I do not doubt that this result will be confirmed more closely as soon as such observations are made.

Finally, systematical relations among the ratio of the amplitudes of earthquake motion at the free surface of a weak layer and those of bed rocks disclosed, coefficient of solid viscosity, velocities and densities of weak layer and bed rocks, and the thickness of weak layer, were obtained. The results are shown in Fig. 4 and Table I. This chart will be of use in estimating the practical degree of ground against earthquake, when the requisite constants are examined by such means as seismic prospecting.

In conclusion, the author wishes to express his thanks to Miss S. Yoshizawa for her help in preparing this paper.

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## 4. 地表層の性質と地震動振幅との關係

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舊式日本家屋の震害と冲積層の厚さとの間に密接な關係があることは,數多くの地震について指摘されて來たところであるが，近年になつて，ごく地表近くの軟弱層の厚さとか，その中での地震波の速度，減衰係數などが大分問題にされ出して來た．

本論文の初めの部分では，これらの問題を究明する意味で，地表近くに 2 つの層があり，各層ともに粘性の影響を入れた場合についての彈性波の問題の解式を導いた．

今回は，その特別な場合の地表層が 1 つある場合についてのみ數値計算を行つた．

數値計算例を見ると，關東地震の際の舊東京市における木造家屋の被害と冲積層の厚さとの關係，即ち，冲積層の厚さが 30 m 以上になると被害が急激に增す傾向に似た曲線がある．又，同じ地震の横濱市の被害のように冲積層の厚さが 40 m 位を越すと餘り被害が增さない傾向に似た曲線も見られる．

冲積層及び基盤の常數のはつきりわかつたものが少いので，これらの關係について，これ以上の吟味をすることは現在のところできない．

最後に，地盤に耐震上の良否の階級をつける 1 つの指數とする意味で，地震地下探査法などで得られる地盤の諸常數を使つて,軟弱層の表面での振幅と基盤が露出したところでの振幅との比を見出す圖表を作つておいた．