20. *Electromagnetic Shielding within the Earth and Geomagnetic Secular Variation.*

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1. Introduction

One of the most striking features of the earth's magnetic field is the secular variation. According to the extensive investigation carried out by E. H. Vestine and his collaborators[1),2)], we find a number of foci of the secular variation distributed irregularly on the earth's surface, the foci having likely some tendencies of drift motion towards the west as appears in Vestine's map for four epochs at a distance of ten years during the period from 1905 to 1945. It was also found out that some of the foci disappeared and others newly appeared during the period. More than fifty years ago L. A. Bauer[3)] and V. Carlheim-Gyllensköld[4)] already pointed out the westerly drift of the secular variation. As a result of spherical harmonic analysis, Carlheim-Gyllensköld obtained 3147, 1381 and 454 *years* as the apparent periods of complete revolution respectively for the harmonics P_1^1, P_2^1 and P_2^2.

Though these old investigations were not always accurate as commented by S. Chapman and J. Bartels[5)], it seems that the secular variation of world-wide scale has an apparent period or duration-time of order of 1000 *years*, while the regional one of 100 *years* as recently reconsidered by W. M. Elsasser[6)] and E. C. Bullard.[7)]

Elsasser[6)] and Bullard[7)] studied the electromagnetic induction in the earth's

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1) E. H. VESTINE, L. LAPORTE, C. COOPER, I. LANGE and W. C. HENDRIX, "Description of the Earth's Magnetic Field and its Secular Change, 1905-1945." *Cairnegie Inst. of Washington, Publication No.* **578** (1947).

2) E. H. VESTINE, L. LAPORTE, I. LANGE and W. E. SCOTT, "The Geomagnetic Field, Its Description and Analysis," *Carnegie Inst. of Washington, Publication No.* **580** (1947).

3) L. A. BAUER, *Amer. Journ. Sci.*, **50** (1895), 109, 189 and 314.

4) V. CARLHEIM-GYLLENSKÖLD, *Astron. Jaktagelser och Undersökm, Stockholms Observat.*, **5**, Nr 5 (1896), 36. *Astron. och Fysik.*, **3**, No. 7 Uppsala (1906), 5.

5) S. CHAPMAN and J. BARTELS, *Geomagnetism* (1940), p. 667.

6) W. M. ELSASSER, *Phys. Rev.*, **69** (1946), 106, **70** (1946), 202. *Rev. Mod. Phys.*, **22** (1950), 1.

7) E. C. BULLARD, *M. N. R. A. S. Geophys. Suppl.* **5** (1948), 248. *Proc. Roy. Soc. London* A **197** (1949), 433, **199** (1949), 413.

core with special relation to the origin of the earth's magnetic field and its secular change. According to them, the secular variation was recognized as a perturbation of the main field caused by fluid motion near the surface of the core. However, we have not yet gained sufficient knowledge about the electric properties and the fluid motion in the core, so these theories may be regarded as hypothetical notwithstanding their interesting and suggestive characters.

Meanwhile, the earth's mantle has a finite electrical conductivity as clarified from the studies of geomagnetic variation of short period or duration-time which were developed by S. Chapman[8),9)], A. T. Price[9),10)], B. N. Lahiri[10)], K. Terada[11)] and the writer[12)]. Among the investigations, the writer approximately determined the conductivity-distribution down to a depth of about 1500 *km* by studying various variations such as **Sq** (solar daily variation on quiet days), **S_D** (solar daily disturbance variation), **Dst** (main phase of magnetic storm), bay-type disturbance, S. F. variation (magnetic effect associated with solar eruption or Dellinger effect), sudden commencement of magnetic storm, and 27-day period variation, the period or duration-time of these variations ranging from a few minutes to scores of days. It was found out that the electrical conductivity increases discontinuously at a depth of nearly 400 *km* from 10^{-15} *emu* to 10^{-12} *emu* and then increases gradually with the increase in depth, the most probable distribution being expressed by

$$\begin{aligned} \sigma &= \sigma' & q_1 < \rho < 1 \\ &= \sigma_0 \rho^{-l} & \rho < q_1 , \end{aligned}$$

where σ', σ_0, l, and q_1 is determined to be 10^{-15} *emu*, 1.0×10^{-12} *emu*, 11, and 0.94 respectively. ρ denotes the ratio of the radial distance r to the radius of the earth a.

Since the earth's mantle has some electrical conductivity as mentioned above, it should be borne in mind that the varying magnetic field which is hypothetically considered by Elsasser and Bullard will be shielded to some extent by the existence of the mantle. In his latest memoir, Elsasser[6)] has taken into account a rough estimate of the above mentioned effect. On the basis of the conductivity-distribution obtained by the writer, the writer will calculate here the electromagnetic

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8) S. CHAPMAN, *Phil. Trans. Roy. Soc. London* A, **218** (1919), 1.

9) S. CHAPMAN and A.T. PRICE, *Phil, Trans. Roy. Soc. London* A, **229** (1930), 427.

10) B. N. LAHIRI and A. T. PRICE, *Phil. Trans. Roy. Soc. London* A, **237** (1939), 509.

11) K. TERADA, *Geophys. Mag.*, **13** (1939) 63, **16** (1948), 5.

12) T. RIKITAKE, *Trans. Oslo. Meeting I. U. G. G. A. T. M. E.* (1950), 435. *Bull. Earthq. Res. Inst.*, **28** (1950) 45, **29** (1951) 219 and 263.

shielding within the earth, which will be useful in checking the possibility of Elsasser and Bullards' interesting theories concerning the geomagnetic secular variation.

2. Theory.

We have not yet a concrete knowledge of the electrical properties in the core, whose upper boundary is situated, as determined from seismology, at a depth of 2900 *km* which is too deep for the electric currents induced by geomagnetic variation of external origin to reach as has been shown by the writer[12]. Though it is difficult to attack the problem by geophysical means, a number of investigators believes from the physical standpoint that the electrical conductivity should be appreciably high in the core, amounting to the order of 10^{-6} *emu* as estimated by Elsasser and Bullard. If it is supposed that the core is composed of iron, the conductivity would attain to such order through the effect of high temperature and pressure. Even if we take the view that the substance in the core assumes a state of high energy due to a phase transition of the silicates under enormous compression as suggested by W. H. Ramsey[13], a high conductivity is expected in the core. Taking into consideration again the fact that the conductivity increases with the increase in depth as shown by geophysical studies, it may be allowable to assume that the conductivity takes a high value in the core though the precise order is still unknown.

In that case, the varying magnetic field that appears outside the core should be due to the currents flowing in a thin layer just below the boundary of the core as has been pointed out by Elsasser[6]. He estimated for the variation with a period of 100 *years* that the currents flowing at a depth much larger than 50 *km* from the boundary do not practically produce a magnetic field outside the core wheareas it would be allowed to assume the current-sheet approximation. The writer, for the sake of mathematical simplicity, also considered here a current-sheet that corresponds to the source of geomagnetic secular variation.

As may be seen in Fig. 1, we will take an earth-model in which the electrical conductivity σ is equal to σ'[14], $\sigma_0\rho^{-l}$[15] and σ_c[16] respectively for the regions 1, 2

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[13] W. H. RAMSEY, *M. N. R. A. S.*, **108** (1948), 406. *M. N. R. A. S. Geophys. Suppl.*, **5** (1949), 409.

[14] As already stated in Introduction, σ' amounts to only 10^{-15} *emu* down to a depth of 400 *km*, the value being so small that we can regard the substance to be non-conducting for very slow variation such as secular one.

[15] Though the expression fits for the distribution up to a depth of about 1500 *km*, we tentatively assume that the expression is applicable throughout the mantle.

[16] The conductivity is assumed to be constant in the core for the sake of simplicity.

and 3. The magnetic permeability is assumed as unity everywhere. In the non-conducting region, we have magnetic potential of which the typical term may be written as

$$W_n^m = a i_n(t) \rho^{-n-1} S_n^m , \qquad (1)$$

where the external origin part is not taken into account since the secular variation originates in the earth. S_n^m denotes spherical surface harmonic having degree n and order m. In that case, the components of the magnetic field become

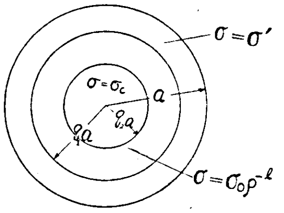

Fig. 1

$$\left.\begin{aligned} H_r &= (n+1)\rho^{-n-2} i_n S_n^m , \\ H_\theta &= -\rho^{-n-2} i_n \partial S_n^m/\partial\theta , \\ H_\phi &= -\rho^{-n-2} i_n \partial S_n^m/(\sin\theta\partial\phi). \end{aligned}\right\} \qquad (2)$$

On the other hand, we have in the conducting medium vector potential $\vec{A}$ which satisfies the relation

$$\nabla^2\vec{A} = 4\pi\sigma(\rho)\partial\vec{A}/\partial t \qquad (3)$$

for the phenomenon is assumed to be quasi-stationary. The typical term of the solutions of (3) becomes

$$\vec{A}_n^m = a f_n(t,\rho)\,[r \operatorname{grad} S_n^m], \qquad (4)$$

where f_n satisfies the differential equation

$$\partial(\rho^2\partial f_n/\partial\rho)/\partial\rho = \{n(n+1) + 4\pi a^2\sigma(\rho)\rho^2\partial/\partial t\} f_n . \qquad (5)$$

Thus the components of the magnetic field are given by

$$\left.\begin{aligned} H_r &= -n(n+1)\rho^{-1} f_n S_n^m , \\ H_\theta &= -\rho^{-1}\frac{\partial}{\partial\rho}(\rho f_n)\partial S_n^m/\partial\theta , \\ H_\phi &= -\rho^{-1}\frac{\partial}{\partial\rho}(\rho f_n)\partial S_n^m/(\sin\theta\partial\phi) . \end{aligned}\right\} \qquad (6)$$

The function f_n becomes respectively for the regions 2 and 3 as follows;

$$f_{n,2} = \rho^{-1/2}\{C_2 K_\nu(z) + D_2 I_\nu(z)\} \quad \text{for region 2,} \qquad (7)$$

where C_2 and D_2 are both functions of time, K_ν and I_ν denote modified Bessel functions while

$$\rho^{1-l/2} = (l-2)z/(2\zeta) , \ \nu = (2n+1)/(l-2) , \ \zeta^2 = 4\pi a^2\sigma_0 p \qquad (8)$$

and

$$f_{n,2} = C_3 \rho^n F_n(\rho) \quad \text{for region 3,} \qquad (9)$$

where C_3 also denotes a function of time. F_n was fully studied in the studies of electromagnetic induction within the earth as given in the well-known book "*G-oe* *magnetism*"[17]. In the above treatment, $\partial/\partial t$ is replaced by p as is usual in operational calculus.

Now, on assuming a current-sheet at $\rho=q_2$, ($q_2=0.545$), the typical term of the current-function being given by

$$J_n^m=K_n^m S_n^m\,, \tag{10}$$

we get the next conditions from the continuity of magnetic field at the boundary of the core or $\rho=q_2$

$$\{\partial(\rho f_{n,2})/\partial\rho\}_{\rho=q_2}-\{\partial(\rho f_{n,3})/\partial\rho\}_{\rho=q_2}=(4\pi/a)K_n^m\,, \tag{11}$$

$$(f_{n,2})_{\rho=q_2}=(f_{n,3})_{\rho=q_2}\,. \tag{12}$$

We have also

$$n(f_{n,2})_{\rho=q_1}=-i_n q_1^{-n-1} \tag{13}$$

$$\{\partial(\rho f_{n,2})/\partial\rho\}_{\rho=q_1}=i_n q_1^{-n-1}\,, \tag{14}$$

from the boundary conditions at $\rho=q_1$.

Then, solving the simultaneous equations (11), (12), (13) and (14), we get' with the aid of recurrence formulae, the relation

$$(4\pi/a)K_n^m=q_1^{-n-1}\frac{2n+1}{n}\,z_1\left\{\frac{z_2}{2\nu}\Big(I_{\nu+1}(z_2)K_{\nu+1}(z_1)-K_{\nu+1}(z_2)I_{\nu+1}(z_1)\Big)\right.$$
$$\left.+\frac{F_{n-1}(q_2)}{F_n(q_2)}\Big(I_\nu(z_2)K_{\nu+1}(z_1)+K_\nu(z_2)I_{\nu+1}(z_1)\Big)\right\}i_n\,,$$

where z_1 and z_2 respectively denote $z_{\rho=q_1}$ and $z_{\rho=q_2}$.

In case of periodic variation having a period $2\pi/\alpha$, the operator p is replaced by $i\alpha$ in the above expressions. When the period is larger than about 10 *years*, I_ν and K_ν are numerically calculated by use of ascending power series such as

$$\left.\begin{aligned}&I_\nu(z)=(\Phi_\nu+i\Psi_\nu)e^{i\nu\pi/4}\,,\\&K_\nu(z)=(\pi/2)\operatorname{cosec}\nu\pi\{(\Phi_{-\nu}+i\Psi_{-\nu})e^{-i\nu\pi/4}-(\Phi_\nu+i\Psi_\nu)e^{i\nu\pi/4}\}\,,\end{aligned}\right\} \tag{16}$$

where

$$\Phi_\nu=\sum_{s=0}^{\infty}\frac{(-1)^s(x/2)^{\nu+4s}}{(2s)!\,\Gamma(\nu+2s+1)}\qquad \Psi_\nu=\sum_{s=0}^{\infty}\frac{(-1)^s(x/2)^{\nu+4s+2}}{(2s+1)!\,\Gamma(\nu+2s+2)} \tag{17}$$

and

$$z=x\sqrt{i}\,.$$

When $x\ll1$, (16) becomes

$$(4\pi/a)K_n^m=q_2^{-n-1}\frac{2n+1}{n}\left[\frac{F_{n-1}(q_2)}{F_n(q_2)}\right.$$
$$\left.+\frac{i}{\nu(\nu+1)}\left\{(x_2/2)^2+(q_2/q_1)^{2n+1}\left(\frac{F_{n-1}(q_2)}{F_n(q_2)}-1\right)(x_1/2)^2\right\}\right]i_n\,. \tag{19}$$

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[17] S. CHAPMAN and J. BARTELS, *loc. cit.*, p. 738.

Further, if σ_c is supposed to be zero, (19) becomes for $\sigma_c=0$

$$(4\pi/a)K_n^m = q_2^{-n-1}\frac{2n+1}{n}\, i_n\,. \tag{20}$$

because F_{n-1}/F_n tends to 1 as σ_c approaches zero. As easily seen, (20) is just the same with the relation between the current and the magnetic potential when the conductivity is considered everywhere null. If we denote the coefficient of magnetic potential in that special case by $i_{n,0}$, the quantity $i_n/i_{n,0}$ is convenient for studying the shielding effect.

On the other hand, I_ν and K_ν are obtaind for smaller period by asymptotic expansions such as

$$\left.\begin{aligned} I_\nu(z)&=(2\pi z)^{-1/2}\, e^{z}\left\{1-\frac{4\nu^2-1^2}{1!8z}+\frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8z)^2}-\ldots\ldots\right\},\\ K_\nu(z)&=(\pi/2z)^{-1/2}\, e^{-z}\left\{1+\frac{4\nu^2-1^2}{1!8z}+\frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8z)^2}+\ldots\ldots\right\},\end{aligned}\right\} \tag{20}$$

while F_{n-1}/F_n is calculated by

$$\frac{F_{n-1}(x)}{F_n(x)}=\frac{2n+1}{x}\left\{1+\frac{n}{x}+\frac{n(n+1)}{2x^2}+\frac{n(n+1)}{2x^3}+\ldots\ldots\right\}, \tag{21}$$

and

$$F_n(x)=1+\frac{x^2}{2(2n+3)}+\frac{x^4}{2\cdot 4(2n+3)(2n+5)}+\ldots\ldots \tag{22}$$

respectively for $|x|=\sqrt{2\pi\sigma_c\alpha}\,q\,a>1$ and $|x|<1$. By use of these expressions, *mod.* $i_n/i_{n,0}$ are calculated for various σ_c as shown in Fig. 2 where n is taken to be 1.

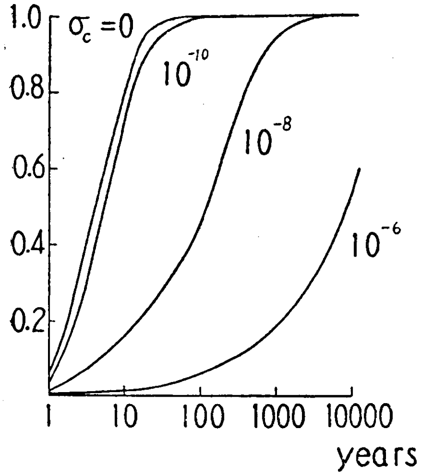

Fig. 2. The shielding effect for secular variation with various period.

3. Discussions

As seen in the figure, periodic variations having smaller period than 1 *year* seem to be almost shielded by the conducting mantle, only 5 *per cent* of its amplitude of 1 *year* period variation being expected at the earth's surface even in the case of non-conducting core. The larger the period is, the smaller becomes the shielding effect. Though the variation whose period is larger than sevral hundred *years* is partly shielded, it may be observed on the earth's surface if the conductivity amounts to the order of 10^{-6} *emu* or less in the core. Hence the geomagnetic secular change having period of several hundred *years* or more may not be shielded perfectly and will penetrate up to the earth's surface provided the action which is considered by Elsasser and Bullard is really occurring in the core. At least, it may be said that the existence of rapid geomagnetic variation having a period of 1 *year* or less is by no means likely.

4. Conclusion.

In order to estimate the shielding effect in the earth's interior to the hypothetical secular change, the writer has executed a calculation on the basis of the conductivity-distribution in the mantle which was previously obtained by the writer himself.

Since, however, the electrical property in the earth's core is ambiguous, no definite conclusion could be obtained except the fact that variation with period smaller than about 1 *year* should be almost shielded. But, it is likely that a part of secular variation with a period larger than several hundred *years* appears on the earth's surface even in case of appreciably high conducting core. Thus, if we assume the action in the core which is taken into account by Elsasser and Bullard, a part of the varying field would be observed on the earth's surface. Taking into consideration the actually observed variation, the result obtained here seems favourable to the hypothetical theories ventured by Elsasser and Bullard.

In conclusion, the writer wishes to express his hearty thanks to Dr. Hans G. Macht for his encouragement in this study.

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20. 地磁氣永年變化に對する遮蔽効果

地震研究所　　力武常次

近年 Elsasser や Bullard によつて，地球内核中の流體運動と磁場との相互作用として，地磁氣永年變化の發生を説明しようという試みが行なわれている．一方筆者は地磁氣變化の解析より，1500 *km* 程度の深さまでの電氣傳導度分布を決定したが，その結果を利用して上述のような機構で永年變化が起るとしたときのシールディングを計算した　その結果は周期 1 年程度より短い變化はほとんど完全に遮蔽されて地表では觀測出來ないことになる．また數百年より長周期のものは大體に於いて遮蔽の程度が少なく，實際に觀測される永年變化を考えるとき，Elsasser や Bullard の云うような機構で永年變化が起つてもよいという結果を得た．
