21. *Changes in Earth Current and their Relation to the Electrical State of the Earth's Crust.*

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(Read Oct. 18, 1949.—Received Mar. 20, 1951.)

1. In the previous papers[1), 2)], the writer has studied electromagnetic induction in semi-infinite earth. It was shown in those papers that the horizontal components of the electric and magnetic forces at the earth's surface, perpendicular to each other, are connected by the next operational equation

$$E=\sqrt{\frac{\mu p}{\pi\sigma}}H \qquad (1)$$

where μ and σ denote respectively magnetic permeability and electrical conductivity. Interpreting the operational equation, we find out that, in the case of periodic variation, (1) the amplitude ratio E/H is proportional to the inverse square root of the period, the so-called Terada's law[3)] being substantiated, and (2) the phase difference is always kept to be $\pi/4$.

According to H. Hatakeyama and M. Hirayamas' study[4)] on the observed results at Toyohara, however, the phase difference between the changes in the EW-component of the earth potential and in the horizontal one of the earth's magnetic field does not follow the above-mentioned simple relation. The phase difference becomes as small as 20°–30° for the changes of comparatively long period, though the difference seems to amount to about 45° for small period as shown in the writer's previous studies. In order to explain the results, Hatakeyama assumed a suitable distribution of the variation over the earth's surface. In this paper, an interpretation will be attempted from another stand-point.

As a matter of fact, the influence of the electrical property of the deep region of the earth's crust on the earth current which is observed at the earth's surface has never been quantitativery discussed. Studying the electromagnetic induction within the earth covered by a superficial layer, the present paper will deal with that

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1) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **24** (1946), 1.
2) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **25** (1947), 9.
3) T. TERADA, *Journ. Col. Sci. Tokyo Imp. Univ.*, **37** (1917), Art. 9,56.
4) H. HATAKEYAMA and M. HIRAYAMA, *Journ. Meteorol. Soc. Japan* **12**, (1934), 449.

problem. Recently, K. Hirao[5] studied the above-mentioned influence, though not generally, in connexion with the anisotropy of the electrical conductivity of the earth's crust. As will be shown later, the writer's study includes Hirao's result as an extreme case.

2. According to the theory[6] of electromagnetic induction in a sphere which has an inner core covered by an outside layer of different conductivity, the typical term of the north component of the magnetic field at the earth's surface and the west component of the electric field in the outside layer are respectively given by the next expression in spherical coordinate,

$$H=\{1+I(p)\}\mathbf{e}_n^m \partial S_n^m/\partial\theta, \tag{2}$$

$$E=ap\{\bar{C}_2(p)\rho^n F_n(k_2\rho a)+\bar{D}_2(p)\rho^{-n-1}G_n(k_2\rho a)\}\mathbf{e}_n^m \partial S_n^m/\partial\theta, \tag{3}$$

where $\mathbf{e}_n^m$, S_n^m, a and ρ denote respectively the coefficient of the external part of the magnetic potential, surface spherical harmonic of degree n and order m, the earth's radius and the ratio of the radial distance to the earth's radius. The conductivity and the boundaries are shown in Fig. 1. In the expressions, I, $\bar{C}_2$ and $\bar{D}_2$ are determined by the boundary conditions being given by

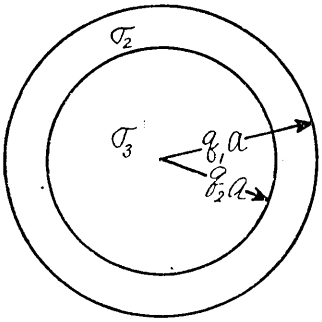

Fig. 1.

$$I(p)=-q_1^{2n+1}\frac{n}{n+1}\frac{\left\{1-\frac{F_{n-1}(2,1)}{F_n(2,1)}\right\}\left\{1-\frac{F_{n-1}(3,2)}{F_n(3,2)}-\frac{G_{n+1}(2,2)}{G_n(2,2)}\right\}+\tau^{2n+1}\left\{\frac{F_{n-1}(3,2)}{F_n(3,2)}-\frac{F_{n-1}(2,2)}{F_n(2,2)}\right\}\frac{F_n(2,2)}{F_n(2,1)}\frac{G_n(2,1)}{G_n(2,2)}}{\frac{F_{n-1}(2,1)}{F_n(2,1)}\left\{1-\frac{F_{n-1}(3,2)}{F_n(3,2)}-\frac{G_{n+1}(2,2)}{G_n(2,2)}\right\}+\tau^{2n+1}\left\{1-\frac{G_{n+1}(2,1)}{G_n(2,1)}\right\}\left\{\frac{F_{n-1}(3,2)}{F_n(3,2)}-\frac{F_{n-1}(2,2)}{F_n(2,2)}\right\}\frac{F_n(2,2)}{F_n(2,1)}\frac{G_n(2,1)}{G_n(2,2)}},$$

$$\bar{C}_2(p)=\frac{1}{n+1}\frac{1}{F_n(2,1)}\frac{1-\frac{F_{n-1}(3,2)}{F_n(3,2)}-\frac{G_{n+1}(2,2)}{G_n(2,2)}}{''},$$

$$\bar{D}_2(p)=q_2^{2n+1}\frac{1}{n+1}\frac{F_n(2,2)}{F_n(2,1)}\frac{\frac{F_{n-1}(3,2)}{F_n(3,2)}-\frac{F_{n-1}(2,2)}{F_n(2,2)}}{''}. \tag{4}$$

5) K. HIRAO, *Zisin* 2nd ser. **1** (1948), 12.
6) T. RIKITAKE, *Trans. Oslo Meeting, I.U.G.G. A.T.M.E.*, (1948), 435.
Kagaku **18** (1948), 469.
Bull. Earthq. Res. Inst., **28** (1950), 45 and **29** (1951), 219 and 263.

where

$$k_\nu^2 = 4\pi\sigma_\nu p$$

and μ is assumed to be unity. For the sake of simplicity, $F_n(k_\nu q_\mu a)$ and $G_n(k_\nu q_\mu a)$ are written respectively as $F_n(\nu,\mu)$ and $G_n(\nu,\mu)$ in the above expressions. We also wrote τ in place of q_2/q_1. F_n and G_n are modified Bessel functions whose nature has been fully investigated in the well-known book "*Geomagnetism*"[7].

With these expressions, the relations between E and H can be obtained for given σ's and q's. If a is supposed to be large, the asymptotic expansions for F_n and G_n are available. After some calculations, we get

$$E = \frac{p}{\rho k_2} e^{-k_2(1-\rho)a} \frac{k_2+k_3+(k_2-k_3)e^{-2k_2(\rho-q_2)a}}{k_2+k_3-\dfrac{n+1}{2n+1}(k_2-k_3)e^{-k_2(1-q_2)a}} H, \tag{5}$$

where the relation $q_1=1$ is taken into account.

Putting $\rho=1$, then, the relation between E and H at the surface is written as

$$E = \frac{p}{k_2} \frac{k_2+k_3+(k_2-k_3)e^{-2k_2 D}}{k_2+k_3-\dfrac{n+1}{2n+1}(k_2-k_3)e^{-2k_2 D}} H \; [8] \tag{6}$$

where D denotes the thickness of the outside layer. Making $n \to o$, (6) becomes the same with that obtained by Hirao.

In the case of a periodic variation of period T, the amplitude ratio and phase difference become respectively as follows:

$$\left.\begin{aligned}
A &= \frac{1}{\sqrt{2\sigma_2 T}} \\
&\times \left\{ \frac{1+2Ke^{-4\pi D\sqrt{\sigma_2/T}}\cos(4\pi D\sqrt{\sigma_2/T}) + K^2 e^{-8\pi D\sqrt{\sigma_2/T}}}{1-\dfrac{2\sqrt{2}(n+1)}{2n+1}Ke^{-4\pi D\sqrt{\sigma_2/T}}\sin(\pi/4-4\pi D\sqrt{\sigma_3/T}) + \left(\dfrac{n+1}{2n+1}\right)^2 K^2 e^{-8\pi D\sqrt{\sigma_2/T}}} \right\}^{1/2}, \\
\psi &= \tan^{-1} \frac{1+\sqrt{2}Ke^{-4\pi D\sqrt{\sigma_2/T}}\sin(\pi/4-4\pi D\sqrt{\sigma_2/T})}{1+\sqrt{2}Ke^{-4\pi D\sqrt{\sigma_2/T}}\sin(\pi/4+4\pi D\sqrt{\sigma_2/T})}, \\
&\quad -\tan^{-1} \frac{\dfrac{n+1}{2n+1}Ke^{-4\pi D\sqrt{\sigma_2/T}}\sin(4\pi D\sqrt{\sigma_2/T})}{1-\dfrac{n+1}{2n+1}Ke^{-4\pi D\sqrt{\sigma_2/T}}\cos(4\pi D\sqrt{\sigma_2/T})},
\end{aligned}\right\} \tag{7}$$

7) S. CHAPMAN and J. BARTELS, *Geomagnetism* Vol. II, p. 738.

8) In the case of a sphere of uniform conductivity, the relation can be written as follows:

$$E = \frac{p}{k} e^{-ka(1-\rho)} H$$

which is the same with that obtained by the writer in the studies of the electromagnetic induction within semi-infinite earth.

where

$$K = (\sqrt{\sigma_2} - \sqrt{\sigma_3})/(\sqrt{\sigma_2} + \sqrt{\sigma_3})$$

Thus we can obtain the amplitude ratio and phase difference for given n, σ_2, σ_3, D and T.

3. In the first place, n is assumed to be unity. Then, A and ψ are calculated for certain combinations of σ_2 and σ_3. Their changes with increase of the period

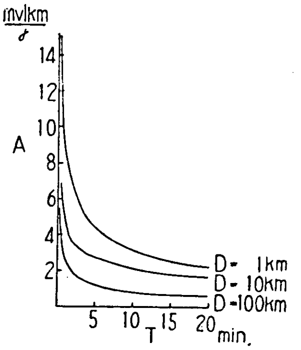

Fig. 2a. Amplitude ratio for $\sigma_2=10^{-13}emu$ and $\sigma_3=10^{-15}emu$.

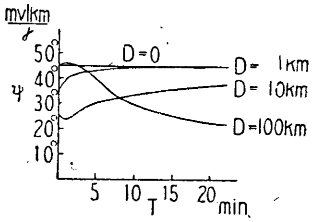

Fig. 2b. Phase difference for $\sigma_2=10^{-13}$ emu and $\sigma_3=10^{-15}emu$.

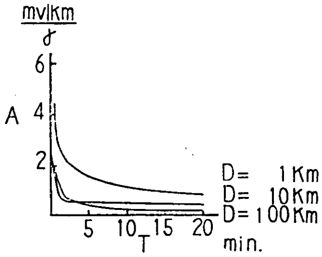

Fig. 3a. Amplitude ratio for $\sigma_2=10^{-12}$ emu and $\sigma_3=10^{-15}$ emu.

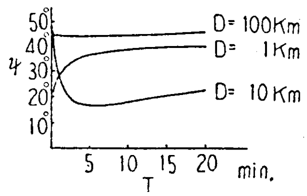

Fig. 3b. Phase difference for $\sigma_2=10^{-12}$ emu and $\sigma_3=10^{-15}$ emu.

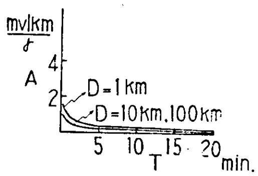

Fig. 4a. Amplitude ratio for $\sigma_2=10^{-11}$ emu and $\sigma_3=10^{-15}$ emu.

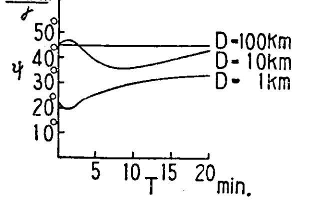

Fig. 4b. Phase difference for $\sigma_2=10^{-11}$ emu and $\sigma_3=10^{-15}$ emu.

are shown in Figs. 2, 3 and 4 where D is taken to be 1, 10 and 100 km. As seen in the figures, the phase difference varies remarkably with the period in some cases though the amplitude ratio is always roughly proportional to $1/\sqrt{T}$. Hence, the best combination of σ_2, σ_3 and D by which we can explain the actually observed $A-T$ and $\psi-T$ curves may be determined. From this point of view, Hatakeyama and Hirayamas' results can be interpreted as the electromagnetic induction within the earth covered by a layer having the conductivity amounting to the order of 10^{-12} *emu* while the thickness of it seems to amount to a few *kilometers*. Since we have made few observations, however, detailed studies in regard to the actual earth will be postponed.

It must be studied, in the next place, how far A and ψ change with n. In

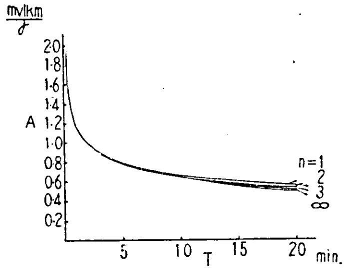

Fig. 5a. Amplitude ratio for various n.

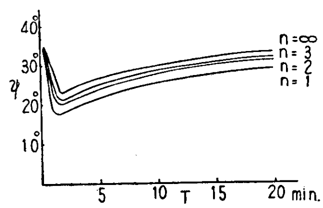

Fig. 5b. Phase difference for various n.

Fig. 5, assuming $\sigma_2=10^{-12}emu$, $\sigma_3=10^{-15}emu$ and $D=5\,km$, A and ψ are calculated for different n. As seen in the figures, the effect of n is rather small. Physically saying, the influence of the wave-length of the variation is not so large as far as we treat the problems as quasi-stationary phenomena.

4. Studying the electromagnetic induction within the earth covered by an outside layer of different conductivity, we found out that the phase difference between magnetic and electric forces was seriously affected by the electrical state of the earth's crust. To sum up, it will be possible to determine approximately the electrical state near the earth's surface by analysing the changes in earth current and geomagnetic field.

21. 地電流變化と地下の電氣的性質

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前論文において，半無限大地中の電磁感應をしらべたが，その際互に直角な地磁氣 H および地電位差 E の成分間には

$$E=\sqrt{\frac{\mu p}{4\pi\sigma}}\,H$$

の如き關係が成立し，周期的變化の場合には振幅比 E/H は周期の自乘根に逆比例し，位相差は常に 45° となることがわかつた．

一方實際に觀測された結果をみると，位相差は周期によつて變化するようにみえる．この事實を說明するために，異なつた電氣傳導度をもつ層に被われた地球内の電磁感應を論じ，半徑を無限大とした極限において，前論文と比較した．その結果，位相差と周期の關係は電氣傳導度のとりかたによつて著しく變化することがわかつた．したがつて地磁氣および地電流變化を同時に觀測することにより，地下の電氣的構造が或程度推測出來ることになる．