34. *Destruction System of Building by Earthquake.*

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1. Introduction.

Buildings are destroyed most easily when the period of earthquake motion approximates to the natural period of the buildings, in other words when the vibration state of buildings comes near the resonance condition.

At the time of earthquake, if a building is at the resonance condition and destruction occurs anywhere in it, its proper period becomes longer. Therefore, when the period of earthquake motion remains constant, the building will be out of the resonance condition. As to the phenomena following, we may consider two cases.

(1) Since the building comes out of resonance condition, the vibration amplitude or the strain of the building will diminish so that the destruction will advance no further, or

(2) though the building comes out of resonance condition, the stiffness of building diminishes. Therefore the vibration amplitude or the strain becomes rather larger and the destruction will advance.

The influence of the building damping is the cause of these two different cases. In the present study, the condition which causes these phenomena practically is pursued through theoretical investigation, and applying the results of this investigation to practical cases, the earthquake damage of building is examined.

2. The influence of the inner-damping of building.

First, in the case of the single-storeyed Rahmen structure with the assumption that the beam is of infinite rigidity and the mass of column is concentrated on the floor, we studied, taking the various coefficients of solid viscosity of column, how the maximum of bending moment varies in accordance with the decrease of the column stiffness or the change of its connection to the beam. In this case the equation of column motion is

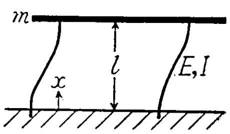

Fig. 1.

$$\frac{\partial^4 y}{\partial x^4}=0, \quad \text{.......................}(1)$$

and its solution is

$$y=(A+Bx+Cx^2+Dx^3)e^{ipt}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2)$$

where A, B, C, and D are the arbitrary constants to be determined by boundary conditions, and $p=2\pi/T$, $T=$period.

As the boundary conditions

$$x=0; \qquad y=\begin{cases}\mathfrak{A}e^{ipt}, \\ 0,\end{cases} \qquad \frac{\partial y}{\partial x}=0, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3),\ (4)$$

$$x=l; \qquad \left(EI+\xi I\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial x^2}=-K\frac{\partial y}{\partial x}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$m\frac{\partial^2 y}{\partial t^2}-\left(EI+\xi I\frac{\partial}{\partial t}\right)\frac{\partial^3 y}{\partial x^3}=\begin{cases}0, \\ Fe^{ipt},\end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where l, E, I and ξ denote the height, Young's modulus, moment of inertia and coefficient of solid viscosity of column respectively, while m is the concentrated mass and K the constant determined by the fixing condition of the column to the beam.

Putting equation (2) into (3)～(6), we get

$$y=\mathfrak{A}e^{ipt}+\frac{p^2 m\mathfrak{A}e^{ipt}}{E_0 I_0 \dot{\Phi}}[-2\{(\eta+\kappa)+i\tau\eta\sqrt{\gamma}\}x^2+3\{(2\eta+\kappa)+2i\tau\eta\sqrt{\gamma}\}x^3], \quad (7a)$$

$$y=\frac{Fe^{ipt}}{E_0 I_0 \dot{\Phi}}[\quad '' \quad], \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

where

$$\left.\begin{aligned}\dot{\Phi}&=\{12\eta(\eta+\kappa)-\gamma(4\eta+\kappa+12\tau^2\eta^2)\}+4i\tau\eta\sqrt{\gamma}\{3(2\eta+\kappa)-\gamma\}, \\ \gamma&=\frac{mp^2l^3}{E_0I_0}, \quad \eta=\frac{EI}{E_0I_0}, \quad \kappa=\frac{Kl}{E_0I_0}, \quad \tau=\frac{\xi}{E}\sqrt{\frac{E_0I_0}{ml^3}}.\end{aligned}\right\} \quad \ldots\ldots (8)$$

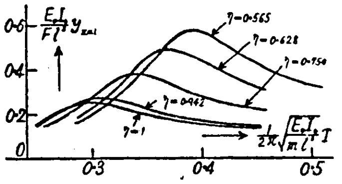

Fig. 2. The resonance curves for the various cases of stiffness of columns, under the condition of the clamped fixing between beam and column, that is to say, $\kappa(=Kl/E_0I_0)=\infty$, and $\tau(=\xi/E\sqrt{E_0I_0/ml^3})=0.1$

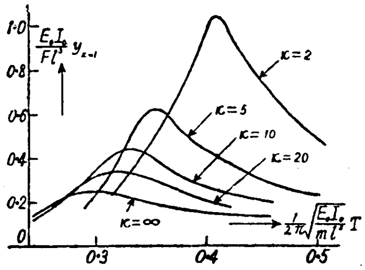

Fig. 3. The resonance curves for the various types of the fixing between beam and column, under the condition $\eta(=EI/E_0I_0)=1$ and $\tau(=\xi/E\sqrt{E_0I_0/ml^3})=0.1$

Using equation (7b), we plot in Fig. 2 the resonance curve, keeping the fixing of beam to column in the state of clamp and varying the stiffness of column. Fig. 3 illustrates the resonance curve at varying condition of fixing between the beam and the column of which the stiffness is kept constant. It is already known that the natural period of building becomes longer and the resonance amplitude larger as the stiffness of column diminishes or when the fixing of the column to the beam is not in good state. In Figs. 2 and 3, if the resonance curve of non-damaged building $\eta=1$ and $\kappa=\infty$, the abscissa 0.3 corresponds to the proper period. When the ordinate represents the ratio of the amplitude of the other curve corresponding to abscissa 0.3 in Figs. 2 and 3 to the resonance amplitude of non-damaged building and the abscissa represents the ratio of the natural period of damaged building (T) to that of the non-damaged one (T_0), the function between them are shown in Fig. 4. These figures show that when the earthquake motion is of constant period and the building is destroyed at the resonance condition, its natural period becomes longer and the building comes out of resonance condition, nevertheless that destruction advances further is a possible phenomenon within a certain range ($T/T_0<1.17$ in case of K=constant, and $T/T_0<1.23$ in case of EI=constant) without the vibration amplitude being diminished.

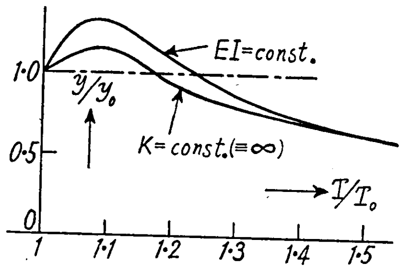

Fig. 4.

Next, the case where the fixing of column to the beam forms a perfect clamp ($K\to\infty$) was studied. Fig. 5 represents the relation of the following two ratios at varying coefficients of solid viscosity: the ratio of the bending moment in the case where the external force of the same period is given in the state of diminished stiffness to that under resonance condition, and the ratio of the natural period of the building with diminished stiffness to that of yet undamaged building.

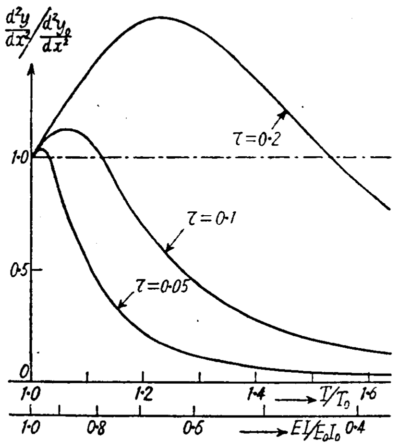

Fig. 5.

From Fig. 5, we have to clarify the relation between the coefficient of solid viscosity and the range within which the ratio of the value of bending moment of damaged building to that at the resonance condition of undamaged building, in case the period of external force is constant, is more than 1. This is shown in Fig. 6.

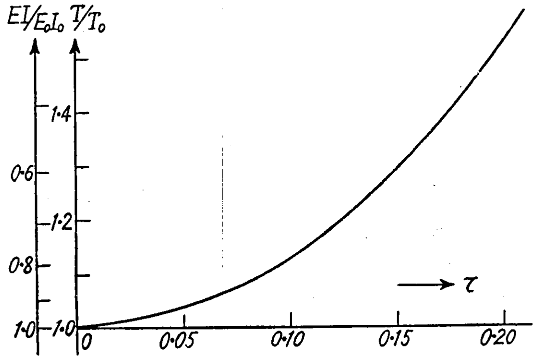

Fig. 6. Range within which destruction will advance.

It may be concluded from Fig. 6 that if a building with small damping diminishes in its stiffness and comes out of the resonance condition even slightly, the bending moment of column becomes smaller than in the resonance condition, but in case of large damping, such a phenomenon does not occur unless the stiffness diminishes considerably and the state of building is far from the resonance condition of original building. In other words, in case the damping of a building is large, although the building may not be destroyed so easily due to its small resonance amplitude, once begun, the destruction advances rapidly. On the other hand, in the case of small damping, there is much possibility of the destruction occurring only sectionally.

3. The influence owing to the vibration energy of building which dissipates to the ground as elastic waves.

Since the vibration damping of a comparatively heavy building is more likely to be caused by its vibration energy which dissipates to the ground as elastic waves than by the solid viscosity of the material, we made the following theoretical investigation, taking this fact into consideration.

As was shown in a previous paper[1], the motion of a tall building with rigid floors subjected to horizontal oscillation of the ground is analogous to the case of

1) K. SEZAWA and K. KANAI, "Some New Problems of Free Vibrations of Structure", *Bull. Earthq. Res.Inst.*, **12** (1934), 819.

shearing vibrations of a simple structure, so we shall deal with the vibration problem of a tall structure with rigid floors subjected to incident transverse waves under the dissipation of vibrational energy in the form of elastic waves into the ground.

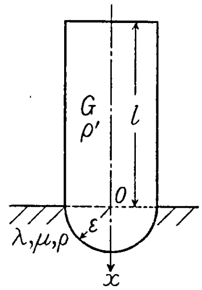

Fig. 7.

Let the incident transverse waves with their displacements orientated vertically be

$$u_0=\cos(pt+kx). \quad \text{..................}(9)$$

The final solution of the stress in a structure is expressed by[2)]

$$S=4\pi\varepsilon^2Gk'\sqrt{\frac{\Gamma_1^2+\Gamma_2^2}{P^2+Q^2}}\sin k'(x+l)$$

$$\cdot\cos\left(pt+\tan^{-1}\frac{\Gamma_2}{\Gamma_1}-\tan^{-1}\frac{Q}{P}\right), \quad (10)$$

where

$$\left.\begin{aligned}
&P=2\Gamma_1\cos k'l+\frac{3G\varepsilon}{\mu l}\Lambda_1 k'l\sin k'l,\\
&Q=2\Gamma_2\cos k'l+\frac{3G\varepsilon}{\mu l}\Lambda_2 k'l\sin k'l,\\
&\Gamma_1=3\left(\frac{\lambda}{\mu}+2\right)+\nu(k'l)^2\left(\frac{\lambda}{\mu}-3\sqrt{\frac{\lambda}{\mu}+2}\right),\\
&\Gamma_2=\sqrt{\nu}\,k'l\left\{3\left(\frac{\lambda}{\mu}+2+\sqrt{\frac{\lambda}{\mu}+2}\right)+\nu(k'l)^2\left(\sqrt{\frac{\lambda}{\mu}+2}-2\right)\right\},\\
&\Lambda_1=\left(\frac{2\lambda}{\mu}+5\right)-\nu(k'l)^2, \quad \Lambda_2=\sqrt{\nu}\,k'l\left(2\sqrt{\frac{\lambda}{\mu}+2}+1\right),\\
&\nu=\frac{\rho G\varepsilon^2}{\rho'\mu l^2}, \qquad k'^2=\frac{\rho'p^2}{G},
\end{aligned}\right\} \quad \text{.........}(11)$$

in which ρ, λ, μ; $\rho'(=m/a_1l_1)$, $G(=12.4\,Ej/l_1^2)$ stand for the density and the elastic constants of the earth, and the effective density and the effective rigidity of the structure, and E, j, l_1, l, a_1 and ε for Young's modulus, radius of gyration of section, length of the column between two adjacent floors, height of the structure, sectional area of the column and radius of the column respectively.

Using equation (10), we studied the change of stress when the building decreases in stiffness and is to be out of resonance condition assuming that the period and the amplitude of the earthquake motion are constant as in the previous chapter. In Fig. 8, the abscissa represents the ratio of rigidity and the ordinate the ratio

2) K. SEZAWA and K. KANAI, "Some Improved Theory of Energy Dissipation in Seismic Vibrations of a Structure", *Bull. Earthq. Res. Inst.*, **14** (1936), 168.

between the stress at resonance condition and that out of it. In this case the ratio between the radius and the height of the building, (ε/l), is 1/2. We see that when the rigidity ratio of the building to the ground is small, the stress becomes smaller than at the resonance condition with a slight decrease in the rigidity of the building, but if the rigidity ratio is comparatively large, the stress be it ever so slightly, becomes larger than at the resonance condition till the building decreases in its rigidity to a considerable degree.

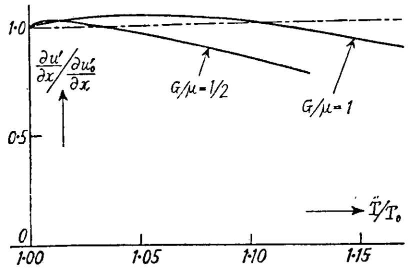

Fig. 8. $\varepsilon/l=1/2$.

Let the abscissa be the rigidity ratio of the ground to the building, and the ordinate the range of the ratio between the stress at the resonance condition and the stress in the state of decreased stiffness (out of the resonance condition), keeping the period and amplitude of the earthquake motion constant, less than 1, the function between them is shown by the full line in Fig. 9.

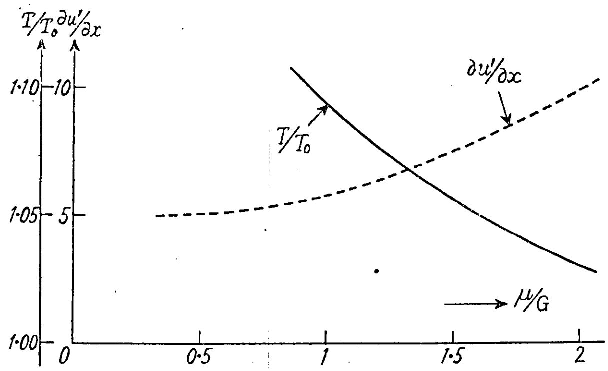

Fig. 9. $\varepsilon/l=1/2$.

Fig. 9 shows that the smaller the rigidity of the ground compared with that of the building is, the smaller the stress at resonance condition becomes, but if the period of earthquake motion is kept constant, the range within which the ratio of the stress at resonance condition to the stress in the state of decreased stiffness is less than 1, becomes narrower. In other words, buildings on soft ground, having larger vibration damping, are more unlikely to collapse than those on hard ground, but once the destruction begins, it advances rapidly.

Keeping the rigidity ratio of building to ground constant and varying the ratio between the building radius and height, we will calculate as we did previously. The result is shown in Figs. 10 and 11.

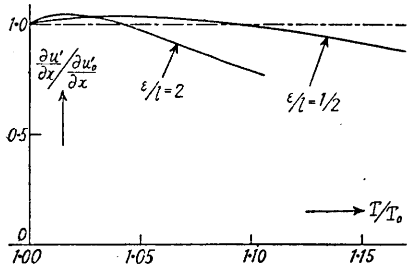

Fig. 10. $G/\mu=1$.

We know from Fig. 11 that as the ratio of ε/l decreases, the stress at resonance condition becomes smaller, nevertheless the range within which the ratio of the stress at resonance condition to the stress in the state of decreased rigidity is less than 1 becomes narrower, when the period of earthquake motion is kept constant. This means that the larger the height-to-area-ratio of the building is, the larger the vibration damping will be, so that destruction is not easily caused, but once begun, it advances very rapidly.

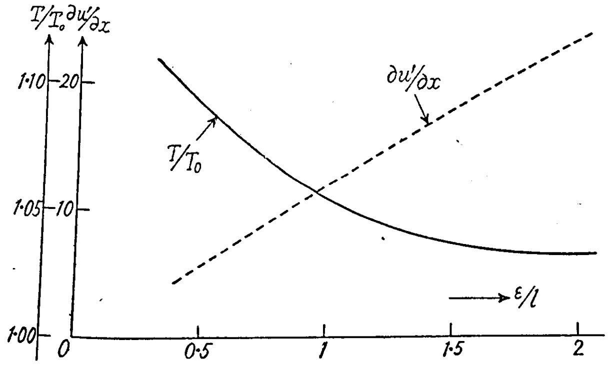

Fig. 11. $G/\mu=1$. Full line; range within which destruction will advance.

4. The practical examination of earthqugke-damage of building.

The broken line in Fig. 12 shows the damage ratio of brick buildings in old Tokyo-shi at the time of the Kwanto Earthquake of 1923. The ratio is divided into three according to the thickness of alluvium. When the ordinate represents the quotient of the ratio between the totally-destroyed and the semi-destroyed buildings divided by the damage ratio, a full line is drawn. The relation shown in Fig. 12 is quite similar to the relation shown in Fig. 9. Thus it has been found that the brick houses on soft ground are not destroyed as easily as those on hard ground, but once the destruction begins, it will not stop till a much damage has been done. From this fact we find that the theoretical investigations coincide at least quantitatively with the practical facts in the study of quake-damage.

In the next place, using the same data, we inquired into the relations among the ratio of the building radius ε' (considering the floor space as a circle) to the height (l), the damage ratio and the quotient of the ratio of the totally-destroyed to the semi-destroyed divided by the damage ratio. The results are shown by the broken line and full line in Fig. 13. These relations resemble those in Fig. 11. The higher the building is in comparison with the floor space, the less destructible it is by earthquake, but once the destruction begins it grows serious.

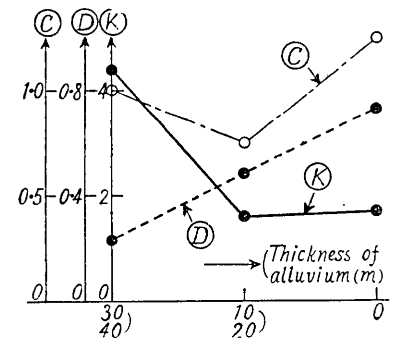

Fig. 12. The case of the damage caused to brick buildings at the Kwanto Earthquake of 1923. Symbols D, C, and K represent the damage ratio, the ratio of the totally-destroyed buildings to the semi-destroyed ones and the ratio of C to D respectively.

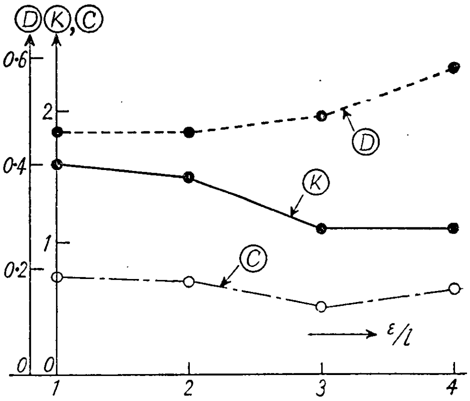

Fig. 13. The case of the damage caused to brick buildings by the Kwanto earthquake of 1923. Symbols D, C and K represent the damage ratio, the ratio of the totally-destroyed buildings to the semi-destroyed ones and the ratio of C to D respectively.

5. Conclusion

From theoretical studies as well as from statistical studies dealing practically with the earthquake-damage, the following facts have been made clear.

The larger the building damping is, the less destructive the building is, but once the destruction begins, it advances rapidly.

The result of theoretical study which has been developed from the idea that the vibration energy dissipates into the ground as elastic waves at the time of earthquake is ascertained quantitatively by the coincidence with results obtained by the practical study of earthquake damage concerning five points: i.e. the relation between the nature of ground and damage ratio[3], ground and the degree of damage, the height of building and damage ratio[4], the height and the degree of damage, and finally the ground and the height of the place which suffered most damage[5]. Therefore by developing this problem qualitatively, it may become possible to construct the most reasonable earthquake-proof building according to the property of the ground.

In conclusion, I wish to express my thanks to Miss S. Yoshizawa for the valuable assistance she offered in the present investigation.

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34. 地震による建物の破壞過程について

地震研究所　金　井　清

建物の振動減衰性が大きい程，地震による破壞は起り難いが，一旦どこかに破壞が起つた場合には，小破壞に止らないで，大破壞に進んで行く傾向があることが數理的研究で明かになつた．又この數理的研究結果は實際の震害に關する統計的研究結果とよく合うことが確められた．

地震の際に，建物の震動勢力が彈性波として地中に逸散するという思想から出發した數理的研究結果は實際の震害の統計的研究結果と少くとも定性的には一致することが次の 5 つの關係で確められた．即ち，地盤と被害率，被害程度及び建物の最大被害を受ける場所の高さとの關係；建物の高さと被害率，被害程度との關係がそれである．

從つて，この研究を定量的な方向に進めることができれば，地盤の性質を考慮に入れた經濟的で合理的な耐震構造を作ることができる筈である．

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3) K. KANAI and S. YOSHIZAWA, "Relation between the Earthquake Damage of Non-wooden Buildings and the Nature of the Ground. II", *Bull. Earthq. Res. Inst.*, **29** (1951), 219.

4) *loc. cit.*, 3).

5) K. KANAI, "*ditto.*", *Bull. Earthq. Res. Inst.*, **27** (1949), 97.