1. *Study on Surface Waves I. Velocity of Love-Waves.*

By Yasuo SATÔ,

Earthquake Research Institute.

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1. Introduction.

The world in which we live is limited. However great it may seem to us, it always has its own limits. Of course the theory of elastic waves was at first studied under the assumption of isotropic, homogeneous, perfect and infinite elastic body. And this is very natural from the point of view of practical treatment, such a theory being most simple, fundamental and useful. And yet our elastic body is neither perfect nor infinite. Our globe is a spherical body with finite dimension, and the elastic waves generated must necessarily receive the effect of boundary surfaces. Then, how is the effect of boundaries? What will happen when waves come to these surfaces? Such doubts naturally lead us to the investigation of boundary waves or surface waves. First, G. Green[1] treated this problem, and fifty years later, C. G. Knott[2] took up the same problem from the standpoint of a seismologist. And after that, many prominent authors[3] have studied the same kind of problems; that is the reflection and refraction of plane waves at the contact serface of two semi-infinite elastic bodies, or at the free surface of them.

On 1885 Lord Rayleigh[4] published a famous paper which opened a new age in our study of surface waves. Since then the theory of surface waves upon elastic solids became one of the main battle-ground of seismologists, and a number of authors were engaged in the investigations how to com-

1) G. GREEN, *Cambridge Phil. Soc. Trans.*, **7** (1839). The author has not yet read this paper.

2) C.G. KNOTT, "Earthquake and Earthquake Sounds; As Illustration of General Theory of Elastic Vibrations." *Trans. Seism. Soc. Japan*, **12** (1888), 115.

3) e.g. B. GUTENBERG, Über Erdbebenwellen IV, *Göttingen Nachrichten*, (1912), 321. G.W. WALKER, "Surface Reflection of Earthquake Waves." *Phil. Trans.*, **218** (1919), 373. H. JEFFREYS, "The Refection of Elastic Waves." *M.N.R.A.S. Geo. Sup.*, **1** (1926), 321. T. MATUZAWA, *Zisin*, **4** (1938), 7, (in Japanese). H. KAWASUMI and T. SUZUKI, *Zisin*, **4** (1932), 277 (in Japanese).

4) Lord RAYLEIGH, "On Waves Propagated along the Plane Surface of an Elastic Body." *Proc. Math. Soc. London*, **17** (1885) or *Scientific Papers*, **2**, p. 441.

prehend and explain the nature of these waves.

In the following study we shall develop several calculations in the said part of elastic waves.

2. Love-waves.

When we study the property of elastic waves, we usually take up the velocity of propagation first of all. For, it is one of the most fundamental quantities which show the nature of waves, and also has a strong point that scarcely suffers fatal change by observation instruments and by the nature of soil or rock on which the station is situated. Consequently, when we intend to discriminate a special phase in seismograms, the arrival time or in other words the velocity of propagation always affords the best key for our purpose.

Since the celebrated paper of Lord Rayleigh was published,[5] various cases were investigated in which some boundary waves can exist, and the velocity in question is given as an eigen-value or some eigen-values of equation which is introduced, in our problems, as boundary conditions.

Of these investigations, the most important and interesting one is the so-called Love-waves discovered by A.E.H. Love in 1911.[6] In view of the observational facts, the existence of such waves is today doubted by nobody, and yet the study of Love-waves is far behind that of Rayleigh-weves. This is partly because the theory of Love-waves is mathematically simple and has hardly attracted the interest of mathematicians. However, there still remain a number of problems to be clarified and moreover it is one of the fundamental type of surface waves, whose importance the writer wishes to assert.

3. One explanation of the characteristic equation of Love-waves.

Throughout this study, we will only treat the two-dimensional problems. The mathematical analysis is so complicated that we must make every possible effort to simplify the problem, and fortunately the nature of phenomena in three dimensional space may be fully understood through this simplified treatment.

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5) *Ibid.*

6) A.E.H. LOVE, *Some Problems of Geodynamics* (1911).

We take x-axis along the lower boundary plane of the stratum and z-axis vertically upwards. Further, we employ the following notations ;

	Within stratum	Within substratum medium
Density	ρ	ρ'
Lamé's constants	$\lambda,\ \mu$	$\lambda',\ \mu'$
Velocity of S waves	V_s	V_s'
and	$b = 1/V_s$	$b' = 1/V_s'$
	$k = bp$	$k' = b'p$
	$\beta = \sqrt{(f^2 - k^2)} = i\tilde{\beta}$	$\beta' = \sqrt{(f^2 - k'^2)}$

in which p means circular frequency and $f = 2\pi/(\text{wave-length})$. These notations will be always used hence.

Displacement v in the upper layer is expressed by using the above notations ;

$$v = A\{\exp(i\tilde{\beta}z) + \exp(-i\tilde{\beta}z)\}\exp(ifx + ipt) \qquad (3.1)$$

From the boundary conditions necessary for the existence of SH waves in the above medium (continuity of displacement and stress at the boundary surface and vanishing of stress at the free surface), we get the following characteristic equation

$$(\mu\beta + \mu'\beta')\exp(\beta H) - (\mu\beta - \mu'\beta')\exp(-\beta H) = 0 \qquad (3.2)$$

which is the well-known velocity equation of Love-waves.

Slightly modifying the form of equation, we get

$$\exp\{-i2\Theta + i2\tilde{\beta}H\} = 1$$

or

$$-2\Theta + 2\tilde{\beta}H = 2n\pi \qquad (3.3)$$

where

$$\exp\{i2\Theta\} = -(\chi\beta' - i\tilde{\beta})/(\chi\beta' + i\tilde{\beta})$$

$$\Theta = \arctan(\chi\beta'/\tilde{\beta})$$

It is well known that Love-waves can only exist when $V_s < V_s'$, and the phase velocity V satisfies the relation $V_s < V < V_s'$; therefore $\tilde{\beta}$ and β' are both real and $\chi\beta' - i\tilde{\beta}$ and $\chi\beta' + i\tilde{\beta}$ are conjugate complex to each other, so that $|\exp\{i\Theta\}| = 1$ and Θ takes real value.

This last eqation may be interpreted as follows. (See Fig. 1.)

Consider a wave which is just passing a point P lying on the lower boundary of the layer. (For example we may notice the first term in the parentheses { } of (3.1).) This wave suffers the variation of phase by the angle $\tilde{\beta}H$ before reaching the upper surface of the layer. When it is reflected at the free surface the phase angle becomes reverse and takes the value $-\tilde{\beta}H$, (cf. the second term of (3.1)) and when it reaches the lower boundary again, the phase angle becomes $-2\tilde{\beta}H$. However, when it is reflected by the boundary surface, the phase angle again undergoes a change in sign and becomes $2\tilde{\beta}H$, and furthermore the jump of phase (by the angle Θ) occurs and the final value becomes $2\tilde{\beta}H-\Theta$. The second expression of (3·3) requires that this (change of) phase angle is the multiple of 2π. And it may be regarded that when the waves satisfying the said condition are superposed and predominate, they constitute Love-waves. This interpretation reminds us of the Lummer-Gehrcke plate in spectroscopy. The Lummer-plate is an apparatus using the interference of wave-trains such as A, B, C... in Fig. 3, and Love-waves

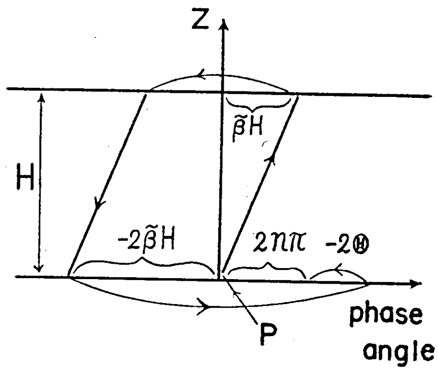


Fig. 1.

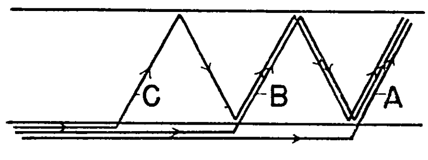


Fig. 2

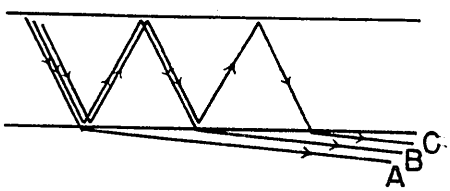


Fig. 3

are, as is clear from the above explanation, also a kind of interference phenomena between the wave-trains such as A, B, C... in Fig. 2.[7] Of course this sort of interpretation is not always applicable to every velocity equation of surface waves, therefore the above nature is not the essential property of characteristic equation obtained from boundary conditions. For, the interpretation of that of Rayleigh-waves is quite different from above, and also the Rayleigh-waves in stratified medium are not interpreted in the same way.

7) Y. SATÔ, *Kagaku* **19** (1949), 311, (in Japanese).

Table I.

Value of ρ'/ρ corresponding to every pair of χ and $\mathrm{Sin}^2\,\theta_0$.

$\chi = \mu'/\mu$

$\mathrm{Sin}^2\,\theta_0 = \left(\frac{\mu}{\rho}\right) \Big/ \left(\frac{\mu'}{\rho'}\right) = V_s^2/V_s'^2$

$\mathrm{Sin}\,\theta_0$ = ratio of the velocity of S waves in the upper layer to that of the lower medium.

θ_0 = minimum value of the angle between wave surface of Love-waves and free surface.

$\chi=\mu'/\mu$ \ $\mathrm{Sin}^2\,\theta_0$	1/100	1/20	1/10	1/5	1/3	1/2	2/3	5/6	10/11
$\mathrm{Sin}\,\theta_0$	.1	.22361	.31623	.44721	.57735	.70711	.81650	.91287	.95346
$\mathrm{Cosec}\,\theta_0$	10	4.4721	3.1623	2.2361	1.73205	1.41421	1.22474	1.09545	1.04881
θ_0	5°44′20″	12°55′15″	18°26′06″	26°33′54″	35°15′52″	45°0′0″	54°44′10″	65°54′20″	72°27′06″
0.9				.18	.30	.45	.60	.75	.81818
1.0			.10	.20	.33333	.50	.66667	.83333	.90909
1.1			.11	.22	.36667	.55	.73333	.91667	1.0
1.2			.12	.24	.40	.60	.80	1.00	1.09091
1.5			.15	.30	.50	.75	1.0	1.25	1.36364
2		.10	.20	.40	.66667	1.00	1.33333	1.66667	1.81818
3		.15	.30	.60	1.00	1.50	2.00	2.50	2.72727
5		.25	.50	1.00	1.66667	2.50	3.33333	4.16667	4.54545
10	.10	.50	1.00	2.00	3.33333	5.00	6.66667	8.33333	9.09091
20	.20	1.00	2.00	4.00	6.66667	10.00			
100	1.00	5.00	10.00						

Table II.

Minimum group velocity of Love-waves $\mathfrak{V}_{min}$ (unit; $V_s=\sqrt{(\mu/\rho)}$) and the corresponding values of phase velocity $\mathfrak{v}$, period τ (unit; H/V_s) and wave-length λ (unit; H) [Branch L_0].

χ \ $\sin^2\theta_0$		1/100	1/20	1/10	1/5	1/3	1/2	2/3	5/6	10/11
0.9	$\mathfrak{V}_{min}$				.9674	.9738	.9812	.9880	.9942	.9969
	$\mathfrak{v}$				1.113	1.089	1.058	1.036	1.017	1.008
	τ				2.7207	2.4371	1.9519	1.5448	1.0570	.7397
	λ				3.028	2.654	2.065	1.600	1.075	.746
1.0	$\mathfrak{V}_{min}$			.9567	.9628	.9703	.9788	.9864	.9936	.9965
	$\mathfrak{v}$			1.149	1.120	1.095	1.063	1.039	1.018	1.009
	τ			3.0171	2.7167	2.4355	1.9857	1.5830	1.0896	.7543
	λ			3.467	3.043	2.667	2.111	1.645	1.109	.761
1.1	$\mathfrak{V}_{min}$			.9514	.9584	.9669	.9763	.9848	.9928	.9961
	$\mathfrak{v}$			1.158	1.129	1.101	1.068	1.042	1.020	1.010
	τ			3.0122	2.7397	2.4424	2.0123	1.5977	1.1214	.7745
	λ			3.488	3.093	2.689	2.149	1.665	1.144	.782
1.2	$\mathfrak{V}_{min}$			.9462	.9540	.9634	.9739	.9835	.9920	.9956
	$\mathfrak{v}$			1.175	1.140	1.109	1.072	1.045	1.022	1.011
	τ			3.1004	2.7846	2.4861	2.0321	1.6177	1.1439	.7897
	λ			3.643	3.174	2.757	2.178	1.690	1.169	.798
1.5	$\mathfrak{V}_{min}$			.9309	.9413	.9536	.9671	.9791	.9901	.9947
	$\mathfrak{v}$			1.211	1.170	1.127	1.085	1.051	1.024	1.012
	τ			3.1795	2.8962	2.5394	2.1000	1.6579	1.1542	.8145
	λ			3.850	3.389	2.862	2.279	1.742	1.182	.824
2	$\mathfrak{V}_{min}$		.8998	.9079	.9219	.9386	.9569	.9726	.9869	.9930
	$\mathfrak{v}$		1.292	1.260	1.209	1.156	1.102	1.060	1.028	1.015
	τ		3.4101	3.2558	2.9737	2.6264	2.1753	1.7035	1.1853	.8725
	λ		4.406	4.102	3.595	3.036	2.397	1.806	1.218	.886
3	$\mathfrak{V}_{min}$		.8562	.8686	.8897	.9140	.9399	.9622	.9821	.9904
	$\mathfrak{v}$		1.400	1.346	1.272	1.198	1.129	1.076	1.034	1.019
	τ		3.5455	3.3619	3.0786	2.7113	2.2673	1.7866	1.2137	.9188
	λ		4.964	4.525	3.916	3.248	2.560	1.922	1.255	.936
5	$\mathfrak{V}_{min}$		.7918	.8110	.8431	.8787	.9158	.9474	.9752	.9866
	$\mathfrak{v}$		1.562	1.480	1.363	1.255	1.164	1.096	1.042	1.023
	τ		3.6568	3.4863	3.1826	2.8056	2.3587	1.8771	1.2830	.9541
	λ		5.712	5.160	4.338	3.521	2.746	2.057	1.337	.976
10	$\mathfrak{V}_{min}$	.6686	.6969	.7268	.7759	.8283	.8816	.9264	.9652	.9816
	$\mathfrak{v}$	1.976	1.831	1.673	1.492	1.333	1.209	1.117	1.053	1.027
	τ	3.8752	3.7484	3.5626	3.2754	2.8982	2.4514	1.9335	1.3590	.9852
	λ	7.657	6.863	5.960	4.887	3.863	2.964	2.160	1.431	1.012
20	$\mathfrak{V}_{min}$	.5634	.6049	.6465	.7119	.7804	.8490			
	$\mathfrak{v}$	2.421	2.103	1.877	1.610	1.407	1.249			
	τ	3.9254	3.7690	3.6107	3.3266	2.9736	2.5238			
	λ	9.503	7.926	6.777	5.356	4.184	3.152			
100	$\mathfrak{V}_{min}$	.3660	.4370	.5022						
	$\mathfrak{v}$	3.742	2.816	2.323						
	τ	3.9523	3.8217	3.6828						
	λ	14.790	10.762	8.555						

Table III.

Minimum group velocity of Love-waves $\mathfrak{V}_{min}$ (unit; $V_s=\sqrt{(\mu/\rho)}$) and the corresponding values of phase velocity $\mathfrak{v}$, period τ (unit; H/V_s) and wave-length λ (unit; H) [Branch L_1].

χ	$\sin^2\theta_0$	1/100	1/20	1/10	1/5	1/3	1/2	2/3	5/6	10/11
0.9	$\mathfrak{V}_{min}$				.7670	.8364	.8948	.9378	.9718	.9852
	$\mathfrak{v}$				1.748	1.445	1.254	1.138	1.059	1.030
	τ				1.4661	1.2646	1.0398	.8119	.5542	.3993
	λ				2.563	1.827	1.304	.924	.587	.411
1.0	$\mathfrak{V}_{min}$			.6970	.7768	.8422	.8978	.9391	.9722	.9854
	$\mathfrak{v}$			2.203	1.710	1.424	1.244	1.133	1.058	1.028
	τ			1.6032	1.4247	1.2245	1.0064	.7840	.5399	.3791
	λ			3.532	2.436	1.744	1.252	.888	.571	.390
1.1	$\mathfrak{V}_{min}$			.7096	.7842	.8458	.8992	.9394	.9722	.9854
	$\mathfrak{v}$			2.144	1.666	1.407	1.233	1.128	1.055	1.027
	τ			1.5680	1.3783	1.1889	.9714	.7603	.5185	.3675
	λ			3.362	2.296	1.673	1.198	.858	.547	.377
1.2	$\mathfrak{V}_{min}$			.7194	.7890	.8480	.8999	.9397	.9723	.9853
	$\mathfrak{v}$			2.069	1.639	1.390	1.225	1.126	1.054	1.027
	τ			1.5254	1.3444	1.1548	.9444	.7451	.5092	.3602
	λ			3.156	2.203	1.605	1.157	.839	.537	.370
1.5	$\mathfrak{V}_{min}$			.7341	.7941	.8485	.8986	.9379	.9713	.9848
	$\mathfrak{v}$			1.919	1.572	1.365	1.215	1.123	1.053	1.028
	τ			1.4223	1.2550	1.0900	.8962	.7147	.4853	.3569
	λ			2.729	1.973	1.488	1.089	.803	.511	.367
2	$\mathfrak{V}_{min}$		.6953	.7333	.7880	.8413	.8924	.9336	.9691	.9837
	$\mathfrak{v}$		2.089	1.807	1.537	1.348	1.211	1.123	1.053	1.028
	τ		1.4113	1.3181	1.1820	1.0302	.8573	.6910	.4697	.3487
	λ		2.948	2.382	1.817	1.389	1.038	.776	.495	.358
3	$\mathfrak{V}_{min}$		.6735	.7101	.7660	.8228	.8790	.9250	.9650	.9814
	$\mathfrak{v}$		2.006	1.784	1.540	1.365	1.219	1.125	1.055	1.028
	τ		1.3166	1.2462	1.1325	1.0022	.8395	.6699	.4640	.3387
	λ		2.641	2.223	1.744	1.368	1.023	.754	.490	.348
5	$\mathfrak{V}_{min}$		.6246	.6654	.7288	.7939	.8587	.9123	.9588	.9781
	$\mathfrak{v}$		2.103	1.856	1.592	1.393	1.239	1.135	1.060	1.031
	τ		1.2807	1.2186	1.1182	.9943	.8400	.6694	.4672	.3443
	λ		2.693	2.262	1.780	1.385	1.041	.760	.495	.355
10	$\mathfrak{V}_{min}$	.4894	.5500	.5994	.6756	.7534	.8306	.8950	.9507	.9738
	$\mathfrak{v}$	2.803	2.323	2.018	1.689	1.450	1.271	1.146	1.066	1.034
	τ	1.3202	1.2682	1.2160	1.1233	1.0060	.8547	.6846	.4762	.3496
	λ	3.701	2.946	2.454	1.897	1.459	1.086	.785	.508	.361
20	$\mathfrak{V}_{min}$	.4175	.4806	.5396	.6277	.7171	.8055			
	$\mathfrak{v}$	3.326	2.613	2.195	1.784	1.509	1.300			
	τ	1.3182	1.2705	1.2220	1.1334	1.0235	.8710			
	λ	4.384	3.320	2.682	2.022	1.544	1.132			
100	$\mathfrak{V}_{min}$	.2750	.3640	.4393						
	$\mathfrak{v}$	4.810	3.236	2.549						
	τ	1.3193	1.2802	1.2368						
	λ	6.346	4.143	3.153						

However, a similar explanation will be seen in the investigation of bodily waves.

4. Velocity of Love-waves.

Using new notations

$$f = k\sin\theta, \quad k' = k\sin\theta_0, \quad kH = \omega \dots\dots\dots\dots (4.1)$$

we have from (3.2)

$$\omega_n = \sec\theta[\mathrm{Tan}^{-1}\{\chi\sec\theta\sqrt{(\sin^2\theta - \sin^2\theta_0)}\} + n\pi] \dots\dots (4.2)$$

Tan^{-1} means the principal value of inverse tangent function and the suffix n implies the number of node in the layer.

Displacement in the surface layer is expressed as

$$v \propto \exp\{ik(x\sin\theta + z\cos\theta - V_s t)\}$$
$$+ \exp\{ik(x\sin\theta - z\cos\theta - V_s t)\} \dots\dots\dots\dots (4.3)$$

therefore θ is the angle between the direction of wave path and z-axis. By means of this variable, the velocity is expressed as cosec θ, and the period of waves as $2\pi/\omega$.

Now, we will show the results of calculations; here two parameters $\chi(=\mu'/\mu)$ and $\sin\theta_0(=k'/k=V_s/V_s')$ are used.

On considering the physical constants of elastic substances which commonly exist around us, we notice that the values of elastic constants vary very widely. On the contrary it is very seldom that the ratio of density of two substances arbitrarily chosen amounts to as large as ten. While the value of $\sin^2\theta_0$ is always smaller than unity, consequently the possible range of values of two parameters is naturally determined.

Under the condition $1/10 \leq \rho'/\rho \leq 10$ we chose 76 pairs of χ and $\sin^2\theta_0$ which is shown in Tab. I, and calculated the value of $\mathrm{cosec}\,\theta$ ($= \mathfrak{v} =$ velocity of Love-waves whose unit is V_s) and ω_0 and ω_1 (=circular frequency, time unit is H/V_s) with every degree of θ from θ_0 to 90°. The reason why we did not calculate the branch $\omega_2, \omega_3 \dots$ and so on, will be found in the later part of our study.

4.1 Minimum value of group velocity.

The importance of the minimum value of group velocity has been already noticed by several authors; for example, K. Sezawa and K. Kanai's

M_2 waves[8] correspond to the minimum group velocity of certain branch of layered Rayleigh-waves, and recently an interesting investigation was published by a number of authors in America.[9] We will here show, using the above numerical data, the minimum group velocity and the corresponding phase velocity and period of every branch of Love-waves. (Tabs. II~III and Figs. 4~7.)

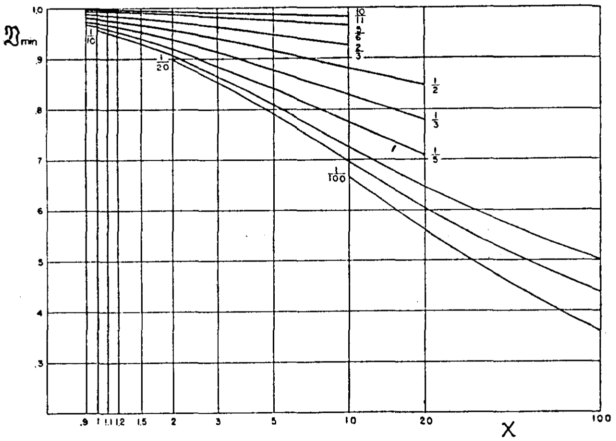


Fig. 4 Minimum group velocity $\mathfrak{V}_{min}$ of Love-waves.
Abscissa ; $\chi = \mu'/\mu$
Parameter ; $\mathrm{Sin}^2\theta_0 = \left(\frac{\mu}{\rho}\right) / \left(\frac{\mu'}{\rho'}\right)$ (Branch L_0)

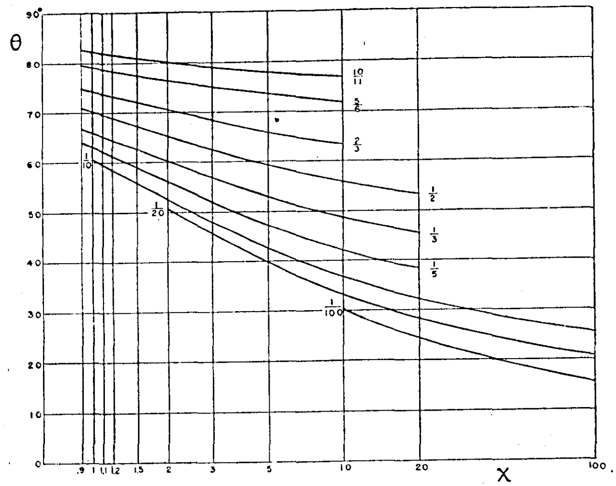
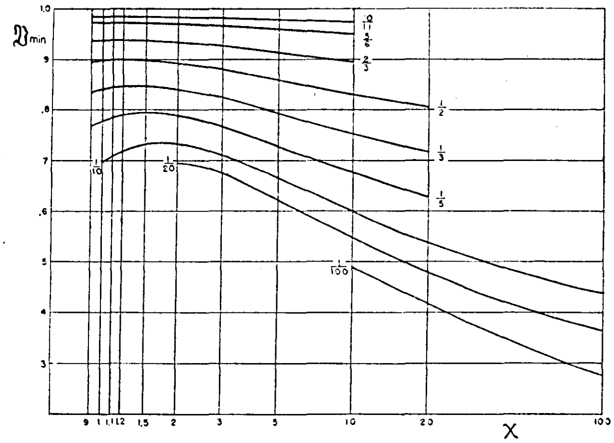


Fig. 5 Value of θ (=cosec^{-1} (phase velocity)) corresponding to minimum group velocity. (Branch L_0)



Flg. 6 Minimum group velocity $\mathfrak{B}_{\min}$ of Love-waves.
Abscissa ; $\chi = \mu'/\mu$
Parameter ; $\mathrm{Sin}^2\,\theta_0 = \left(\frac{\mu}{\rho}\right) \Big/ \left(\frac{\mu'}{\rho'}\right)$ (Barnch L_1)

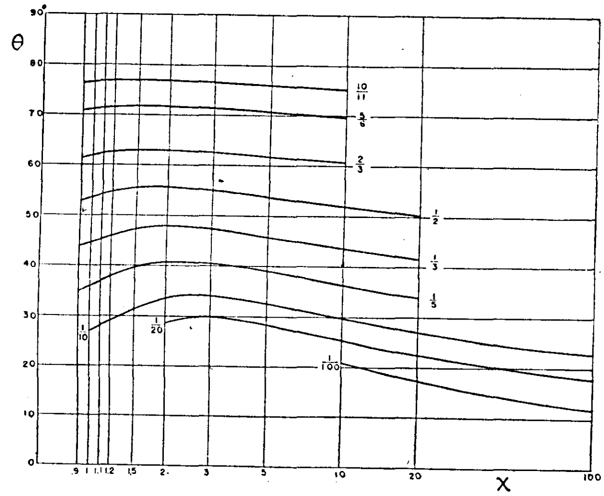


Fig. 7 Value of θ ($=\mathrm{cosec}^{-1}$ (phase velocity)) corresponding to minimum group velocity. (Branch L_1)

8) K. KANAI, "On the Existence of the M_2 Waves in Actual Seismic Disturbance", *Bull Earthq. Res. Inst.*, **26** (1948), 57. Also See. K. SEZAWA and K. KANAI, *Proc. Imp. Acad.*, **11** (1935), 13, 96; *Bull. Earthq. Res. Inst.*, **13** (1935), 237, 471; K. SEZAWA, *Bull. Earthq. Res. Inst.*, **16** (1938), 1.

9) F. PRESS, M. EWING and I. TOLSTOY, "The Airy Phase of Shallow Fucus Submarine Earthquakes." *Bull. Seism. Soc. Amer.*, **40** (1950), 111. C. ECKART, "The Approximate Solution of One-Dimensional Wave Equations." *Rev. Mod. Phys.*, **20** (1948), 399.

Further for convenience of calculations concerning Love-waves, we will show various forms of characteristic equation.

One of the most fundamental forms is

$$(\mu'\beta' + i\mu\tilde{\beta}) \exp (i\tilde{\beta}H) + (\mu'\beta' - i\mu\tilde{\beta}) \exp (-i\tilde{\beta}H) = 0 \quad \ldots\ldots (4.4)$$

Considering that $\tilde{\beta}$ and β' are real, with slight transformation we get the three following forms;

$$\begin{aligned} &\exp (i2\tilde{\beta}H) = -(\chi\beta' - i\tilde{\beta})/(\chi\beta' + i\tilde{\beta}), \ \chi = \mu'/\mu \\ &\tan \tilde{\beta}H = \chi\beta'/\tilde{\beta} \quad \ldots\ldots (4.5) \\ &-\Theta + \tilde{\beta}H = n\pi, \ \Theta = \mathrm{Tan}^{-1}(\chi\beta'/\tilde{\beta}) \end{aligned}$$

For numerical calculations, non-dimensional notations are desirable; therefore putting

$$fH = \xi, \ kH = \omega, \ V/V_s = \mathfrak{v} = \mathrm{cosec}\, \theta$$

we have

$$\begin{aligned} &\tan (\xi\sqrt{(\mathfrak{v}^2 - 1)}) = \chi\sqrt{(1 - \mathfrak{v}^2 \sin^2\theta_0)} / \sqrt{(\mathfrak{v}^2 - 1)} \\ &\qquad \sin^2\theta_0 = k'^2/k^2 = V_s^2/V_s'^2 \quad \ldots\ldots (4.6) \\ &\omega_n = \sec\theta[\mathrm{Tan}^{-1}\{\chi \sec\theta\sqrt{(\sin^2\theta - \sin^2\theta_0)}\} + n\pi] \end{aligned}$$

We used this last form of equation for our calculation.

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1. 表面波の研究 I

ラブ波の速度

地震研究所　佐藤泰夫

半無限彈性體，又はそれに表面層がある場合の表面波の研究は，從來多くの大家によつて研究されてゐるが，なほ十分とはいへない．

ここでは先づラブ波について考へ，その速度方程式を變形して調べてみることにより，分光器として使はれる Lummer-Gehrcke 板の作用と極めて似てゐる事を示した．

次に上層と下部の速度の比 $\mathrm{Sin}\,\theta_0 = V_s/V_s'$ 及び剛性率の比 $\chi = \mu'/\mu$ を色々に與へて，實在しうるやうな組合せ 76 通りについて，ラブ波の基本分枝 L_0 と節を一つもつ分枝 L_1 の位相速度を求め，さらにこれからラブ波の極小群速度を出した，この速度に相當する時よりも後に到着した波は，ラブ波としては解釋されない．