9. *On Magnetization of Volcanoes.*

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1. Introduction.

It is a well known fact that geomagnetic anomalies of particular distribution are found out in the neighborhood of volcanoes as clarified, for examples, by the magnetic surveys of Volcano Mihara[1,2], Volcano Stromboli[3], Volcano Miyake-sima[4,5] Volcano Omuro[6], Volcano Usu[7], Volcano Asama[8], Volcano Fuji (Huji or Fuzi)[9] Volcano Sakura-jima[10,11] and others. These anomalies are all interpreted as due to the magnetization of the volcanoes, the direction of the magnetization being roughly the same with the geomagnetic force there. In order to determine the intensity of magnetization, for example, we take, as T. Minakami[8,10] did, one or two rotational ellipsoids of suitable dimension having uniform magnetization in the direction of the present geomagnetic force. Then, by comparing the magnetic field caused by the ellipsoids with the observed anomaly, the intensity of magnetization is determined. Thoug the intensity thus determined is thought to be reasonable from the results of the experimental study[12] on the intensity of natural remanent magnetization and induced one, we should pay attention to the fact that the dimension of the magnetic mass is chosen rather arbitrarily. From the theoretical standpoint, it is desirable to determine simultaneously both

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1) R. TAKAHASI and T. NAGATA, *Bull. Earthq. Res. Inst.*, **15** (1937), 441.

2) T. RIKITAKE, I. YOKOYAMA, A. OKADA, and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **29** (1951), 583.

3) M. BOSSOLASCO, *Geofisica pura e applicata*, **5** (1943), 11.

4) T. MINAKAMI, *Bull. Earthq. Res, Inst.*, **19** (1941), 356.

5) R. TAKAHASI and K. HIRANO, *Bull. Earthq. Res. Inst.*, **19** (1951), 82 and 373.

6) T. NAGATA, *I.U.G.G.A.T.M.E. Trans. Oslo Meeting*, (1950).

7) T. NAGATA, *I.U.G.G. Assoc. Volcanology, Brussel Meeting*, (1951).

8) T. MINAKAMI, *Bull. Earthq. Res. Inst.*, **16** (1938), 100 and **18** (1940) 178.

9) T. MINAKAMI, not published.

10) H. TSUYA and T. MINAKAMI, *Bull. Earthq. Res. Inst.*, **18** (1940), 318.

11) Y. HARADA, T. HATAKEYAMA and T. OBAYASHI, *Bull. Earthq. Res. Inst.*, **24** (1946), 207.

12) T. NAGATA, *Bull. Earthq. Res. Inst.*, **21** (1943), 1.

the intensity of magnetization and the extent of magnetic mass that is effective to the anomaly. The effective region of volcano thus determined may have some geophysical significances for the structure of volcano. The writer will attempt here to analyse the anomalies near volcanoes from the standpoint stated above.

2. Theory.

As studied by T. Terada[13], a volcano is more or less a conical heap of rocky mass, the essential tendency of the geometrical form of it being expressible by a mean profile curve. For the sake of mathematical simplicity, the writer also deals with the mean topography of a volcano which is obtained after Terada's method.

As already studied in the writer's earlier paper[14], the components of the magnetic field due to a uniformly magnetized circular cone having the radius a_0 at the bottom and the height h is obtained as follows;

$$\left.\begin{aligned} \Delta X &= J\left\{\cos\theta\left(\frac{F}{r}\cos 2\phi - G\cos^2\phi\right) - H\sin\theta\cos\phi\right\}, \\ \Delta Y &= J\left\{\cos\theta\sin 2\phi\left(\frac{F}{r}-\frac{G}{2}\right) - H\sin\theta\sin\phi\right\}, \\ \Delta Z &= J\left\{H\cos\theta\cos\phi + G\sin\phi\right\}, \end{aligned}\right\} \qquad (1)$$

where

$$\left.\begin{aligned} F &= \int_z^h aI_1 dz_1 + \int_0^z aI_1' dz_1, \\ G &= \int_z^h aI_2 dz_1 + \int_0^z aI_2' dz_1, \\ H &= \int_z^h aI_3 dz_1 - \int_0^z aI_3' dz_1, \end{aligned}\right\} \qquad (2)$$

and

$$\left.\begin{aligned} \left.\begin{matrix} I_1 \\ I_1' \end{matrix}\right\} &= \int_0^\infty e^{\mp a(z_1-z)} J_1(ar) J_1(aa)\, da, \\ \left.\begin{matrix} I_2 \\ I_2' \end{matrix}\right\} &= \int_0^\infty a e^{\mp a(z_1-z)} J_0(ar) J_1(aa)\, da, \\ \left.\begin{matrix} I_3 \\ I_3' \end{matrix}\right\} &= \int_0^\infty a e^{\mp a(z_1-z)} J_1(ar) J_1(aa)\, da. \end{aligned}\right\} \qquad (3)$$

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[13] T. TERADA, *Bull. Earthq. Res. Inst.*, **7** (1929), 207.

[14] T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **29** (1951), 161.

Here, we assume that the cone is magnetized in a direction in xz-plane with inclination θ. J and a denote respectively the intensity of magnetization and the radius at $z=z_1$. The origin of the cylindrical coordinate r, ϕ and z is taken at the centre of the base plane of the cone.

At a point on the central axis of the cone where $r=0$ and $z=h+d$, we have

$$\frac{2F}{r}=G=\int_0^h \frac{(a_0-kz_1)^2}{\{(a_0-kz_1)^2+(h+d-z_1)\}^{3/2}}\,dz_1\,,\quad H=0\,, \tag{4}$$

where

$$k=\cot\lambda\,, \tag{5}$$

λ denoting the slope of the cone. In that case, it becomes simply that

$$\left.\begin{aligned} \Delta X&=-\pi JG\cos\theta\,, \\ \Delta Y&=0\,, \\ \Delta Z&=2\pi JG\sin\theta\,, \end{aligned}\right\} \tag{6}$$

where, after integrating (4), G is given by

$$G=-\frac{m}{(m^2+1)^{3/2}}\log\frac{\alpha+m\beta+m^2+1+\sqrt{m^2+1}\sqrt{(1+\alpha)^2+(m+\beta)^2}}{\alpha+m\beta+\sqrt{m^2+1}\sqrt{\alpha^2+\beta^2}}$$

$$+\frac{m}{m^2+1}\left\{\frac{m^2-1-(\alpha-m\beta)}{\sqrt{(1+\alpha)^2+(m+\beta)^2}}+\frac{\alpha-m\beta}{\sqrt{\alpha^2+\beta^2}}\right\}, \tag{7}$$

and where

$$m=1/k\,,\quad \alpha=c_0m/h\,,\quad \beta=dm/h\,. \tag{8}$$

c_0 denotes the radius of the top plane of the cone.

With (6) and (7), it is possible to calculate the magnetic anomaly on the central axis due to the cone if the form of the cone and J are given. Corresponding to dip anomaly ΔI, if ΔI is small, we define a function $f(\Delta I)$ which is given by

$$\begin{aligned} f(\Delta I)&=\{\tan(I_0+\Delta I)-\tan I_0\}/\tan I_0 \\ &=\Delta Z/Z_0-\Delta X/X_0 \\ &=\pi JG\left(\frac{2\sin\theta}{Z_0}+\frac{\cos\theta}{X_0}\right), \end{aligned} \tag{10}$$

where I_0, Z_0 and X_0 denote respectively the normal value of I, Z and X there.

Since we have a good number of geomagnetic dip surveys in the neighborhood of various volcanoes, the writer would like to deal with the dip anomaly in the main.

Theoretically speaking, we can calculate G just on the top of the cone from the shape of the cone for any h. Hence, it is possible to determine h by comparing the observed f on the top with the one calculated from (10) where we take certain value of J from the experiment of rocks composing the volcano. According to the experimental results, however, J is considerably scattered even in case of samples taken from the same lava flow. And, moreover, a volcano is not composed of a sort of lava but is usually composed of lava and pyroclastic ejecta whose magnetic properties are by no means the same. Thus it is impossible in practice to determine the effective height of the cone by taking a value of J from experiment.

In order to avoid this difficulty, the writer calculates f for various heights on the central axis of the cone from the distribution of dip anomaly. By taking the ratio of f to that just on the top, the effect of J will be almost approximately eliminated. From the potential theory, the components of the anomaly on a plane can be expressed by

$$\left.\begin{aligned} \Delta X &= \sum_k \sum_n e^{-kz} (A_{kn} \cos n\phi + B_{kn} \sin n\phi)\, J_n(kr), \\ \Delta Z &= \sum_k \sum_n e^{-kz} (C_{kn} \cos n\phi + D_{kn} \sin n\phi)\, J_n(kr), \end{aligned}\right\} \qquad (11)$$

where the anomaly is assumed to occur from the source beneath $z=0$. $f(\Delta I)$ at $z=d$ is calculated from (11) as follows;

$$f(\Delta I) = \sum_k \sum_n e^{-kd} \left\{ \left(\frac{C_{kn}}{Z_0} - \frac{A_{kn}}{X_0} \right) \cos n\phi + \left(\frac{D_{kn}}{Z_0} - \frac{B_{kn}}{X_0} \right) \sin n\phi \right\} J_n(kr) . \qquad (12)$$

Thus it is easily obtained from (12) that

$$f_{z=d} = \sum_k \alpha_{k0}\, e^{-kd} \qquad (13)$$

at $r=0$ and $z=d$, while we have on $z=0$

$$f_{z=0} = \sum_k \sum_n (\alpha_{kn} \cos n\phi + \beta_{kn} \sin n\phi)\, J_n(kr). \qquad (14)$$

Since the distribution of dip anomaly on a conical volcano is roughly circular symmetric, we can, to a fair degree of approximation, express the distribution by

$$f_{z=0} = \sum_k \alpha_{k0}\, J_0(kr)\,, \qquad (15)$$

where the terms for $n>0$ are all ignored. Hence, we can obtain f at a height $z=d$ through (13). However, it should be borne in mind that we assume that the dip anomaly is given on a plane notwithstanding the fact that we measure the anomaly on the slope of the volcano. The influence of this treatment will be discussed in the following section.

In the next place, we compare $\frac{f_{z=d}}{f_{z=0}}$ thus obtained from the observation with the calculated one or $\frac{G_{z=d}}{G_{z=0}}$ that is readily given by (10). As $\frac{G_{z=d}}{G_{z=0}}$ is of course a function of h, we easily determine h from the curve $\frac{G_{z=d}}{G_{z=0}}$ *vs* h so as to make $\frac{f_{z=d}}{f_{z=0}}$ equal to $\frac{G_{z=d}}{G_{z=0}}$. Since the effective height of the volcano is thus obtained, the mean intensity of magnetization J is readily determined from (10) where θ is assumed to agree with the inclination of the present geomagnetic force as approximately proved in the previous investigations.

As mentioned above, it is possible to determine simultaneously both the effective height and mean intensity of magnetization while the shape of volcano is idealized by preferring its mean topography.

3. Examination of the method.

In order to examine and test the method described in the former section, the dip anomaly near Volcano Mihara in Ooshima Island will b analysed in the following. According to the writer's survey[14] in 1950, the dip-angle increases with the height above sea level as reproduced in Fig. 1, the general tendency of increase being expressed by the curve in the figure where the curve being determined as a quadratic expression of the height by means of the least square method. On the other hand, the height above sea level is read off at every 500 m on the map of 1/50,000 scale along the eight azimuthal lines radiating from the centre of the volcano. By averaging the height thus read off, we get the mean profile as shown in Fig. 2. As seen in the figure, the idealized form of Volcano Mihara is fairly expressed by a circular cone, the slope of

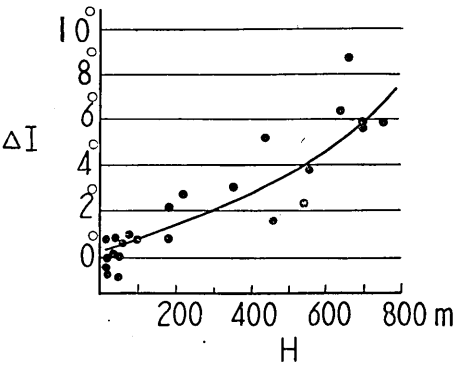

Fig. 1. The relation between dip-anomaly and height on Volcano Mihara.

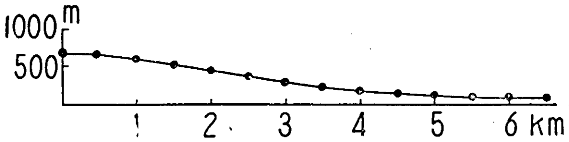

Fig. 2. Mean profile of Volcano Mihara. Ordinate: Height above sea level, abscissa: Distance from the centre of the volcano.

which amounts to about 9°, while the radius of the top plane amounts to almost 500 *m*. Combining, then, the relation between the dip anomaly and height with the mean topography, the relation between $f(\Delta I)$ and the horizontal distance from the centre is obtained as shown in Fig. 3.

From the relation shown in Fig. 3, the coefficients α_{10}, α_{20}, …… in (5) are determined by solving certain simultaneous equations. As fully studied by E. H. Vestine and H. Davids[15], we select a value $r=r_1$ sufficiently large so that f becomes negligibly small. From f at $r=0$, $\frac{1}{6}r_1$, $\frac{2}{6}r_1$, … , $\frac{5}{6}r_1$, it is possible to determine α_{10}, α_{20}, …… , α_{60}. With these coefficients, f at any height on the central axis of the cone is obtained from (13) as shown in Fig. 4.

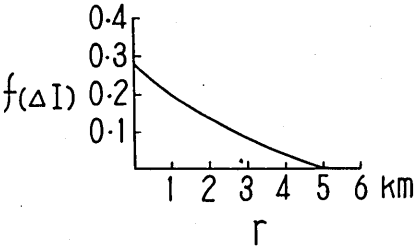

Fig. 3. The relation between $f(\Delta I)$ and the horizontal distance from the centre of the volcano.

Meanwhile, $G_{d=0}$, $G_{d=1\,km}$, $G_{d=4\,km}$ and $G_{d=5\,km}$ for the various value of *h* are calculated by (7) and (8) from which $\frac{f_{d=1\,km}}{f_{d=0}}$, $\frac{f_{d=4\,km}}{f_{d=0}}$, and $\frac{f_{d=5\,km}}{f_{d=0}}$

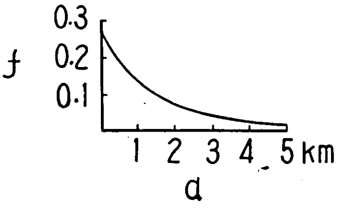

Fig. 4.

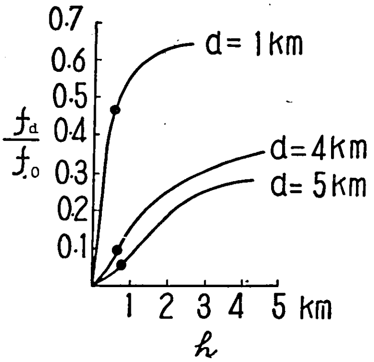

Fig. 5.

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[15] E. H. VESTINE and N. DAVIDS, *Terr. Magn.*, **50** (1945), 1.

are obtained as shown in Fig. 5. On the curves in the figure, the actually determined value of $\frac{f_{d=1\,km}}{f_{d=0}}$, $\frac{f_{d=4\,km}}{f_{d=0}}$, and $\frac{f_{d=5\,km}}{f_{d=0}}$ are respectively shown with small circles. Thus we graphically get, as the effective height of the volcano, $h=0.58$, 0.73 and 0.75 *km* respectively for $d=1$, 4, and 5 *km*.

Now we must examine to what extent the determination of h is affected by the assumption that the anomaly is treated as a distribution on a plane. Since the height above sea level of Volcano Mihara reaches as high as 750 *m*, it would be seriously influenced by the fact that the anomaly is given on the slope if we should adopt a small value of d such as 1 *km*. The effect, however, would become small as d increases. As obtained above, h for $d=1$ *km* seems rather small, while the values for $d=4$ and 5 *km* nearly agree with each other. Hence, it may be said that the said effect is almost avoided by adopting d several times larger than the topographical height of the volcano. It is not possible, however, to take a very large d because f at such great height becomes practically zero.

Next, we calculate $G_{d=0}$ for $h=750\,m$ by (7) and, taking $\theta=47.°9$, $X_0=0.305$ and $Z_0=0.337$ *emu*, J is determined to be 0.037 *emu*.

After all, it is concluded here that the magnetization of a conical body, from the summit to just the sea level, is responsible for the dip anomaly in the neighborhood of Volcano Mihara, while the mean intensity obtained is 0.037 *emu*. The conclusion is just the same with the one obtained in the writer's previous paper[11].

4. Magnetization of various volcanoes in Japan.

Since the method is applicable to conical volcano as tested in section 3, we will examine the magnetization of various volcanoes here. In so far as the writer collect the magnetic data, the results of the magnetic surveys in the neighborhood of Volcano Miyake-sima[4], Volcano Asama[8], and Volcano Sakura-jima[10] surveyed by Minakami, Volcano Omuro by Nagata[6], and Volcano Akagi surveyed by the writer are available, these volcanoes being famous because of their beautiful conical forms. The changes in geomagnetic dip on these volcanoes with the height above sea level or base plane[16] are respectively shown in Figs. 6–10 where

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[16] The base plane of a volcano is defined from the mean profile curve. It is taken at the height where topography becomes almost flat. As to the volcanic island, the height above sea level is adopted as did in section 3.

the general tendency of increase in dip angle is also shown by the curves in the figures, the curves being determined as quadratic expressions of height as in section 3. The mean profiles of topography are also calculated by the same method as in section 3. The relations

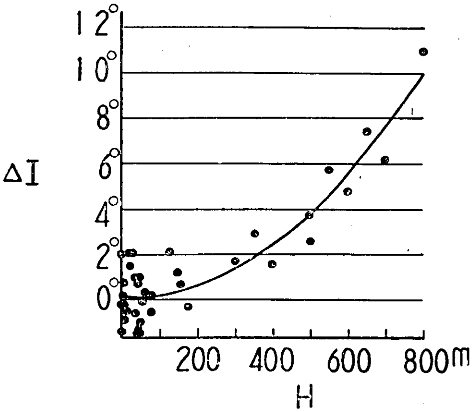

Fig. 6. The relation between dip-anomaly and height on Volcano Miyake-sima.

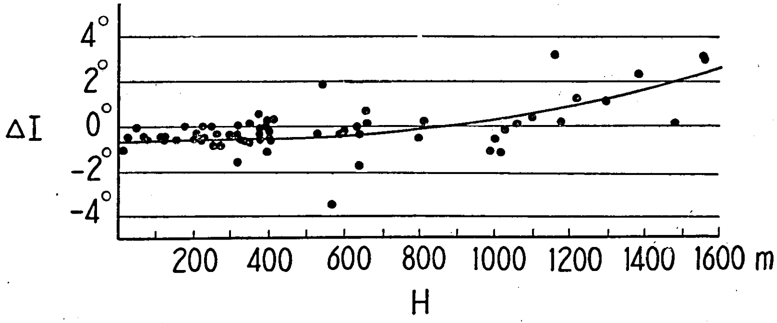

Fig. 7. The relation between dip-anomaly and height on Volcano Asama.

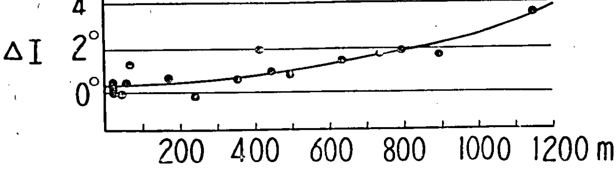

Fig. 8. The relation between dip-anomaly and height on Volcano Sakura-jima.

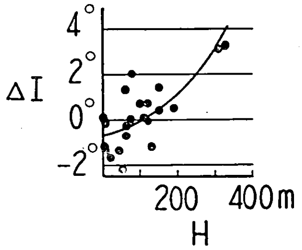

Fig. 9. The relation between dip-anomaly and height on Volcano Omuro.

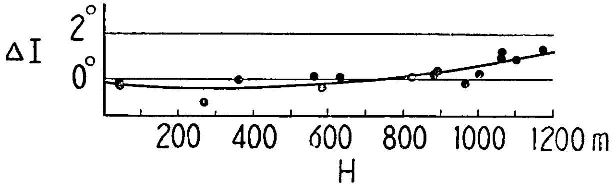

Fig. 10. The relation between dip-anomaly and height on Volcano Akagi.

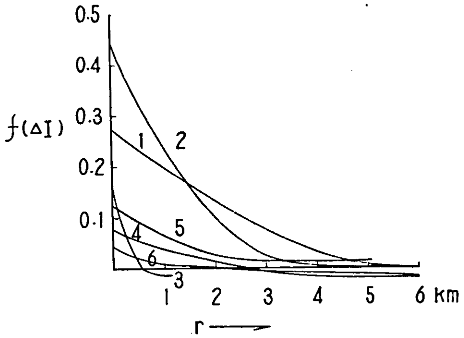

Fig. 11. The relations between and horizontal distance on various volcanoes. 1 Mihara, 2 Miyake-sima, 3 Omuro, 4 Asama, 5 Sakura-jima, 6 Akagi.

between $f(\Delta I)$ and horizontal distance from the centre of the volcano are shown in Fig. 11. From these data the height h of the conical mass that is effective to the dip anomaly and the mean intensity of magnetization J are both determined for respective volcanoes as tabulated in Table I where the specific intensity of natural remanent magnetization J_n experimentally measured and J previously obtained by another investigator are also shown.

Table I. The magnetization of various volcanoes.[17]

Volcano	Rock	Max. height	Survey	h	J	J_n	J by another author
Mihara	basalt	755 *m*	1950 by Rikitake	750*m*	0.037 *emu*	0.016 ∼0.036 *emu*	0.068 *emu* by Nagata
Miyake-sima	basalt	813	1940 by Minakami	520	0.052	0.0047 ∼0.0224	0.015∼0.020 by Minakami
Omuro	basalt	581	1942 by Nagata	270	0.0050	0.0012 ∼0.0032	—
Asama	andesite	2532	1937 by Minakami	600	0.0012	0.0013 ∼0.0025	0.0022 by Minakami
Sakura-jima	andesite	1230	1939 by Minakami	1250	0.0009	0.0015 ∼0.0073	0.0018 by Minakami
Akagi	andesite	1828	1951 by Rikitake	400	0.001	—	—

As revealed in the table, the values of the mean intensity of magnetization J differ considerably from each other for the respectiv volcanoes, while, taking into account the specific intensity of natural remanent magnetization J_n of the rocks composing the volcanoes, we see in general that the larger J_n is the larger J. According to Nagata[12], however, J of the central cone of Volcano Mihara was determined to be 0.0058 *emu* from the anomalous distribution of geomagnetic declination around the crater and J/ρ (ρ denotes the density) of the whole volcano to be 0.027 from the results of the magnetic and torsion balance surveys. The former seems to be too small and the latter, from which we get $J = 0.068$ *emu* by assuming $\rho = 2.5$, too large compared with the writer's value. According to Minakami[4], J of Volcano Miyake-sima amounted to 0.015∼0.020 *emu* as obtained from his dip survey. As to Volcano Asama, Minakami[8] got 0.0022 *emu* for J where the anomaly is replaced by the field caused by a rotational ellipsoid of suitable dimension. He[10] also obtained $J = 0.0018$ *emu* for Volcano Sakura-jima. Summing up these earlier studies, we may say that the writer's values of J agree roughly in their order with the ones previously obtained.

As has been pointed out by Nagata[12], the order of J obtained here cannot be explained by the induced magnetism of volcano in geomagnetic field as clarified by the measurements of the magnetic susceptibility of rocks composing these volcanoes.

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[17] Volcano Mihara, Miyake-sima, Asama and Sakura-jima were active at the time of the surveys, while Omuro and Akagi are Quarternary volcanoes and now are dormant.

The scales of effective mass of volcano as expressed by the values of h are different for respective volcanoes, especially h of Asama is as small as $600\,m$ while, on the contrary, h amounts to more than $1200\,m$ in the case of Sakura-jima. As seen in Figs. 8 and 9, there spreads an extensive area of negative anomaly in dip-angle at the foot of Mt. Asama while we find only the positive anomaly on Volcano Sakura-jima though we cannot measure the anomaly in the sea that surrounds the volcano. The difference in the distribution of anomaly may probably correspond to that in the scale of effective mass determined. However, it is not clear why these differences occur. Since we do not expect, as tested in section 3, any great error of determination, the difference should be ascribed to the structure of volcano though no marked geological evidence for such differences is to be found.

5. Conclusion.

A method of determining simultaneously the mean intensity of magnetization and effective scale of magnetic mass of a volcano from the distribution of anomaly in the earth's magnetic field observed at the place. Applying the method to the results of the magnetic surveys of various volcanoes in Japan, the mean intensity of magnetization is determined, the values agreeing roughly with the experiments on the natural remanent magnetization of rocks composing the volcanoes and the results previously obtained by the analyses of magnetic survey. The effective scales of magnetic mass are also found to differ markedly for respective volcanoes, the difference being probably due to the one in the structure of the volcanoes though we find no marked geological evidences for such differences.

In conclusion, the writer wishes to express his sincere thanks to Miss Y. Hishiyama for her kind assistance. The study was done with the financial aid of the research grant from the Department of Education, and the writer wishes to express his cordial thanks for the grant received.

9. 火山の帯磁

地震研究所　力武常次

從來多くの火山について磁氣測量が實施され，火山附近の地磁氣異常は山體の帯磁に基ずくことが明らかにされた．またその帯磁はほゞ現在の地球磁場と一致していることも知られてきた．そして，磁氣測量の結果より火山の平均の帯磁の强さが求められている．

しかしながら，火山の山體のどの範圍が地磁氣異常に對して有效であるかという點については，山體の規模の定め方がやゝ任意的であるように思われる．本論文に於いては，山體の有效範圍と帯磁の强さを同時に求める方法を考案し，三原山，三宅島火山，大室山，淺間山，櫻島火山および赤城山等の圓錐形の諸火山に應用した結果をのべてある．

まず山體を圓錐で近似し，その圓錐の一様な帯磁を考える時は，火山頂上の地磁氣要素は直ちに帯磁の强さ J および有效な高さ h の函數として求められる．もし J が岩石試料についての實驗値より知られているとすれば，h は直ちに求められるわけであるが，J の實驗値は多くの場合相當のばらつきを示すので，全體の平均を求めるには別の方法を考えねばならない．

そこで，火山直上のある高さの地磁氣要素と山頂の値との比をとることによつて J を消去する．前者は磁氣測量の結果をつかつて，ポテンシャル論により求めることが出來る．實際には，測量は同一平面上に於いて行われているのではないから，山體の數倍の高さに於ける値を採用してその影響をさける．このような比は，有效高度とともに變化するが，その曲線を計算によりえがいておいて，その上に實測より計算された値をプロットすることにより，有效高度 h が定められ，從つて J が求まる．

この方法によつて求められた J は從來の研究者の求めた値および岩石の自然殘留磁氣の强さと大體に於いて調和する．h としては，各火山について異つた値が得られたが，その意味については，現在のところ明確でない．なお上記の考察は伏角の測定結果について行われたものである．