47. *The Result of Observation of Wave-velocity in the Ground.*

By Kiyoshi KANAI,

Earthquake Research Institute.

(Read Mar. 20, May 15, June 19, 1951.—Received June 20, 1951.)

The problem concerning the velocity distribution of seismic waves in the earth already was paid attention to by many seismologist from the early part of this century. They had believed that as there are two or three discontinuities in the earth and two layers in the earth crust, in each layer the velocity of waves increases as they advance further into the interior. Since seismic prospecting has been carried on very actively recently, the velocity-depth relation of seismic waves at the part very near the earth surface has been studied considerably[1].

It was quite recently that it became necessary for us to know the velocity distribution of seismic waves in the interior of the earth in order to promote the study of underground earthquake motions[2]. For this purpose, the velocity of longitudinal waves were observed by causing artifical earthquake. The range of measurement was from the earth-surface to 450 meter under the ground and the place was at Hitachi Mine, Ibaraki Prefecture in Japan.

The geological formation of the neighbouring part of observing station is shown in Fig. 1. The shot points and observing points in the first and second experiments is indicated in Fig. 2 and 3 as well as in Table I and II. Owing to the circumstance of the place, the observing point could not be taken on a line veritcal to the shot point, thus making the analysis of the observation very complicated.

In the first observation the Yokogawa-type 3-elements oscillograph was used and in the second a 6-elements oscillograph of the same type. The speed

1) N. NASU, "Studies on the Propagation of an Artificial Earthquake Wave....", *Bull. Earthq. Res. Inst.*, **18** (1940), 289, (in Japanese).

N. HASKELL, "The Relation Between Depth, Lithology and Seismic Wave Velocity in Tertiary Sandstones and Shales", *Geophysics*, **6** (1941), 318.

L. FAUST, "Seismic Velocity as a Function of Depth and Geologic Time," *Geophysics*, **16** (1951), 192.

2) K. KANAI, T. TANAKA, "Observations of the Earthquake-motion at the different Depths of the Earth. I", *Bull. Earthq. Res. Inst.*, **29** (1951), 107.

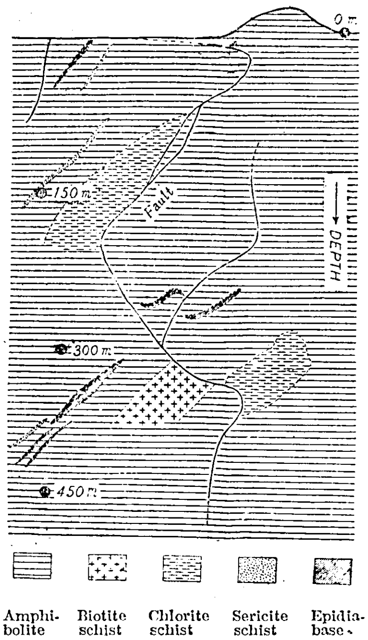


Fig. 1. Geological formations of the neighbouring part of observing stations at Hitachi Mine, Ibaraki Prefecture in Japan.

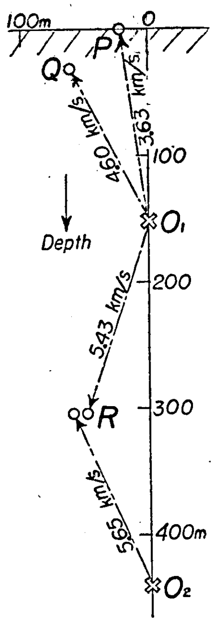


Fig: 2. Vertical section view of the positions of shot points (O_1, O_2) and observing points (P, Q, R) in the first experiment.

of recording paper was 4 m/sec—2 m/sec, with 1/1,000 sec time marking. As an explosive, 1 kg of dynamite was used.

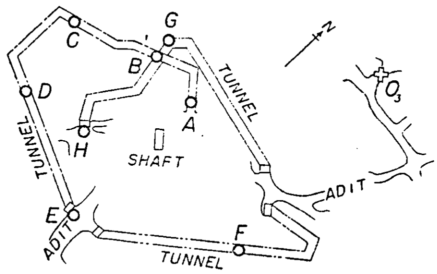
The result of the first observation is shown in Fig. 2 and denoted in Table I. From Fig. 2 and Table I it is found that, within the depth of about 250 m, seismic wave increases its velocity according as it goes further. Beyond that

Table I. The relation between shot points and observing points, and the result of the first experiment.

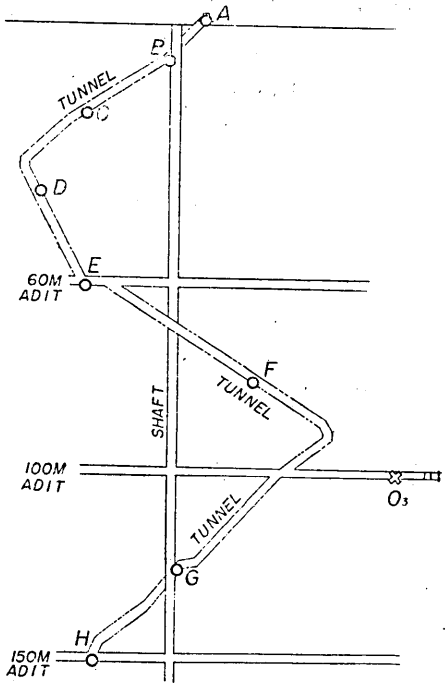
Shot points		Observing points					Travel-time (sec)	Mean velocity (km/sec)
	Depth (m)		Depth (m)	Distance from the shot point (m) Horiz.	Vert.	Direct		
O_1	152.5	P	0	22	152.5	154	0.0424	3.63
〃	〃	Q	31.0	61	121.5	136	0.0296	4.60
〃	〃	R	303.7	49	151.2	159	0.0293	5.43
O_2	439.2	〃	〃	60	135.5	148	0.0262	5.65

depth velocity seems to remain constant. Its value may be estimated at about 5.5 km/sec.

Table II shows the result of the second observation which was carried out for the purpose of studying the velocity distribution within the depth of 150 m precisely. Seeking the velocity distribution in the ground from Table II, we analysed under the conditions that the velocity in one horizontal plane is



a. Horizontal section view.



b. Vertical section view.

Fig. 3. The positions of shot point (O_3) and observing points (A, B,....G, H) in the second experiment.

Table II. The relation between shot point and observing points, and the result of the second experiment.

Shot point		Observing points					Travel-time (sec)		
	Depth (m)		Depth (m)	Distance from the shot point (m)			Observ.		Calcul.
				Horiz.	Vert.	Direct	1 st	2 nd	
O_3	108.0	A	0	46.0	108.0	117.4	0.0368	0.0366	0.0357
〃	〃	B	9.8	53.9	98.2	112.0	0.0309	0.0299	0.0298
〃	〃	C	22.8	75.0	85.2	113.5	0.0298	—	0.0294
〃	〃	D	41.7	85.0	66.3	107.8	0.0267	0.0259	0.0271
〃	〃	E	63.7	81.0	44.3	92.3	0.0232	0.0229	0.0230
〃	〃	F	86.1	54.0	21.9	58.3	0.0148	0.0140	0.0143
〃	〃	G	130.6	51.5	22.6	56.2	0.0135	0.0133	0.0134
〃	〃	H	153.1	73.0	45.0	85.8	0.0200	0.0196	0.0204

constant and the velocity distribution concerning depth is also constant in the layer between two horizontal planes passing through each observing point. That is to say, the horizontal planes passing through A, B....G, H and O_3 are assumed to be discontinuous planes of velocity. Then assuming that the path of waves ranging from O_3F to O_3G is straight, that the path between O_3E and O_3H refracts once at the horizontal planes which passes F and G, and the pathes passing through O_3D, O_3C, O_3B and O_3A refract 2, 3, 4 and 5 times respectively, we will calculate the velocity of longitudinal waves in each assumed layer. Now representing the travel time taken from the shot point to observing point by T, the horizontal distance between those points by x, and the thickness and velocity of the assumed horizontal layer by z_s and v_s we obtain

$$x=\sum_1^n \frac{z_s(cv_s)}{\sqrt{1-(cv_s)^2}}=\sum_1^n \frac{z_s V_s}{\sqrt{1-V_s^2}}, \qquad \text{(1)}$$

$$T=\sum_1^n \frac{z_s}{v_s\sqrt{1-(cv_s)^2}}=c\sum_1^n \frac{z_s}{V_s\sqrt{1-V_s^2}}, \qquad \text{(2)}$$

where

$$V_s(\equiv \sin i_s)=cv_s, \qquad \text{(3)}$$

c is a constant determined by the path of waves which implies $(\sin i_s)/v_s$, i_s the normal angle, and n the number of assumed horizontal layers between the shot point and the observing point.

By means of the successive approximation which we adopted, the velocity of O_3—F layer is calculated using the equation $v_1=\overline{O_1F}/T$. Next the velocity

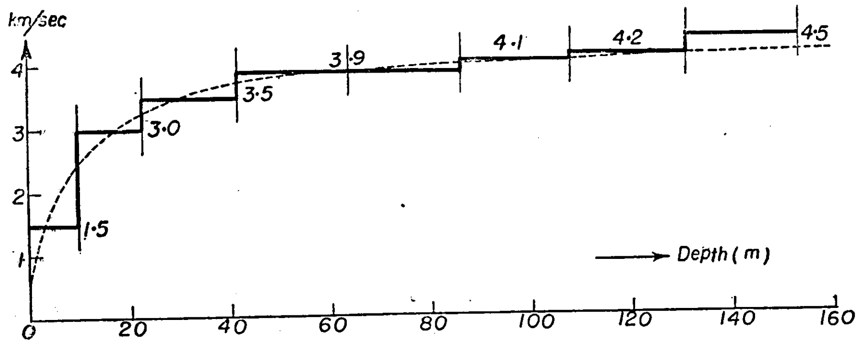


Fig. 4. Velocity of longitudinal waves versus depth from the earth's surface obtained mathematically using the values of experimental result. Broken line represents the calculated result using the empirical formula of v (km/sec) $=0.6\ \{1+718z\,(\text{km})\}/\{(1+96z\,(\text{km})\}$

of F—E layer is obtained from equation (1) applying the trial-and-error method on V_1 and V_2. By placing those values into equation (2), c can be determined. Then after placing c and V_1 into equation (3), we repeat the trial-and-error method till v_1 which was calculated before can be satisfied. By repeating this method the velocity of each layer is given one after another. The results of the calculation are shown in Fig. 4.

When the stepped velocity-depth distribution shown in Fig. 4 is replaced by a curve denoted by broken line in the same figure, its empirical formula becomes

$$v = 0.6 \times \frac{1+718z}{1+96z} \qquad \text{.......................(4)}$$

where v is the velocity at the depth z. The units of v and z are km/sec and km respectively.

Next let us examine to what degree the empirical formula satisfies the observation values. Equation (4) results in

$$V = k\frac{1+az}{1+bz} \qquad \text{..........................(5)}$$

where $a=718$, $b=96$, $k=0.6\,c$, and $c=(\sin i)/v$.

Now, putting the relation of equation (5) into x (the horizontal distance between the shot point and observing point) and T (its travel-time) we get

$$x = \int_{z_1}^{z_2} \frac{Vdz}{\sqrt{1-V^2}} = k\int_{z_1}^{z_2} \frac{(1+az)dz}{\sqrt{\pm z^2(\pm b^2 \mp a^2k^2)+2z(b-ak^2)+(1-k^2)}}$$

$$= \frac{k}{\sqrt{\pm b^2 \mp a^2k^2}}\left[(1\mp a\alpha)\begin{cases}\log\{(z+\alpha)+\sqrt{(z+\alpha)^2-\beta^2}\} \\ \sin^{-1}\left(\dfrac{z-\alpha}{\beta}\right)\end{cases} \pm a\sqrt{\pm(z\pm\alpha)^2 \mp \beta^2}\right]_1^{z_2}$$

$$\text{........(6)}$$

$$T = \int_{z_1}^{z_2} \frac{cdz}{V\sqrt{1-V^2}} = \frac{c}{k}\int_{z_1}^{z_2} \frac{(1+bz)^2dz}{(1+az)\sqrt{\pm z^2(\pm b^2 \mp a^2k^2)+2z(b-ak^2)+(1-k^2)}}$$

$$= K\left[\pm b^2\sqrt{\pm(z\pm\alpha)^2\mp\beta^2} + \left(2b-\frac{b^2}{a}-b^2\alpha\right)\begin{cases}\log\{(z+\alpha)+\sqrt{(z+\alpha)^2-\beta^2}\} \\ \sin^{-1}\left(\dfrac{z-\alpha}{\beta}\right)\end{cases}\right.$$

$$\left. + \frac{\pm\left(\dfrac{2b}{a}-\dfrac{b^2}{a^2}-1\right)}{\sqrt{\pm\left(\alpha\mp\dfrac{1}{a}\right)^2\mp\beta^2}} \log\left\{\frac{2}{z+1/a}\left\{\pm\left(\alpha\mp\frac{1}{a}\right)^2\mp\beta^2\right\}\right.\right.$$

$$+2\left(\alpha\mp\frac{1}{a}\right)+2\sqrt{\pm\left(\alpha\mp\frac{1}{a}\right)^2\mp\beta^2}$$

$$\times\sqrt{\left\{\pm\left(\alpha\mp\frac{1}{a}\right)^2\mp\beta^2\right\}\frac{1}{(z+1/a)^2}+\frac{2(\alpha\mp1/a)}{z+1/a}\pm1}\Bigg]_{z_1}^{z_2} \quad \ldots\ldots\ldots(7)$$

where

$$K=\frac{c}{ka\sqrt{\pm b^2\mp a^2k^2}},\quad \alpha=\frac{b-ak^2}{\pm b^2\mp a^2k^2},\quad \beta^2=\alpha^2-\frac{1-k^2}{b^2-a^2k^2} \quad \ldots\ldots\ldots(8)$$

in (6)~(8) the upper and the lower notation represent the case of $(b^2-a^2k^2)>0$ and $(b^2-a^2k^2)<0$ respectively.

And in the case of $(b^2-a^2k^2)=0$, we get

$$=\frac{k}{b-ak^2}\left[\left\{1-\frac{a(\{1-k^2)-z(b-ak^2)\}}{3(b-ak^2)}\right\}\sqrt{2z(b-ak^2)+(1-k^2)}\right]_{z_1}^{z_2}, \quad \ldots(6')$$

$$T=\frac{1}{a(b-ak)}\left[\left\{\left(2b-\frac{b^2}{a}\right)-\frac{b^2\{(1-k^2)-z(b-ak)\}}{3(b-ak)}\right\}\sqrt{2z(b-ak)+(1-k^2)}\right]_{z_1}^{z_2}$$

$$+\frac{\frac{1}{a}\left(1-\frac{2b}{a}+\frac{b^2}{a}\right)}{\sqrt{(1-k^2)-\frac{2}{a}(b-ak)}}\left[\log\frac{\sqrt{P}+\sqrt{Q}}{\sqrt{P}-\sqrt{Q}}\right]_{z_1}^{z_2}, \quad \ldots(7')$$

where

$$P=\frac{2(b-ak)}{z+1/a}+\left\{(1-k^2)-\frac{2}{a}(b-ak)\right\},\qquad Q=\left\{(1-k^2)-\frac{2}{a}(b-ak)\right\}\quad [Q>0] \quad \ldots\ldots\ldots\ldots(9)$$

From equation (6) we will calculate, using the trial-and-error method, the value of k that can satisfy x, and which is put into equation (7) in order to get T. In other words, the path ranging from the shot point to the observing point, and the travel time taken in this path are given from (6) and (7) respectively. The travel time thus obtained is denoted in the right column of Table II. In this table it is found that the observed value agrees very well with the calculated one within the depth of about 130 m. Therefore it may be said that the empirical formula (4) represents approximately the natural state.

We presume that a discontinuity surface of velocity lies near the depth of 130 m, but the lack of data prevents us from making more complicated discussion on the distribution beyond that depth.

As the result of this investigations is inexplicable by the velocity versus pressure relation for rock[3], we will discuss in the near future the causes of such distribution by examining the property of elastic constants and other properties referring to the samples found there.

In conclusion the author wishes to express his hearty thanks to the personnel of the Motoyama Office, Hitachi Mine, for their kind assistance, and also to Messrs: T. Tanaka, T. Suzuki, K. Osada and Miss S. Yoshizawa without whose help this work could not be done.

47. 地下における弾性波速度の測定結果

地震研究所　金　井　清

日立鑛山における地下 130 m 位までの縦波の速度分布は次式の關係で表し得ることがわかつた。

$$v = 0.6 \times \frac{1+718z}{1+96z} \qquad (1)$$

ここに v は縦波の速度で單位は km/sec, z は地表面よりの深さで單位は km である。

250m 位以下の平均速度は 5.4 km/sec ぐらいとなり，130m 附近に速度の不連續面があるようにも見えるが，データが不足で，これ以上のことは現在のところ言えない。

(1)式の關係は壓力のみでは説明が困難なことがらであるから，そのうちに試驗片について種々の物理的常數を調べてみる豫定である。

なお，日立鑛山一帶の地質構造は，秩父古生層で，御荷鉾層に對比される所謂赤澤層より秩父古生層上部に屬する鮎川層上部に至る一般走向 N 45°E，西又は東に 70° 内外の傾斜を存する累層より成り，これを東北及び西北の三方より屏擁する火成岩とから構成される。岩石は成層岩の角閃片岩及び角閃岩を主とし絹雲母片岩，綠泥片岩，黑雲母片岩，硅質片岩，結晶質石灰岩等種々の變成岩より成り，火成岩は花崗閃綠岩を主として閃綠岩，變輝綠岩，石英斑岩等の岩脈又は岩床を作ひ又西方には橄欖岩の大岩脈を見る。

3) D. S. Hughes and H. J. Jones "Elastic Wave Velocities in Sedimentary Rocks", *Trans. Amer. Geophs. Union*, **32** (1951), 173.