8. *Dynamical Measurements of the Elastic Constants of Rocks subjected to Initial Stresses.*
1. *Measurement of Young's Modulus under Uniaxial Compression.*

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1. Introduction.

The experimental studies concerning the elastic and plastic properties of rocks have recently made remarkable developments. Among these studies, measurements of cubic compressibility, Young's modulus or rigidity under elevated temperature and hydrostatic pressure have extended our knowledge on the behaviour of rocks in the earth's interior. But if the earth's crust is subjected, in addition to hydrostatic pressure, to such stresses as simple tension, compression or shearing stress, the elastic properties of the crust may differ from those with no such additional stress. This difference in the elastic constants and density of the crust will in turn produce a change in the velocity of seismic waves. In effect, some Japanese observations of the earth tides[1] or of the velocity of seismic waves[2] seem to show the change in elasticity of the earth's crust with time. They seem also to have some relations with the occurrence of earthquakes.

The author intends to study experimentally (1) the variation of the elastic constants of rocks due to the change in type or amount of the initial stress, and (2) the relations of these elastic behaviours of rocks to their fine structures. For geophysical applications, rock samples must be subjected to a differential force under the confining hydrostatic pressure. As the first step of the study, however, igneous rock bars were simply kept compressed parallel to the axis, and the flexural vibrations of the bars were observed in this condition. Young's modulus was calculated from the resonance frequency of the flexural vibrations. Relations among the rate of change in Young's modulus, the amount

1) NISHIMURA, E., *Trans. Amer. Geophys. Union*, v. **31**, no. 3, 1950.
2) HAYAKAWA, M., *Rep. Geol. Surv., Japan*, special number, 1950.

of stress and the fine structures of rocks are also discussed in this paper.

2 Method of Experiment.

A rock specimen, cut into a rectangular form, was compressed between the upper and lower shafts of a testing machine through a pair of knife-edge so as to admit pure flexural vibrations of the specimen. If the ends of specimen are cramped or simply supported, the mode of the actual vibration becomes sometimes complicated owing to the mechanical imperfectness of the end conditions. The dimensions and densities of the rock specimen were as follows;

Specimen	Locality	Dimension (cm)	Density
Augite-basalt	Ajiro, Izu	1.75×1.90×24.55	3.00
Biotite-granite	Inada	1.95×2.45×19.93	2.69
Augite-hypersthene-andesite	Suwa	2.00×2.50×19.96	2.66
Pitchstone	Wadatoge	2.00×2.50×19.98	2.36

The method adopted here is to calculate Young's modulus from the measured natural frequency of the flexural vibration of the specimen. Therefore, it was necessary that the shape of the specimen should be accurately known, otherwise the value of Young's modulus calculated from the resonance frequency would be incorrect. The driving force was applied at the middle of the specimen by means of a light aluminium rod cemented to the voice coil of a dynamic speaker and contacted with the specimen. The voice coil was drived by a variable frequency vacuum tube oscillator. Both of the mass and electric reactance of the driving system were so small that they did not affect the vibration frequency of the specimen. Thus the specimen was forced into the flexural vibration of variable frequency. At the opposite side of the driving system, a rochelle salt pick-up of twister bimorph type touched the specimen at the middle. (cf. Fig. 4). When the natural frequency of the specimen coincides with the driving frequency, the resonance is observed by means of a pick-up and an amplifier. As the natural frequency of the pick-up is 1,2000 cps when cramped at the middle, and that of the specimen several thousands cps, we could easily distinguish the resonance frequency of the specimen from that of the pick-up.

Thus at each stage of the increasing compression, the respective resonance frequency was measured.

3. Method of Calculation of Young's Modulus.

From the natural frequency of the specimen, we must calculate Young's modulus or the velocity of longitudinal wave as the function of the applied stress. As the mass and stiffness of the testing machine are very large compared with the vibrating specimen, the natural frequency of the specimen would not be affected. Strictly speaking, the finite strain theory must be adopted in calculating the natural frequency, but for the present case, the common theory was used as a first approximation.

The equation of the motion of the flexural vibration of the bar subjected to the axial compression, neglecting the rotatory inertia and the internal friction, is

$$E\kappa^2 \frac{\partial^4 y}{\partial x^4} + \frac{T}{a}\frac{\partial^2 y}{\partial x^2} + \rho \frac{\partial^2 y}{\partial t^2} = 0$$

End conditions are

$$x = \pm \frac{l}{2}: \quad y = 0, \quad Ei\frac{\partial^2 y}{\partial x^2} = I_0 \frac{\partial^2}{\partial t^2}\left(\frac{\partial y}{\partial x}\right),$$

where E: Young's modulus,
y: lateral displacement,
a: sectional area,
κ: radius of giration,
ρ: density,
l: length,
i: moment of inertia of the specimen,
I_0; 〃 of the end masses (knife-edge),
T: compressional force per cm^2.

The general solution of the equation of motion is

$$y = e^{ipt}\{A\cos mx + B\cosh nx + C\sin mx + D\sinh nx\},$$

in which

$$m = \frac{\sqrt{\sqrt{\alpha^2+\beta^2}-\alpha}}{l}, \qquad n = \frac{\sqrt{\sqrt{\alpha^2+\beta^2}+\alpha}}{l},$$

$$\alpha = -\frac{Tl^2}{2aE\kappa^2}, \qquad \beta^2 = \frac{\rho l^4 p^2}{E\kappa^2}.$$

By eliminating A, B, C, D by means of the above end conditions, we obtain the following equation finally,

$$(m^2-n^2)^2 \sinh\frac{nl}{2}\cosh\frac{nl}{2} - \left(\frac{I_0}{Ei}\right)p^4\left(-m\cosh\frac{nl}{2}\tan\frac{ml}{2}+n\sinh\frac{nl}{2}\right)\left(m\sinh\frac{nl}{2}\cot\frac{ml}{2}-n\cosh\frac{nl}{2}\right)=0$$

From this equation, by trial-and-error method, E was decided graphically for each value of T.

4. Discussion.

Fig. 1a shows an example of the resonance curve of granite, and Fig. 1b that of basalt. Exciting frequency is plotted against amplified current. The former curve is a typical resonance curve, but the latter is abnormally rugged. As basalt has usually many remarkable cracks along its flow structure, the zig-zags in the resonance curve are attributed to these cracks. Smooth resonance curves were obtained in the case of other specimens.

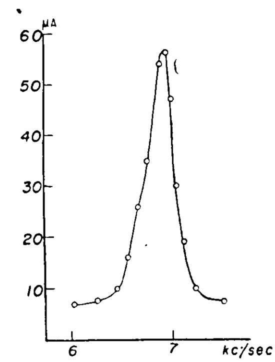

Fig. 1a. Resonance curve of Granite.

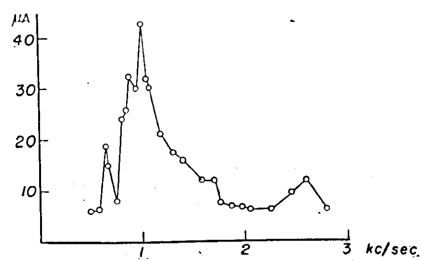

Fig. 1b. Resonance curve of Basalt.

In Fig. 2 Young's modulus E, as calculated from the resonance frequency, was plotted against the applied compressional stress. When the specimen is free from stress, Young's modulus of basalt is the smallest, and that of pitchstone the largest.

Next, let us consider the rate of change of Young's modulus with the increase in stress. At small compression, the rate of increase in E with stress ($\partial E/\partial T$) has, to E itself, the greatest value in the case of basalt, and the smallest value for pitchstone. This is just contrary to that of Young's modulus when specimens are free from stress. The fact leads to the conclusion that a rock which has a large initial value of E shows little change in E with stress.

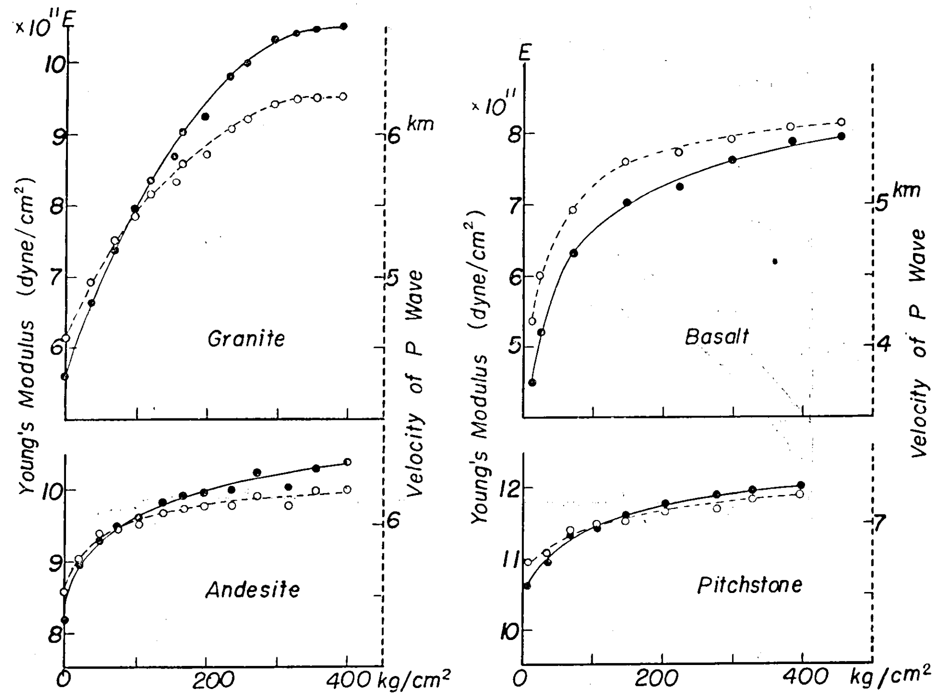

Fig. 2. Young's modulus vs stress. Solid line is Young's modulus. Broken line is the velocity of longitudinal wave.

Fig. 5 shows the photomicrographs of these four rocks. In granite, there are many small cleavages in or between the crystals, and there is no ground-mass. Basalt shows a notable flow structure and has a holocrystalline ground-mass rich in augite. Andesite has a glassy groundmass, and pitchstone is glass including feldspar. Even when the applied force is small, cavities or cracks in basalt would be compressed and the rock will become relatively compact as a whole. The conspicuous rate of increase of E in the case of basalt may thus be explained. On the contrary, pitchstone which has no cracks or cavities, exhibits little change of E with stress. Thus the considerable steepness of the initial slope in E-T diagram in the case of basalt and granite is not due to the compression of rock-forming minerals, but depends largely on the diminishing of many cracks in the rocks. At considerably large stress, $\partial E/\partial T$ becomes small in basalt, andesite and pitchstone with the exception of granite. This tendency may probably be due to the existence of a torelably compact ground-mass. Fig. 3 represents the rela-

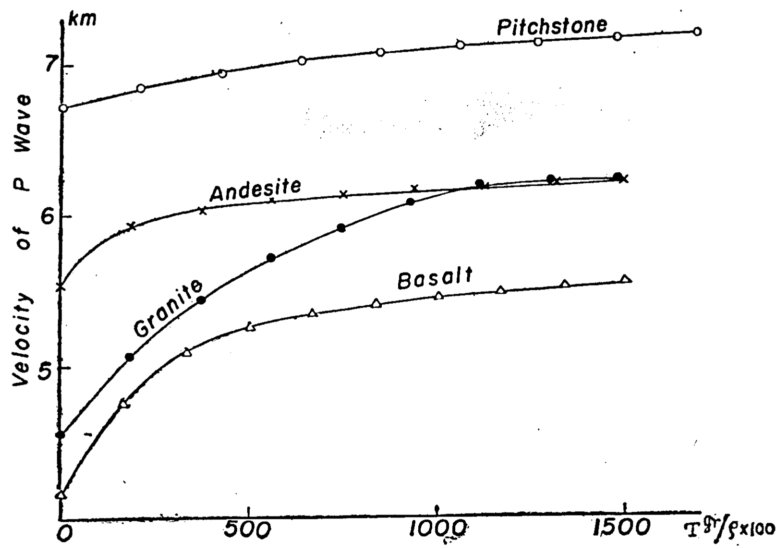

Fig. 3.

tion between the velocity of longitudinal wave and $T/\rho \times 100$ (which corresponds to the depth in meter, if T was not the compressional stress, but hydrostatic confining pressure.). Thus rocks which have compact ground-mass and no cracks should exhibit little change of E with stress.

5. Acknowledgment.

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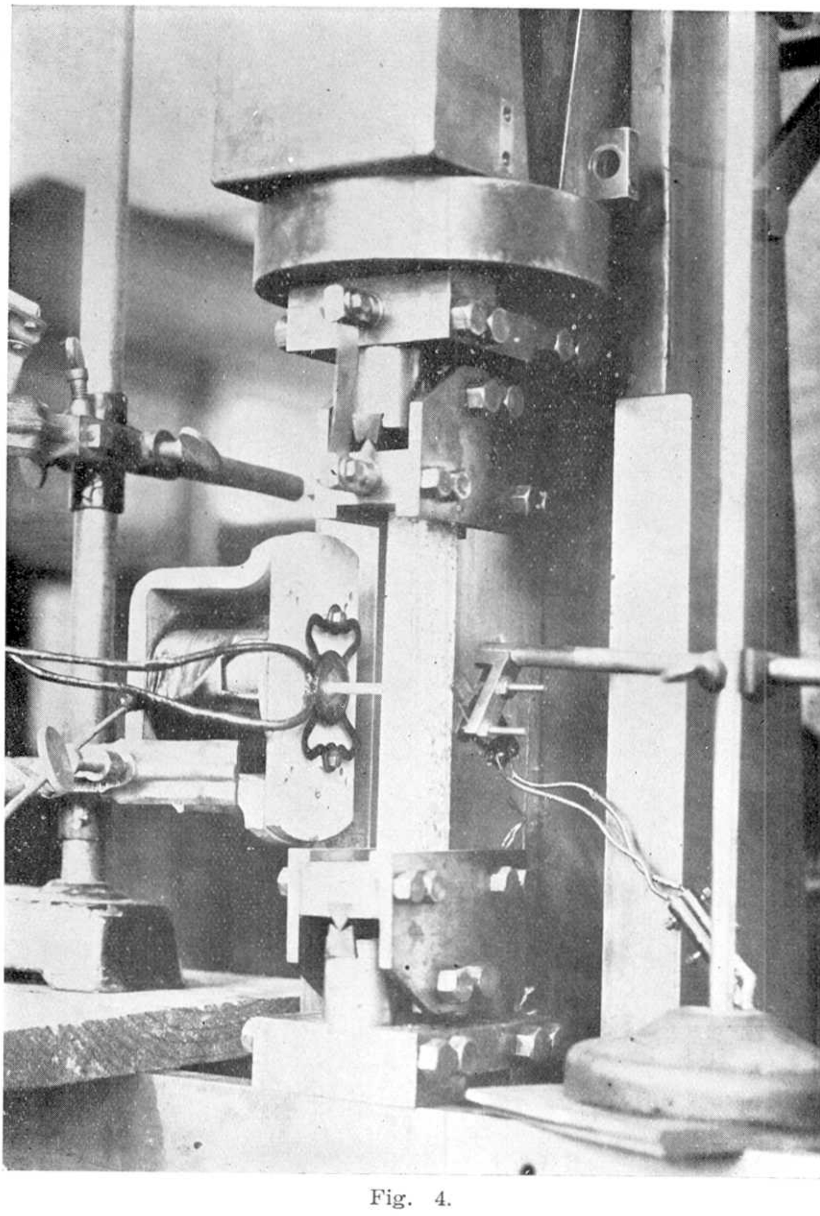

Fig. 4.

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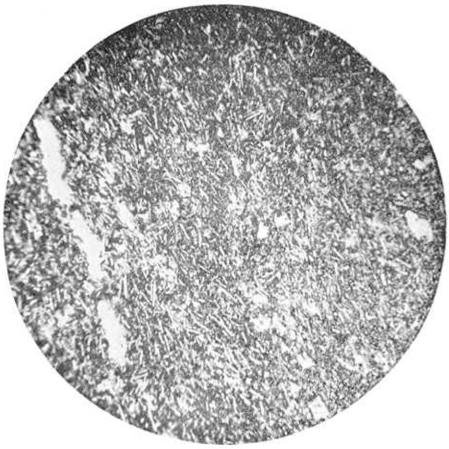
Granite

Andesite



Basalt

Pitchstone

Fig. 5.

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8. 應力がかゝつた狀態に於ける岩石彈性率の動力學的研究
1. 軸壓のかゝつた狀態に於けるヤング率の測定

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最近十年間に於ける岩石の彈性に關する實驗的研究は多くの成果をもたらし,地球内部の岩石の模樣の記述に或る程度成功したと云へる．然し,我々の地殼に,破壞限界内に於て應力が作用してゐる時,彈性が變化し,從つて,歪んだ領域を通過する地震波の速度が變化する．即ち,地殼の或る深さに於て,どの樣な種類のどの位の應力が作用した時に,その場所の彈性率はどれだけ變化するであらうかといふ事を研究する事は興味ある事である．その爲めには,岩石の試片に高溫のもとで,靜水壓をかけて,その狀態で,應力を作用させ,動力學的にその岩石の彈性率又は P 波, S 波の速度を測定すれば良い譯であるが,先づ豫備的實驗として,四種類の代表的火山岩をとり,室溫,一氣壓のもとで,軸方向の壓力をかけた狀態で,電氣的方法に依り,撓み振動を勵起させ,その固有振動數からヤング率を計算し,壓力に依るヤング率の變化の模樣をしらべた．その結果,明らかになつた事は, 1) ヤング率一應力曲線 (第二圖) を見ると判る樣に,應力が小なる時に,玄武岩と,花崗岩のヤング率の割合が他に比して,大きい．この理由として造岩鑛物が壓縮されるよりはむしろ,結晶中の割れ目が先づ壓縮される事が考へられる事と, 2) 又,全體として,應力零の時のヤング率の大きいものは,應力に依るヤング率の變化の割合が小さい．3) 更に,應力が增加すると,非常に緻密な石基を持つた岩石程,ヤング率の增加する割合は少ないと云ふ事である．將來は他の電氣的方式に依つて,成るべく地球内部の狀態に近い條件のもとに,應力の形を色々變へて,彈性率を測定しようと思ふ．