6. *Electromagnetic Induction within the Earth and Its Relation to the Electrical State of the Earth's Interior. Part III.*

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1. Introduction.

In the previous papers,[1] the writer studied electromagnetic induction by various variations in the earth's magnetic field, the slowest of them being the main phase of magnetic storm (D_{st}) with a duration-time of a few days. In order to study the electric and magnetic properties in the deeper part of the earth's interior, however, it was considered necessary to investigate some variations, the periods of which are longer than the ones treated in the previous papers.

Taking into consideration the fact that geomagnetic condition has the 27-day recurrence-tendency, it may be possible to find out an apparent geomagnetic variation of about 27-day period with suitable means. In this paper, the 27-day period variation will be analyzed in regard to the electrical state of the earth's interior.

2. Magnetic data and the analysis.

In order to avoid irregularities, the daily means of the magnetic field are superposed with 27-day interval throughout nearly half a year. The reports during the years 1924 and 1925 from several magnetic observatories are used, the names of the observatories being given in Table I together with their geomagnetic latitudes and longitudes.

The mean variations in the horizontal and vertical components superposed throughout the period I (from Jan. 1 to July 7, 1924), II (from July 8 to Dec. 16, 1924), III (from Dec. 17, 1924 to June 23, 1925) and IV (from June 24 to Dec. 29, 1925) are respectively shown in Figs. 1, 2, 3, and 4. As the variations in the declination are usually very small, we may approximately consider the variations in the horizontal component to be those in the

1) T. RIKITAKE, *Bull. Earthq. Res. Inst.* **28** (1950), 45, 219 and 263.

Table I. List of the magnetic observatories.

Observatory	Geomagn. latitude	Geomagn. longitude
Sitka	60.°0	275.°4
Val-Joyeux	51.3	84.5
Cheltenham	50.1	350.5
Tucson	40.4	312.2
Vieques	29.6	3.9
Kakioka	26.0	206.0
Honolulu	21.1	266.5
Alibag	9.5	143.6
Rio de Janeiro	−12.5	24.3

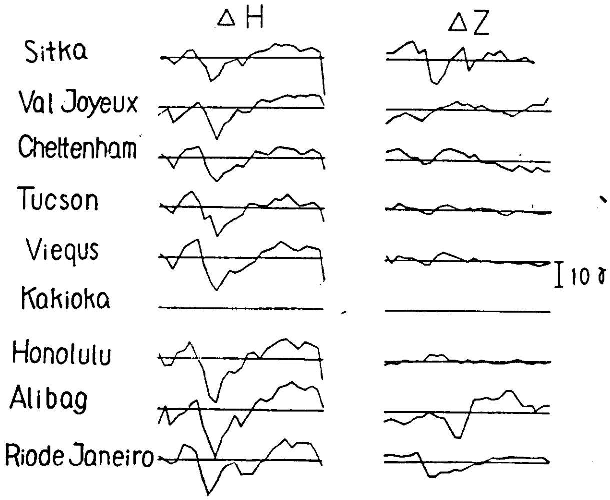

Fig. 1. Mean variation in the components of the earth's magnetic field during the period I. The left and right ends of the curves correspond respectively to 0 and 27 day.

geomagnetic north component. The variations at Kakioka during the periods I and II are omitted because the observations on many days were missing. Meanwhile, the reports in the year 1925 from Tucson and Vieques were not available.

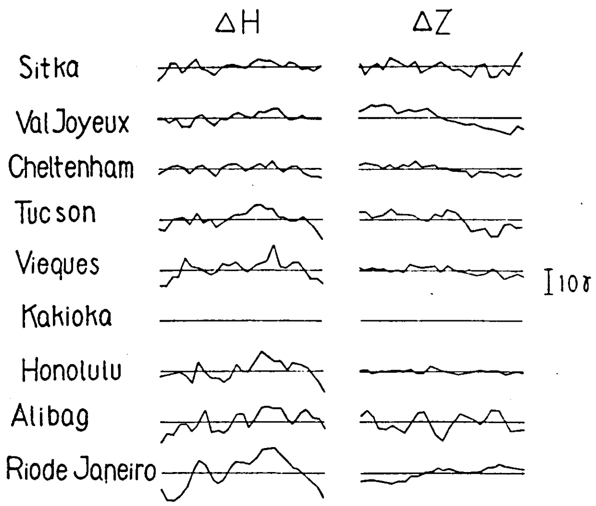

Fig. 2. Mean variation in the components of the earth's magnetic field during the period II.

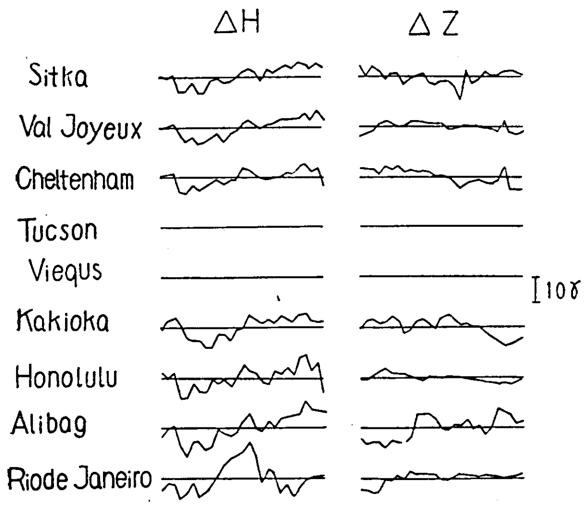

Fig. 3. Mean variation in the components of the earth's magnetic field during the period III.

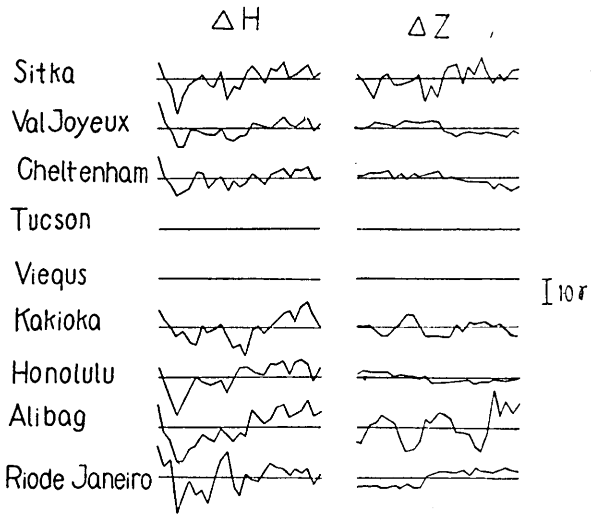

Fig. 4. Mean variation in the components of the earth's magnetic field during the period IV.

As seen in the figures, the geomagnetic north component appears to be small at high latitude. It increases equatorwards, however, attaining its maximum near the equator. On the other hand, the vertical one is becoming smaller near the equator, where a reversal of sign seems to occur. Near the auroral zone, it becomes large and irregular as shown by the curve at Sitka, while the cause of the abnormally large amplitude at Alibag is unknown.

Table II. The amplitude and phase angle during the period I.

Observatory	Horizontal comp.		Vertical comp.	
	C_H	α_H	C_Z	α_Z
Sitka	3.8γ	276°	(1.6)γ	(342)°
Val-Joyeux	5.4	287	1.5	210
Cheltenham	3.5	298	2.7	112
Tucson	3.6	307	1.1	102
Vieques	4.6	306	0.9	110
Honolulu	6.9	297	0.8	137
Alibag	9.0	296	(5.1)	(273)
Rio de Janeiro	5.8	321	−3.6	151
Mean		299		137

The variations thus obtained are subjected to Fourier analysis, the amplitude and phase angle[2] of the first harmonic during the period I, for example, are shown in Table II. The values of the vertical component at Sitka and Alibag are omitted for the next calculation.

The distribution of C_H and C_Z bearing reference to the geomagnetic latitude are illustrated in Fig. 5. As seen in the figure, it will be possible to consider that the greatest part of the magnetic potential of the variation is expressible, to a fair degree of approximation, with the term including only $P_1(\cos\theta)$, where θ denotes the geomagnetic colatitude. Assuming, then, that the distribution of the amplitude can be expressed by the first spherical harmonic such as

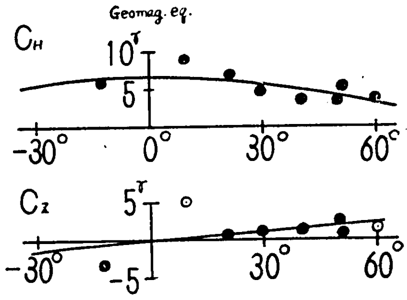

Fig. 5. The distribution of C_H and C_Z referred to the geomagnetic latitude.

$$C_H = A_H \sin\theta, \quad C_Z = A_Z \cos\theta, \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

the coefficients are determined by means of the least square method as given in Table III, while the phase angles are obtained by using arithmetical means as stated above.

Table III.

Period	A_H	A_Z	α_H	α_Z
I	6.8γ	2.7γ	299°	137°
II	4.0	3.2	226	106
III	5.5	3.4	275	122
IV	5.5	3.2	298	93

As to the higher harmonics of the Fourier analyses, the coefficients are almost meaningless for their small amplitudes. Hence, only the first term will be dealt with in this paper.

Now, we take the magnetic potential of the variation which is expressed by

$$W = a\{(r/a)e\cos(t-\epsilon) + (a/r)^2 i\cos(t-\iota)\}P_1(\cos\theta) \ldots\ldots\ldots\ldots (2)$$

where a denotes the radius of the earth. e and i denote respectively the amplitude of the coefficient of the potential of external and internal origins,

2) The first harmonic is expressed by $C\cos(t-\alpha)$.

while ϵ and ι denote that of the phase angle. The geomagnetic north component and vertical component at $r=a$ are derived from (2) as follows:

$$\left.\begin{aligned} \Delta H &= -\{e\cos(t-\epsilon)+i\cos(t-\iota)\}\sin\theta, \\ \Delta Z &= \{e\cos(t-\epsilon)-2i\cos(t-\iota)\}\cos\theta. \end{aligned}\right\} \qquad (3)$$

Hence, comparing (3) with (1), we get

$$\left.\begin{aligned} e &= (1/3)\{4A_H^2 + A_Z^2 + 4A_H A_Z \cos(a_H - a_Z)\}^{1/2}, \\ i &= (1/3)\{A_H^2 + A_Z^2 - 2A_H A_Z \cos(a_H - a_Z)\}^{1/2}, \\ \epsilon &= \tan^{-1}\{(2A_H \sin a_H + A_Z \sin a_Z)/(2A_H \cos a_H + A_Z \cos a_Z)\}, \\ \iota &= \tan^{-1}\{(A_H \sin a_H - A_Z \sin a_Z)/(A_H \cos a_H - A_Z \cos a_Z)\}. \end{aligned}\right\} \qquad (4)$$

Using the values given in Table III, e, i, ϵ, and ι are calculated in regard to every period as shown in Table IV where the amplitude ratio and phase difference are also given.

Table IV.

Period	I	II	III	IV
e	5.4γ	3.4γ	4.8γ	4.7γ
i	1.5	1.2	1.0	1.0
ϵ	122°	62°	101°	112°
ι	107	−3	64	146
e/i	3.7	2.8	4.8	4.7
$\iota-\epsilon$	15°	65°	37°	−34°

The most probabe values of e/i and $\epsilon-\iota$ are respectively obtained to be 4.0 ± 0.4 and $22°\pm16°$.

As the amplitude ratio and phase difference for each period are somewhat scattered, a more complete analysis with abundant data will be needed for a more accurate research. However, it may be of some interest to investigate tentatively the process of the 27-day period variation by the present analysis.

3. The relation to the electrical state of the earth's interior.

In Part II of this paper, the writer determined approximately the distribution of the electrical conductivity from the studies of various geomagnetic variations, the distribution being determined to be

$$\sigma = \sigma' \quad \text{for} \quad 1 > \rho > q,$$

$$\sigma = \sigma_0 \rho^{-l} \quad \text{for} \quad q > \rho,$$

σ', σ_0, q and l being 10^{-15} *emu*, 1.0×10^{-12} *emu*, 0.94 and 11 respectively, where ρ denotes the radial distance divided by the earth's radius.

In the first place, electromagnetic induction by 27-day period variation within the earth having the electrical conductivity cited above will be dealt with. As often studied in the previous papers, the influence of the low conducting layer is negligible for variation of such long period as 27-day. In that case, the amplitude ratio and phase difference of the magnetic potential of external and internal origins are obtained from the *mod.* and *arg.* of $I(i\omega)$. $I(i\omega)$ is given by

$$I(i\omega) = \frac{n}{n+1} q^{2n+1} \frac{K_{\nu-1}(z_1)}{K_{\nu+1}(z_1)}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where

$$z_1 = x\sqrt{i},\ x = \frac{4a\sqrt{\pi\sigma_0\omega}}{|l-2|},\ \nu = \frac{2n+1}{|l-2|} \text{ and } \omega = \frac{2\pi}{T}.$$

K_ν is the modified Bessel function, while n and T denote respectively the degree of the spherical harmonic and the period of the variation. As x amounts to 1.09 in the present case, an expansion in ascending power of $x/2$ is available for the calculation as given in the following:

$$K_\nu(x\sqrt{i}) = (1/2)\pi \operatorname{cosec} \nu\pi \{(\Phi_{-\nu}+i\Psi_{-\nu})e^{-i\nu\pi/4} - (\Phi_\nu + i\Psi_\nu)e^{i\nu\pi/4}\} \ldots (6)$$

where

$$\Phi_\nu = \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{\nu+4r}}{(2r)!\,\Gamma(\nu+2r+1)}, \quad \Psi_\nu = \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{\nu+4r+2}}{(2r+1)!\,\Gamma(\nu+2r+2)}. \quad (7)$$

Thus the amplitude ratio and phase difference for the 27-day period variation are obtained as

3.52 and 14.°2

which agree with the values obtained from the analysis within the errors of determination.

Hence, it may be said that the conductivity-distribution obtained in the previous papers is approximately compatible with longer period variation such as 27-day period one.

4. The distribution of the induced currents.

In order to study how deep the induced currents penetrate into the earth, the distribution of the induced currents is calculated in the same way as in the previous studies.

According to B. N. Lahiri and A. T. Price,[3] the current intensity is proportional to v which is given by

$$v = \rho^{-l-1/2} K_\nu(\rho^{-l/2+1} x\sqrt{i}) / K_\nu(x\sqrt{i}) \, . \quad \ldots\ldots\ldots\ldots (8)$$

mod. v is easily calculated with the aid of (6) and (7) for various values of ρ. Thus the distribution of current intensity is obtained as shown in Fig. 6 in arbitrary unit. Strictly speaking, the phase of the induced currents differs from each other at different depths, but its difference is neglected here as well as in Part II. As the period of the variation is larger than any other variation hitherto treated, the induced currents flow in appreciably

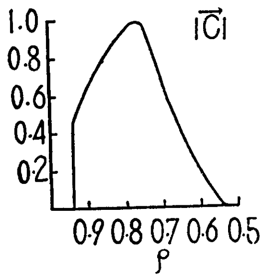

Fig. 6. The distribution of the induced currents in the earth.

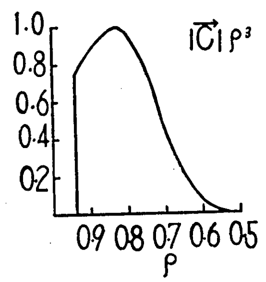

Fig. 7. The contribution of the currents to the magnetic field at the earth's surface.

deeper part, the maximum occuring at as deep as about 1500 *km*. The contribution of the current at any depth to the magnetic field at the earth's surface is also calculated as shown in Fig. 7. As done in Part I, we define the "effective radius" of the electrical conductivity by $\int \rho\varphi(\rho)\, d\rho / \int \varphi(\rho)\, d\rho$ where $\varphi(\rho)$ denotes the contribution of the current. In the present case, the "effective depth" is calculated to be 1200 *km*. Hence, we may consider that the electrical state at a depth of about 1200 *km* is the most effective for the 27-day period variation.

3) B. N. LAHIRI and A. T. PRICE, *Trans. Roy. Soc. London* A **237** (1939), 509.

5. Summary and conclusion.

In addition to the electromagnetic induction by various variations which were already studied in the previous papers, the 27-day period variation, the period of which is larger than any other variation treated before, is studied here in regard to the electrical state of the earth's interior. Though the results of the analysis for various epochs are somewhat scattered, the relation between the external and internal parts is fairly consistent with the conductivity-distribution which was determined in the former studies. From the investigation of the induced currents, it is concluded that the electrical state at a depth of about 1200 *km* is the most effective for the 27-day period variation. Taking into consideration, then, the increase in the electrical conductivity in greater depth and the accompanying screening effect for current penetration, it will be of great difficulty to investigate the electrical properties at further depth even in case of much longer period variation.

In conclusion, the writer wishes to express his hearty thanks to Dr. T. Nagata for his helpful discussions. To Miss Y. Hishiyama who always assisted the writer throughout the numerical work the writer's hearty thanks are also due.

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6. 地球内の電磁感應およびその地球内部の電氣的性質との關係　第3報

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第 1, 2 報で解析した各種の地磁氣變化のほかに，地球磁場の 27 日週期變化を取扱つた。8 箇所の觀測所の資料を 27 日毎に半年間かさねあわせたものについて解析したところ，内外磁場の振幅比および位相差としてそれぞれ 4.0±0.4, 22°±16° という結果を得た。これは前報における地球内部の電氣的性質を考慮するとき，よく調和する。この結果より 27 日という長週期變化によつても，深さ 1500 *km* 程度以上の地球内部の電氣的性質をしらべることは困難なことが推察される。