49. *Study on Surface Waves IV.*
Equivalent Single Layer to Double Superficial Layer.

By Yasuo SATÔ,

Earthquake Research Institute.

(Read Sept. 18, 1951.—Received Sept. 19, 1951.)

1. Introduction.

Evidences hitherto revealed by the perusal of seismograms have lead us to believe that the surface crust of our earth has at least a doubly stratified structure in most places. But notwithstanding this almost universal fact, we often treat the problems of seismic surface waves theoretically under the assumption of semi-infinite elastic body with a single superficial layer[1]. It is certain that this is mainly due to the difficulties of problems in case of multiply stratified medium, but it may be partly due to our vague confidence that the nature of waves and their propagation will not suffer much change by such simplified assumption, since a single layer, if chosen suitably, might be equivalent to the double layer. Indeed this confidence does not seem unreasonable in view of the close resemblance of the shape of dispersion curves for Love-waves propagated on doubly stratified medium and on single layered medium. Thus it seems possible for us to find an equivalent structure of single layer, so long as the velocity of the waves is concerned. In this paper, we will call such a layer as "equivalent single layer", and the structure as "simple structure". But, has this equivalent layer an appropriate value of thickness? Do not the values of density and rigidity of the layer differ markedly from those of two layers? Can we regard them as representing truly the original structure of the medium?

Recently it has been repeatedly suggested that the dispersion curves of surface waves may be used in determining the surface

1) The number of the investigation of surface waves with doubly stratified medium is extremely small compared with the papers treating the media with one or without layer.

structure supplementally to the data of bodily waves[2]. But such doubts as mentioned above must be cleared up before the dispersion curves can be safely used in underground prospecting.

With this aim in mind, the following calculations are made taking some typical examples to clarify the difference of the estimated thickness, density and rigidity of equivalent single layer from those of the original doubly stratified structure.

2. Fundamental equations.

It is well known that we can observe two kinds of surface waves in seismograms: the Rayleigh- and the Love-waves. Although we have as much interest in the former kind of waves as in the latter, we can hardly discuss the dispersion curves of Rayleigh-waves propagated on doubly stratified medium on account of the enormous labour involved in the calculation[3]. Consequently, in this paper, we will confine ourselves to the discussion of Love-waves only.

Characteristic equation of Love-waves assumed to be proportional to $\exp\{ipt-ifx\}$ in doubly stratified medium is as follows;[4]

---

2) B. GUTENBERG, "Dispersion und Extinktion von seismischen Oberflächenwellen und der Aufbau der obersten Erdschichten." *Physikalische Zeits.*, **25** (1924), 377.

P. BYERLY, "Seismic Waves and the Surface Layer of the Earth." *Bull. Nat. Res. Council*, **61** (1927), 88.

R. STONELEY, "The Thickness of the Continental Layers of Europe." *M.N.R.A.S. Geo. Sup.*, **2** (1929), 429.

P. BYERLY, "The Dispersion of Seismic Waves of the Love Type and the Thickness of the Surface Layer of the Earth under the Pacific." *Beitr. z. Geophys.*, **26** (1930), 27.

P. BYERLY, "The California Earthquake of Nov. 28, 1929, and the Surface Layer of the Earth in California." *Proc. Nat. Acad. Sci.*, **17** (1931), 91.

W. MÜHLEN, "Über seismische Oberflächenwellen und ihre Beziehung zum geologischen Aufbau des eurasiatischen Kontinents." *Zeits. f. Geophys.*, **8** (1932).

D.S. CARDAR, "Seismic Surface Waves and the Crustal Structure of the Pacific Region." *Bull. Seism. Soc. Amer.*, **24** (1934), 231.

J.F. DE LISLE, "On Dispersion of Rayleigh Waves from the North Pacific." *Bull. Seism. Soc. Amer.*, **31** (1941), 303.

J.T. WILSON and ORHAM BAYKAL, "Crustal Structure of the North Atlantic Basin as Determined from Rayleigh Wave Dispersion." *Bull. Seism. Soc. Amer.*, **38** (1948), 41.

3) Rayleigh-waves in double layer was once studied by H. Kawasumi and I. Yamasita. Monthly meeting of the Earthq. Res. Inst. March 1940. Although there are a few investigations concerning the same problem, not even a single paper has been published.

4) Y. SATÔ, "Study on Surface Waves III. Love-waves with Double Superficial Layer." *Bull. Earthq. Res. Inst.*, **29** (1951), 435.

$$D \equiv D^c + D^s = 0$$

where

$$D^c/\mu_2^2 f^2 \equiv \quad \mathfrak{A}_2 \cos\{\xi h_1 \mathfrak{A}_1\}\,[-\chi_{32}\,\mathfrak{A}\cos\{\xi h_2 \mathfrak{A}_2\} + \mathfrak{A}_2 \sin\{\xi h_2 \mathfrak{A}_2\}]$$
$$D_s/\mu_2^2 f^2 \equiv \chi_{12}\,\mathfrak{A}_1 \sin\{\xi h_1 \mathfrak{A}_1\}\,[\quad \chi_{32}\,\mathfrak{A}\sin\{\xi h_2 \mathfrak{A}_2\} + \mathfrak{A}_2 \cos\{\xi h_2 \mathfrak{A}_2\}] \qquad \ldots\ldots\ldots(2.1)$$

in which

$$\mathfrak{A}_k \equiv \sqrt{(r_k^2 \mathfrak{v}^2 - 1)}\ , \quad k=1,\ 2,\ 3$$
$$\mathfrak{A} \equiv \sqrt{(1-\mathfrak{v}^2)}\ ,$$
$$\mathfrak{v}_k \equiv V/V_k\ , \quad k=1,\ 2,\ 3$$
$$\mathfrak{v} \equiv \mathfrak{v}_3\ ,$$
$$r_k \equiv V_3/V_k\ , \quad k=1,\ 2.$$
$$\xi \equiv fH\ ,$$
$$\chi_{jk} \equiv \mu_j/\mu_k\ ,$$

where V is the velocity of Love-waves while the other notations are illustrated in Fig. 1.

On the other hand, the characteristic equation in the simple structure is[5)]

$$\omega = \sec\theta\,[\mathrm{Tan}^{-1}\{\chi\sec\theta\cdot\sqrt{(\sin^2\theta - \sin^2\theta_0)}\}] \ldots\ldots\ldots(2.2)$$

in which

$$\sin\theta_0 \equiv V_e/V_e'\ ,$$
$$\sin\theta_0/\sin\theta \equiv \mathfrak{v}_e\ ,$$
$$\omega \equiv pH_e/V_e\ ,$$
$$\chi \equiv \mu_e'/\mu_e\ ,$$

where suffix e denote the value in equivalent simple structure, and $\mathfrak{v}_e V_e'$ the phase velocity of Love-waves in simple structure, while the other notations are illustrated in Fig. 2.

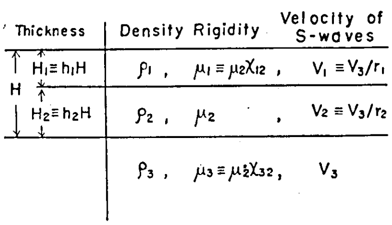

Fig. 1.

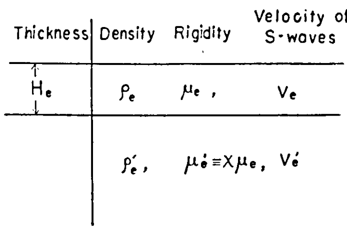

Fig. 2.

5) Y. SATÔ, "Study on Surface Waves I. Velocity of Love-Waves." *Bull. Earthq. Res. Inst.*, **29** (1951), 1. Expression (4.6).

3. Examples.

It seems so difficult to slove the problem described in § 1 generally that we will discuss this subject by taking some typical examples in this section.

Example 1.

As is given in the upper rows of Table II the velocity of S-waves in the upper two layers are assumed to be equal, although the density and rigidity vary discontinuously at the depth H from the free surface. The relation between the velocity and period of Love-waves are calculated by the expression (2.1) and is given in Table I and also by the full line in Fig. 3.

Table I.

Relation between $\mathfrak{v}^2$ and ξ in the example 1.

$3\mathfrak{v}^2$	ξ
3	0
2.96	.1572
2.8225	.3316
2.69	.4312
2.44	.6116
2.21	.7562
2	.918
1.81	1.086
1.64	1.286
1.49	1.531
1.36	1.848
1.25	2.282
1.16	2.926
1.1229	3.382
1.09	3.989
1.0625	4.836
1.04	6.105
1	∞

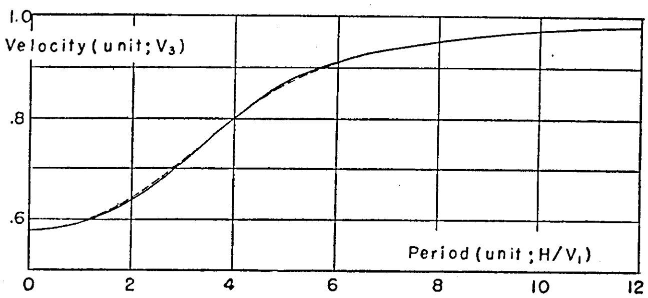

Fig. 3.

On the other hand we searched for a dispersion curve of simple Love-waves which fits well with the above curve. The broken line in Fig. 3 gives the result, showing extremely good coincidence with the full line. Material constants and the thickness of the layer employed for writing this curve is

$$\chi \equiv \mu_e'/\mu_e = 3, \qquad H_e/H = 1. \quad \ldots\ldots(3.1)$$

Namely the estimated structure by the idea of equivalent single layer is as follows[6]:

6) To fit the two curves we always assume as

$$V_e = V_1 \quad \text{and} \quad V_e' = V_3, \qquad \ldots\ldots(3.2)$$

which are necessary to secure the equality of the velocities of extremely short and long waves in two structures.

Further, we adopt the assumptions

$$\mu_e = \mu_1 \;, \qquad \rho_e = \rho_1 \;, \qquad \ldots\ldots(3.3)$$

because the surface values are directly determined by other observations and measurements.

Under such conditions we search for a most appropriate values of μ_e'/μ_e and H_e/H. In this study, however, the word "most appropriate" is not used in the sense of "least squares" principle, but just in the macroscopic sense.

Table II.

		Thickness	Density	Rigidity	Velocity of S-Waves
Real struture with double layer	Upper layer	$\frac{1}{2}H$ (together H)	$\frac{3}{10}\rho$	$\frac{1}{10}\mu$	$\sqrt{\left(\frac{1}{3}\frac{\mu}{\rho}\right)}$
	Middle layer	$\frac{1}{2}H$	ρ	$\frac{1}{3}\mu$	$\sqrt{\left(\frac{1}{3}\frac{\mu}{\rho}\right)}$
	Semi-infinite part	–	ρ	μ	$\sqrt{\left(\frac{\mu}{\rho}\right)}$
Simple structure with equivalent single layer	Layer	H	$\frac{3}{10}\rho$	$\frac{1}{10}\mu$	$\sqrt{\left(\frac{1}{3}\frac{\mu}{\rho}\right)}$
	Semi-infinite part	–	$\frac{3}{10}\rho$	$\frac{3}{10}\mu$	$\sqrt{\left(\frac{\mu}{\rho}\right)}$

As is clear from the above table, the estimation of the thickness is quite good, and is equal to the sum of thickness of two layers of real structure. The coincidence however of the physical constants is not so good as that of the thickness. If we estimate the value of density and rigidity by the above method we shall make a grave mistake.

Example 2.

We will next take another example which we gave in the previous paper[7]. The distribution of matters in this case is a very peculiar one, which is to be rarely found in natural state. (cf. Table III, upper rows.)

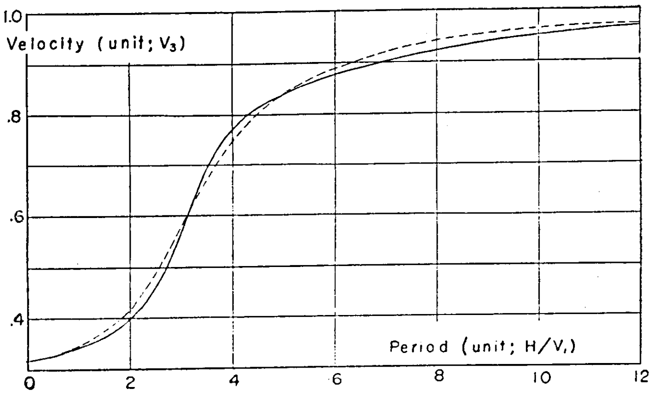

Fig. 4.

Dispersion curve of the original structure is given by a full line in Fig. 4, while the broken line in the same figure—which is constructed

7) *loc. cit.*, 5).

by the constants $\mu_e'/\mu_e=4.5$ and $H_e/H=2/3$—shows that of simple structure[8]. We will give the estimated constants of simple structure in the following table as before, although we could not find a curve which fits so well as in the former case. In this case also, the discrepancy of the values of physical constants is not small as will be seen from Table III.

Table III.

		Thickness	Density	Rigidity	Velocity of S-waves
Real structure with double layer	Upper layer	$\frac{1}{2}H$ } H	ρ	μ	$\sqrt{\left(\frac{\mu}{\rho}\right)}$
	Middle layer	$\frac{1}{2}H$	ρ	$\frac{1}{10}\mu$	$\sqrt{\left(\frac{1}{10}\frac{\mu}{\rho}\right)}$
	Semi-infinite part	—	ρ	μ	$\sqrt{\left(\frac{\mu}{\rho}\right)}$
Simple structure with equivalent single layer.	Layer	$\frac{2}{3}H$	ρ	$\frac{1}{10}\mu$	$\sqrt{\left(\frac{1}{10}\frac{\mu}{\rho}\right)}$
	Semi-infinite part	—	$\frac{9}{20}\rho$	$\frac{9}{20}\mu$	$\sqrt{\left(\frac{\mu}{\rho}\right)}$

Example 3.

Study on the propagation of Love-waves along a doubly stratified earth was performed by T. Matuzawa[9], in which he employed the assumption shown in Fig. 5. His calculation covers the cases $h_1=0.4$, $h_2=0.6$ and $h_1=0.9$, $h_2=0.1$. In this paper, however, we have adopted the

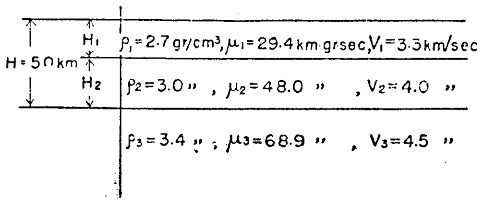

Fig. 5.

Table IV.

Case		a	b	c	d	e
h_1	1.0	.9	.8	.6	.4	.2
h_2	0	.1	.2	.4	.6	.8

8) In this example only, we adopted the assumption

$$V_e = V_2 \,, \qquad \text{and} \qquad \rho_e = \rho_2 \qquad \ldots\ldots\ldots\ldots(3.4)$$

instead of (3.3). This is necessary to secure the equality of the velocity of short waves.

9) T. MATUZAWA, "Propagation of Love Waves along a Doubly Stratified Layer". *Proc. Phys.-Math. Soc. Jap.* [iii], **10** (1928), 25.

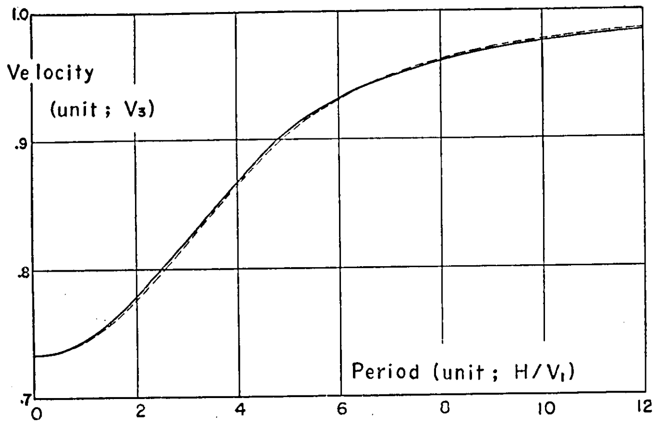

Fig. 6 a.

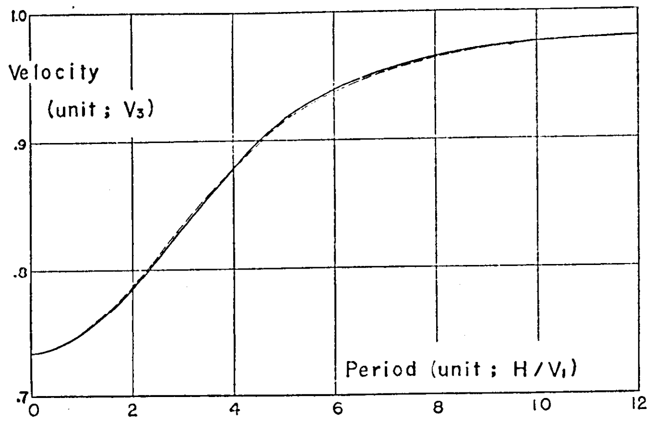

Fig. 6 b.

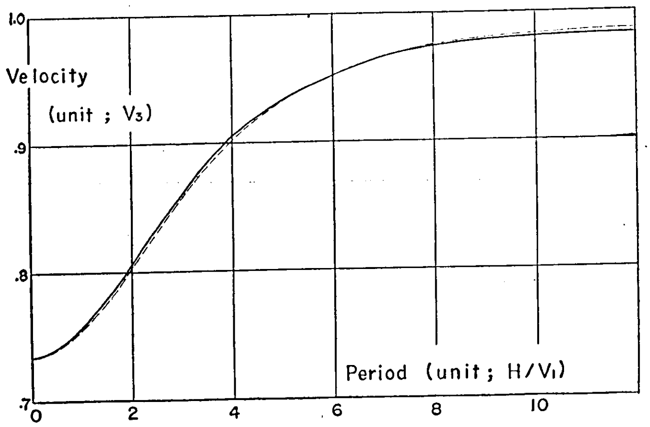

Fig. 6 c.

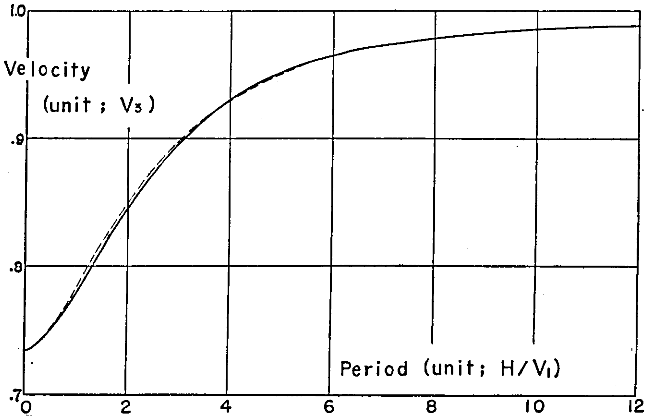

Fig. 6d.

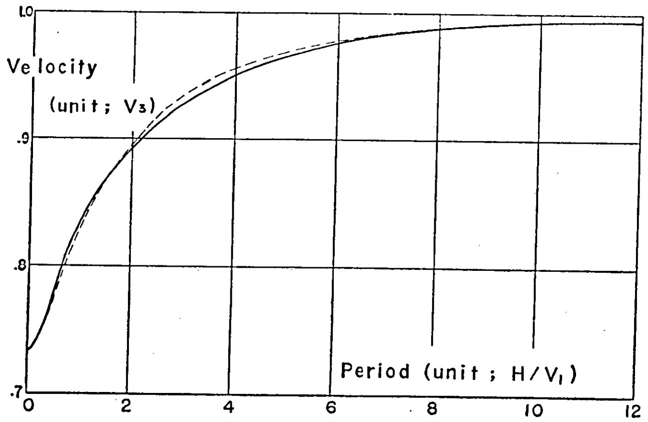

Fig. 6e.

Table V. Relation between $\mathfrak{v}^2$ and ξ in the example 3.

Case	a	b	c	d	e
$\mathfrak{v}^2$	ξ				
1	0	0	0	0	0
.987654	.3187	.3365	.378	.4355	.5115
.916543	.8550	.906	1.040	1.221	1.507
.8217285	1.386	1.476	1.722	2.122	2.935
.7585186	1.762	1.875	2.236	2.873	4.560
.679506	2.496	2.692	3.300	4.582	8.823
.592593	4.554	4.960	6.338	9.388	18.793
.537778	∞	∞	∞	∞	∞

Table VI.

Case		a	b	c	d	e
$h_1 \equiv H_1/H$	1	.9	.8	.6	.4	.2
$h_2 \equiv H_2/H$	0	.1	.2	.4	.6	.8
H_e/H (estimated)	1	4/3	10/9	1/1.05	1/2.1	1/5
H_e (,,) km	50	66.7	55.6	47.6	23.8	10.0
H (original)	50 km					
μ_e'/μ_e (estimated)	2.344	2.5	2.0	2.0	1.05	0.6
μ_e' (,,) km. gr. sec.	68.9	73.5	58.8	58.8	30.9	17.6
μ_3 (original)	68.9 km. gr. sec.					
ρ_e'/ρ_e (estimated)	1.2592	1.344	1.076	1.076	.5647	.3227
ρ_e' (,,) gr/cm³	3.40	3.63	2.90	2.90	1.52	.871
ρ_3 (original)	3.40 gr./cm³					

five ceases in Table IV, and calculated the dispersion curve in every case respectively. The results are given in Fig. 6 a, b, c, d, e (full line) and in Table V.

On the other hand we calculated tentatively the velocity in simple structure, giving various values of μ_e'/μ_e and H_e/H. The obtained results, which seem to fit fairly well with the original curves are the broken lines in the same figures.

The adopted values of μ_e'/μ_e and H_e/H are tabulated in Table VI together with the original real values of the medium.

In spite of the good fitting of full and broken lines, the discrepancies lying between the original values and the estimated ones are generally very large.

4. Conclusions.

In the preceding articles we have examined the possiblity of finding the "simple structure" with an "equivalent single layer." The answer is in the affirmative, and we could find the simple structure whose dispersion curve fits pretty well with that of original structure in every case. However, with regard to the estimated values of the thickness of the layer, the results are not always so satisfactory as in the case of fitting the curve. Moreover, between the density and rigidity calculated by the above method and the original values there is a marked discrepancy. Consequently, judging from the above results, crude and scanty as they are, we are obliged to give a pessimistic conclusion to the questions raised in the introduction of this paper. Namely, the equivalent single layer with regard to dispersion curve does not always give an appropriate value of thickness of the crustal layers; and the estimated density and rigidity are often considerably different from those of two layers; hence we can not regard it as a good representation of the original structure of the medium. So long as we are ignorant of the correct structure, which is quite difficult to grasp, the knowledge of the dispersion curve of the surface waves will be of little use for our practical purpose of prospecting the underground structure.

The above discussion is limited only to the theory of Love-waves; however, we cannot expect to obtain a greatly differing conclusion even if we extend our study to the case of Rayleigh-waves.

---

49. 表面波の研究 IV

二重の表面層と同等な單一層

地震研究所　佐藤泰夫

我我の地球が，表面に二つの層をもつた構造をなす事は，今日多くの人の承認する所である。しかしながら，理論的な研究は主として單一層のものについて行われている狀態である。これは一つには計算の簡單ということもあるが，又一方には，適當な厚さの層を一つ考へれば，二重の層の場合とほぼ同等の效果のある事が，暗に豫期されてゐるからでもあろう。

そこで以上の考への當否を調べるために，まづ二重の表面層を持つた構造をいくつか假定し，この中を傳はるラブ波の分散曲線を正確に求める。次に，單一の表面層を持つた構造で，その中のラブ波が上に求められた分散曲線となるべく似た形の分散曲線をもつように，表面層のあつさや，密度，剛性率などを色色加減する。このようにすると，いづれの場合にも，二つの曲線はかなりよく——我我が地震記象から分散を求める場合の精度よりはるかによい程度に——重なり合ふ。

しからば，このようにして求められた構造は，もとのものに比べてどんな關係を示して居るであろうか。層の厚さについて言ふならば，極めてよくあふ場合もあるけれども，全然ことなつた値を示すこともある。又，兩方の構造において，表面での密度，剛性率をひとしく置くと，半無限體內での値は，多くの場合において非常なへだたりを示して居り，到底單一層の構造をもつて，もとのものの代表と考へる事は困難である。

從つて我我は，上のやうな考へを以てしては，表面波の分散から地球表面の構造を推定しようといふくわだてに，悲觀的な結論を下さざるを得ない。

以上の議論はラブ波に關するものであるが，レーリー波についても，さほど異つた結果は出ないであろう。