22. *The Form of Tsunami-waves outside a Bay as Inferred from the Motion of Bay-water.*

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(Read Sept. 20, 1949.—Received Mar. 31,1951.)

1. The motion of bay-water caused by tsunami-waves had been studied in detail by the members of the Institute[1),2),3),4)]. Among these studies, R. Takahasi investigated the motion of sea-water in a rectangular bay of uniform depth. He showed that the motion outside a bay could be inferred from the motion within the bay.

In this paper, a method of obtaining the motion outside a bay from the motion of bay-water will be described, the method being applicable to a bay of variable section.

2. According to the theory of long wave in a rectangular canal of uniform depth, the elevation of surface satisfies the differential eqation of the type

$$\frac{d^2\eta}{dx^2}-\frac{p^2\eta}{gh}=0 \tag{1}$$

where p denotes the operator $\partial/\partial t$. g and h denote respectively the acceleration of gravity and depth.

The solution by which $\partial\eta/\partial x$ becomes 0 at the head of the bay $(x=0)$ and $\eta=u(t)$ at the mouth $(x=a)$ is given by

$$\eta=\frac{\cosh(px/\sqrt{gh})}{\cosh(pa/\sqrt{gh})}\psi(p), \tag{2}$$

where $\psi(p)$ denotes the operational form of $u(t)$, that is the original form of tsunami-waves outside the bay. If the operational form of $\eta(t)$ is supposed to be $N(p)$, we have the operational equation

$$u(t)=\frac{\cosh(pa/\sqrt{gh})}{\cosh(px/\sqrt{gh})}N(p). \tag{3}$$

By solving (3), we can infer the tsunami-waves out of the bay.

Generally speaking, the method just mentioned is equivalent to the method

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1) G. NISHIMURA and K. KANAI, *Bull. Earthq. Res. Inst. Suppl. Vol.* **1**, Part I (1934), 182.

2) G. NISHIMURA, K. KANAI and T. TAKAYAMA, *Bull. Earthq. Res. Inst.*, **31**(1935), 46

3) R. TAKAHASI, *Bull. Earthq. Res. Inst.*, **25** (1947), 1.

4) T. RIKITAKE and S. MURAUCHI, *Bull. Earthq. Res. Inst.*, **25** (1947), 21.

of the case wherein $u(t)$ is given as a solution of an integral equation, the equation being obtained by the well-known Stokes' method.

In practice, it is convenient to expand the righthand-side of (3) into the form

$$u(t)=\{exp[-p(x-a)/\sqrt{gh}]-exp[-p(3x-a)/\sqrt{gh}]+exp[-p(5x-a)/\sqrt{gh}]-\cdots +exp[-p(x+a)/\sqrt{gh}]-exp[-p(3x+a)/\sqrt{gh}]+exp[-p(5x+a)/\sqrt{gh}]-\cdots\}N(p). \quad (4)$$

Then interpreting term by term, we get

$$u(t)=\sum_{n=0}^{\infty}(-1)^n\Phi\{t-(2n+1)x/\sqrt{gh}+a/\sqrt{gh}\}, \quad (5)$$

where

$$\Phi(t)=\eta(t-2a/\sqrt{gh}). \quad (6)$$

The result agrees with R. Takahasi's. But he got a more general expression that is applicable to imperfect reflections at the head and mouth of the bay.

In the special case where $x=0$, it becomes

$$u(t)=(1/2)\{exp(-pa/\sqrt{gh})+exp(pa/\sqrt{gh})\}N(p) \quad (7)$$

or

$$u(t)=(1/2)\{\eta_0(t-a/\sqrt{gh})+\eta_0(t+a/\sqrt{gh})\}. \quad (8)$$

3. In the next place, we shall study a bay whose breadth and depth vary respectively as

$$b=b_0x/a \text{ and } h=h_0x/a. \quad (9)$$

In many cases, this approximation seems to be the most appropriate one for actual bays.

In that case, the differential equation becomes

$$\frac{d^2\eta}{dx^2}+\frac{2}{x}\frac{d\eta}{dx}-\frac{ap^2}{gh^2}\frac{\eta}{x^2}=0 \quad (10)$$

the solution of which is given by

$$\eta=\sqrt{a/x}\,\frac{I_1(2p\sqrt{ax/gh_0})}{I_1(2p\sqrt{a^2/gh_0})}\psi(p). \quad (11)$$

provided $\partial\eta/\partial x=0$ at $x=0$ and $\eta=u(t)$ at $x=a$. Hence, we have

$$u(t)=\sqrt{x/a}\,\frac{I_1(2p\sqrt{a^2/gh_0})}{I_1(2p\sqrt{ax/gh_0})}N(p). \quad (12)$$

We shall assume, for the sake of simplicity, that $\eta(t)$ or $N(p)$ is given at the head of the bay. In that case (12) becomes

$$u(t)=I_1(2p\sqrt{a^2/gh_0})N(p)/(p\sqrt{a^2/gh_0}). \quad (13)$$

As proved in the footnote[5], we get

$$I_1(pA)=\begin{cases}0 & \text{for } t<-A \quad ,\\ (1/\pi A)\sqrt{A^2-t^2} & \text{for } -A<t<A \ ,\\ 0 & \text{for } A<t \quad ,\end{cases} \tag{14}$$

where

$$A=2\sqrt{a^2/gh_0}.$$

Consequently, adopting Borel's theorem, $u(t+A)$ becomes zero for $t<0$ and

$$u(t+A)=\begin{cases}(2/\pi A^2)\displaystyle\int_A^{t+A}\sqrt{A^2-(t-\tau)^2}\eta_0(\tau)d\tau & \text{for } 0<t<2A,\\ (2/\pi A^2)\displaystyle\int_{t-A}^{t+A}\sqrt{A^2-(t-\tau)^2}\eta_0(\tau)d\tau & \text{for } 2A<t.\end{cases} \tag{15}$$

Thus the motion of sea-water outside the bay can be obtained from that at the head of bay.

4. As examples of the application of the method to an actual bay, the tsunamis of Aug. 5, 1897 and Sept. 8, 1918 in Ayukawa Bay, Miyagi Prefecture are studied as follows.

As shown in Fig. 1, the approximate values of a and h_0 is respectively given by $a=1.3\,km$ and $h_0=16\,m$. The tide-gauge was installed at the head of the bay.

The mareograms[6] are reproduced in Figs. 2 and 3. The disturbances by the tsunami-waves are also shown in the figures, where the tide is eliminated by mean

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5) Let us consider a function

$$f(t)=\begin{cases}\sqrt{A^2-(t-A)^2} & \text{for } t<2A,\\ 0 & \text{for } 2A<t.\end{cases}$$

Then, Carson's integral of $f(t)$ becomes

$$\begin{aligned}p\int_0^\infty e^{-pt}f(t)dt &= p\int_0^{2A} e^{-pt}\sqrt{A^2-(t-A)^2}dt\\ &= pe^{-pA}\int_{-A}^{A} e^{-pt}\sqrt{A^2-t^2}dt\\ &= 2A^2pe^{-pA}\int_0^{\pi/2}\cosh(pA\cos\theta)\sin^2\theta d\theta\\ &= 2A^2pe^{-pA}\times\frac{\pi I_1(pA)}{2pA}\\ &= \pi Ae^{-pA}I_1(pA),\end{aligned}$$

whence we have

$$I_1(pA)=\begin{cases}0 & \text{for } t<-A,\\ (1/\pi A)\sqrt{A^2-t^2} & \text{for } -A<t<A,\\ 0 & \text{for } A<t.\end{cases}$$

6) A. IMAMURA and M. MORIYA, *Jap. Journ. Astr. Geophys.*, **17** (1939), 119.

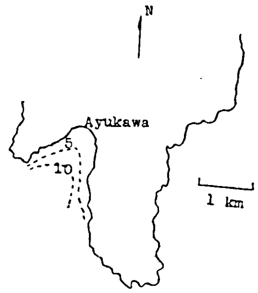

Fig. 1. Topography of Ayukawa Bay.

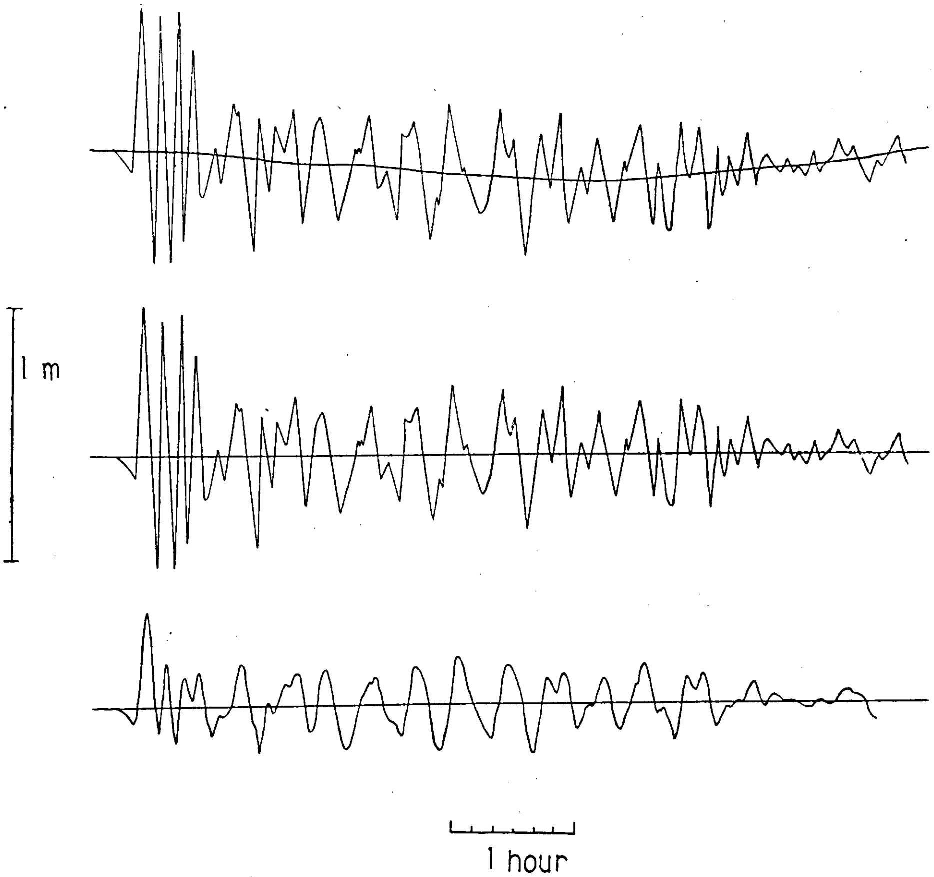

Fig. 2. The tsunami of Aug. 5, 1897 in Ayukawa Bay. Upper: Original record of tide gauge. Middle: Disturbance of bay-water from which the tide is eliminated. Lower: The elevation of sea-water at the mouth of the bay which is calculated by the present method.

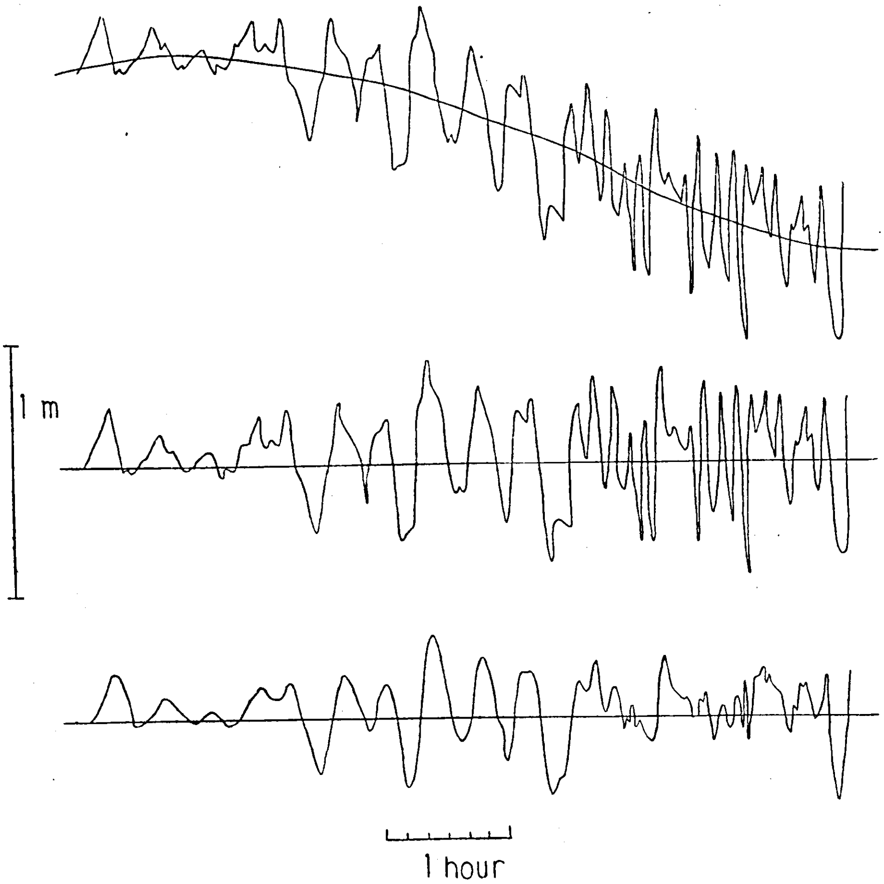

Fig. 3. The tsunami of Sept. 8, 1918 in Ayukawa Bay. Upper: Original record of tide gauge. Middle: Disturbance of bay-water from which the tide is eliminated. Lower: The elevation of sea-water at the mouth of the bay] which is calculated by the present method.

of running-average. Then, applying (15), the elevations at the mouth of the bay are calculated by means of numerical integration, the results being shown in the figures.

The decrease of the amplitudes at the mouth is clearly shown in the figures, especially in the case of the tsunami of Aug. 5, 1897 which was caused by an earthquake about 100 *km* away. On the contrary, the influence of the bay is not so remarkable in the case of the tsunami of Sept. 8, 1913 which came from Uruppu Island situated at a distance of more than 1000 *km*. Taking into consideration the fact that the period of the seiche of the bay amounts to several minutes, it is natural that the shorter the period of the tsunami-waves is, the greater the influence of the bay on the wave-forms will be.

As shown in the figures, however, the oscillatory motion of sea-water is so remarkable that even the influence of the bay is eliminated. Hence, we must take into account seriously the reflections of the tsunani-waves at various parts of the sea.

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22. 灣水の運動と灣外に於ける津浪の波形

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演算子法を用いて，灣内の水の運動より灣外に於ける津浪の波形をしらべる方法を考案した．一様な深さをもつ矩形の灣については，高橋教授の得られた結果に一致する．ここでは深さおよび幅が直線的に變化する灣についても考察し，鮎川灣の検潮記錄について解析を行つた．
