10. *On the Coda Waves of Earthquake Motions.* (*Part 6*)

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(Read Dec. 18, 1945.—Received Dec. 10, 1950.)

CHAPTER 7. ON THE FREE OSCILLATION PERIODS OF THE EARTH'S SURFACE LAYERS REVEALED IN THE OSCILLATION PERIODS OF CODA WAVES.

§ 28. Waves that have the Periods of T_2 and T_3.

In the preceding Chapters 1 to 6[1], analyses have been carried out on the periods of waves in the coda oscillations of earthquake motions. Especially, from the analysis by Takahasi-Husimi's method explained in the previous chapter, it became clear that three sorts of waves of different periods, denoted T_1, T_2 and T_3, are to be found in the coda oscillations. Of these, T_1 becomes longer and longer as the epicentral distance increases, while T_2 and T_3 remain constant, their approximate values being $T_2 = 20$ sec., $T_3 = 16$ sec., irrespective of the difference of epicentral distances. These constant periods have not only resulted from the analysis by Takahasi-Husimi's method, but also have been obtained by other methods of analysis. Here we will compare the values of T_2 and T_3 periods arrived at in the respective chapters by different methods of analysis. In Table I-a and I-b they are tabulated in their corresponding portions. In this table we find first that T_3 period obtained by Takahasi-Husimi's method agrees quite well with the mean period T_m of the second coda waves observed in Chapters 4∼5, and secondly we notice in the T_m column of the same table that, in the portions where the existence of T_2 period has been demonstrated by periodgram analyses and analyses by Takahasi-Husimi's method, there certainly exists the period of 20 sec., though its existence is not so easily perceptible.

The mean value of predominant periods with respect to the entire duration of coda waves of each earthquake is given in Table II. From this Table the values of T_2 and T_3 are found to be $T_2 = 20.0 \pm 0.2$ sec., and $T_3 = 16.0 \pm 0.1$ sec. As these periods have constant values for all the earthquakes irrespective of their different epicentral distances, it is to be suspected

1) S. OMOTE, *Bull. Earthq. Res. Inst.*, **21** (1943), 458; **22** (1944), 140; **23** (1945), 47; **28** (1950), 49; **28** (1950), 285.

that they may represent the periods of the free oscillations of the earth's surface layers.

§ 29. Free Oscillations of the Surface Layers.

Many papers have been written on the free oscillations of the earth's surface layers. F. Omori[2] found the proper periods of the free oscillations to be 0.26, 0.60, 1.25 and 2.6 sec. at the Tokyo University in Hongo, Tokyo, and 0.30, 0.69, 1.34 and 1.88 sec. at the Central Meteorological Observatory

Table I.—a. Periods of coda oscillations in respective portions obtained by different methods.

T_2

Earthq. No.	Portion	T_2	T_m			T_A
	min. min.	sec.	sec.	sec.	sec.	sec.
34	40 ~ 42	19.8	12.7	14.5	14.5	
36	40 ~ 41	18.8	12.3	14.4		
″	41 ~ 42	21.7	14.4	16.9		
″	42 ~ 43	19.4	16.9	17.7		21
″	50 ~ 51	21.7	16.8	14.5		20
″	62 ~ 64	21.2	17.5	15.7	22.0	
37	50 ~ 52	20.8	13.7	16.3	13.5	
″	53 ~ 55	19.7	15.6	14.3	14.8	20
49	72 ~ 73	19.2	17.1	17.1		
″	73 ~ 74	19.7	17.1	17.8		
51	56 ~ 58	19.8	20.8	17.9	18.7	
″	58 ~ 59	19.0	18.7	18.5		
″	70 ~ 72	19.0	18.8	16.9	17.9	18
″	80 ~ 82	20.0	19.9	17.2	15.8	
52	80 ~ 81	19.6	16.6	16.6		
″	81 ~ 83	20.8	16 6	16.0	17.3	
″	67 ~ 63	19.1	18.3	18.4		18
″	80 ~ 81	20.9	18.4	15.9		19
″	82 ~ 83	20.1	17.4	16.1		

in Kanda. After that K. Suyehiro[3], using the wave analyser which he constructed, found that waves of the period of 0.3 sec. predominated at Hongo. Then M. Ishimoto[4], who followed with a detailed study, decided that the predominant period at Hongo was 0.3 sec. and that at Marunouti was

2) F. OMORI, *Publ. Earthq. Invest. Comm.*, **11** (1902), 1.
3) K. SUYEHIRO, *Bull. Earthq. Res. Inst.*, **1** (1926), 59; **7** (1929), 467.
4) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **12** (1934), 234.

about 0.6 sec. According to the study by T. Hagiwara[5] and the present writer the predominant period at Hongo has been found to be 0.2 sec.

In all the studies that have been made until today, the scope has been restricted to the observation of oscillations of short periods, or in other words, the observation of accelerations, as the primary object of the study was in correlating the period of earthquake motions to the destructive force of earthquake motions. Consequently the seismometers used for the above observations were those of relatively short-period types. F. Omori used those of the periods of 5~6 sec. M. Ishimoto and others used two accelerometers of 0.1 sec. and 1.0 sec. each. From the seismograms obtained by these

Table I.—b. Periods of coda oscillations in respective portions obtained by different methods.

T_3

Earthq. No.	Portion	T_3	T_m			T_A
	min. min.	sec.	sec.	sec.	see.	sec.
20	14 ~ 16	18.5	7.4	7.0	6.9	
37	69 ~ 71	16.3	16.4	13.7	14.5	
〃	72 ~ 73	16.7	13.8	16.0		
〃	102 ~ 104	15.9	16.1	16.2	15.8	17
49	99 ~ 101	16.7	16.8	19.2	19.0	
〃	101 ~ 103	16.4	19.0	16.1	18.3	
51	122 ~ 124	16.6	19.0	16.1	18.3	16
〃	150 ~ 151	16.5	16.1	16.6		
〃	151 ~ 153	16.6	16.6	16.5	16.5	
52	107 ~ 108	16.3	15.8	16.4		
〃	108 ~ 110	16.6	16.4	15.7	15.4	
〃	120 ~ 121	16.1	17.4	16.2		
〃	121 ~ 122	16.8	16.2	15.9		
〃	122 ~ 123	16.4	15.9	17.0		
52	138 ~ 139	16.3	16.2	16.3		16
〃	139 ~ 140	16.4	16.3	15.5		
〃	140 ~ 141	16.3	15.5	15.6		
〃	115 ~ 117	16.4	14.7	18.2	15.4	
〃	117 ~ 119	16.3	15.4	15.1	15.2	
〃	130 ~ 131	16.3	16.3	18.0	16.2	
〃	131 ~ 132	16.5	16.5	16.2		
〃	132 ~ 133	16.2	16.2		16.1	
〃	133 ~ 134	16.4	16.4	16.1	17.2	

5) T. HAGIWARA and S. OMOTE, *Bull. Earthq. Res. Inst.*, **16** (1938), 632; **17** (1939), 118.

short-period seismometers, it is very difficult, if not impossible, to find out long period waves such as were described in the preceding chapter, and it would be quite natural that the long-period oscillations with periods of 16 sec. and 20 sec. had long been overlooked and escaped unnoticed.

The question whether free oscillations are made possible by the existence of surface layers was studied by K. Sezawa[6)7)8)] and others some twenty years ago from the point of view of the dispersion of seismic disturbance due to its multiple reflections in these strata. Nevertheless, no satisfactory mechanism has yet been thought of to explain how the free oscillation of the surface layer is generated.

Table II.

T ; Periods determined by means of Takahasi-Husimi's method.
T_M; Periods determined from the curve of successive one minute mean of the periods of coda oscillations.

Earthq. No.	T		T_M
	T_2	T_3	
20	—	18.5	—
54	—	16.4	—
34	19.8	—	—
36	20.8	—	—
37	20.3	16.3	—
49	19.5	16.5	16.9
51	19.5	16.6	16.5
52	20.1	16.4	16.6

§ 30. Free Oscillation Periods of the Surface Layers Calculated by the Rayleigh-Ritz's Method. I. (In Case of Two Layers.)

Thanks to the detailed studies made by T. Matuzawa[9)10)11)12)] and others we have a sufficiently clear knowledge on the structure of the earth's surface layers in the Tokyo district, Japan. Now the question at issue is, when there takes place in a surface layer a stationary oscillation that has the

6) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **8** (1930), 1.
7) K. SEZAWA and G. NISHIMURA, *Bull. Earthq. Res. Inst.*, **8** (1930), 321.
8) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **10** (1932), 1.
9) T. MATUZAWA, K. HASEGAWA and S. HAENO, *Bull. Earthq. Res. Inst.*, **4** (1928), 85.
10) T. MATUZAWA, *ibid.* **5** (1928), 499; **6** (1929), 177.
11) T. MATUZAWA and T. FUKUTOMI, *ibid.* **10** (1932), 499.
12) T. MATUZAWA, K. YAMADA and T. SUZUKI, *ibid.* **7** (1929), 24.

node point at its lower boundary and the loop at the free surface of the earth's crust, what period is to be expected of such an oscillation?

Table III.
Arrangement of surface layers after T. Matuzawa.

Surface layer \ Velocity	V_P	V_S
km km	km/sec	km/sec
0 ~ 4	1.94	1.14
4 ~ 20	5.0	3.15
20 ~ 50	6.1	3.7
50 ~	7.5	4.5

The result of the studies by T. Matuzawa are reproduced in Table III, and Z. Kinosita[13] assumed one more thin layer in the upper most part of Matuzawa's first layer, the thickness d'' of that uppermost layer being $d''=360^m$, provided that $V_P=450$ m/s, and $V_S=240$ m/s. Then according to these studies we will assume that there are three layers in the earth's surface and that a stationary oscillation due to the distortional type of waves has been generated in them.

In the first place, assuming that a stationary wave takes place in the upper layer of Matuzawa, in which the thin layer explained by Kinosita is supposed to be lying in the upper part, we calculated the period of the stationary wave in these two layers by the Rayleigh-Ritz's method.

As seen in Fig. 1, we denote these layers as the first and the second layers according to the order of their vertical arrangement, draw the axis of x upward from the lower boundary of the second layer, and let u', ρ', λ' and μ' represent the displacement, density and Lamé's elastic constants of the lower medium. The similar displacement, density, and the elastic constants of the surface layer of the thickness d'' are to be represented by u'', ρ'', λ'' and μ''. The x-ordinates of the free surface and the lower boundary of the first layer will be taken as H'' and H'. Then, though the mode of the stationary wave that takes place in such layers may not be known precisely, there is the advantage that in calculating the period of such a wave by the Rayleigh-Ritz's method that period will be affected only slightly by the difference of the assumed mode.

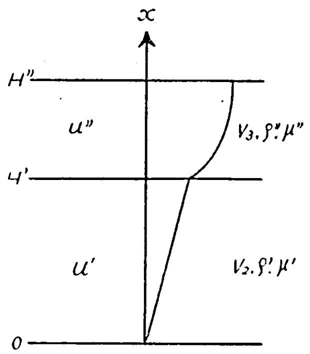


Fig. 1.

Then we will put

$$u'' = A\cos\frac{2\pi}{H''}x+B\,, \quad \ldots\ldots(30\cdot1)$$

$$u' = kx \quad \ldots\ldots(30\cdot2)$$

13) Z. KINOSITA, *Bull. Earthq. Res. Inst.*, **15** (1937), 965.

As the oscillation in question has a node at the lower boundary of the second layer and a loop at the free surface, the boundary conditions will be given by

$$\left.\begin{aligned} &x = 0 \;; && u' = 0 \\ &x = H' \;; && u' = u'', \quad \mu'\frac{\partial u'}{\partial x} = \mu''\frac{\partial u''}{\partial x} \\ &x = H'' ; && \mu''\frac{\partial u''}{\partial x} = 0 \,. \end{aligned}\right\} \qquad \ldots\ldots\ldots(30\cdot3)$$

From these conditions the wave forms in the respective layers will be expressed by

$$u' = kx\sin pt\,, \qquad \ldots\ldots\ldots(30\cdot4)$$

$$u'' = \left\{\frac{H'}{H''} + \frac{\lambda}{2\pi}\left(\cos\frac{2\pi H'}{H''} - \cos\frac{2\pi x}{H''}\right)\right\}kH''\sin pt\,, \qquad \ldots(30\cdot5)$$

where

$$\lambda = \frac{\mu'}{\mu''}\frac{1}{\sin\dfrac{2\pi H'}{H''}}\,.$$

The kinetic and the potential energies of such a wave will be calculated as follows.

1) Kinetic energy.

i) The kinetic energy L' in the second layer will be

$$\begin{aligned} L' &= \int_0^{H'}\frac{1}{2}\rho'\left(\frac{\partial u'}{\partial t}\right)^2 dx \\ &= \frac{1}{2}\rho'\int_0^{H'}k^2x^2p^2\cos^2 pt\,dx \\ &= \frac{1}{6}\rho' k^2 H'^3 p^2\cos^2 pt\,. \qquad \ldots\ldots\ldots(30\cdot6) \end{aligned}$$

ii) The kinetic energy L'' in the first layer will be

$$L'' = \int_{H'}^{H''}\frac{1}{2}\rho''\left(\frac{\partial u''}{\partial t}\right)^2 dx$$

$$= \frac{1}{2}\rho'' k^2 H''^2 p^2\cos^2 pt\left\{\frac{\lambda^2}{4\pi^2}\int_{H''}^{H'}\cos^2\frac{2\pi x}{H''}dx - \frac{\lambda}{\pi}\left(\frac{H'}{H''} + \frac{\lambda}{2\pi}\cos\frac{2\pi H'}{H''}\right)\int_{H''}^{H'}\cos\frac{2\pi x}{H''}dx\right.$$

$$\left. + \left(\frac{H'}{H''} + \frac{\lambda}{2\pi}\cos\frac{2\pi H'}{H''}\right)^2\int_{H''}^{H'}dx\right\}$$

$$=\frac{1}{2}\rho''k^2H''^2p^2\cos^2pt\left[\left\{\frac{\lambda^2}{8\pi^2}+\left(\frac{H'}{H''}+\frac{\lambda}{2\pi}\cos\frac{2\pi H'}{H''}\right)^2\right\}(H''-H')\right.$$

$$\left.+\frac{\lambda}{2\pi^2}\left\{H'+\frac{H''\lambda}{2\pi}\left(\cos\frac{2\pi H'}{H''}-\frac{1}{8}\right)\right\}\sin\frac{2\pi H'}{H''}\right]. \qquad \ldots\ldots\ldots\ldots\ldots\ldots(30\cdot7)$$

The total kinetic energy L_0 of the entire vibrating system will be given from (30·6) and (30·7) by,

$$L_0 = L'+L''$$

$$=\frac{1}{2}k^2p^2H''^2\cos^2pt\left[\frac{1}{3}\rho'\delta^3+\rho''\left\{\frac{\lambda^2}{8\pi^2}+\left(\delta+\frac{\lambda}{2\pi}\cos 2\pi\delta\right)^2\right\}(1-\delta)\right.$$

$$\left.+\frac{\rho''\lambda}{2\pi^2}\left\{\delta+\frac{\lambda}{2\pi}\left(\cos 2\pi\delta-\frac{1}{8}\right)\right\}\sin 2\pi\delta\right], \qquad \ldots\ldots\ldots\ldots\ldots\ldots(30\cdot8)$$

in this δ is put $\delta = H'/H''$.

2) Potential energy.

i) The potential energy V' in the second layer will be

$$V' = \int_0^{H'}\frac{1}{2}\mu'\left(\frac{\partial u'}{\partial x}\right)^2dx$$

$$= \frac{1}{2}\mu'k^2\sin^2pt\int_0^{H'}dx$$

$$= \frac{1}{2}\mu'k^2H'\sin^2pt\,. \qquad \ldots\ldots\ldots\ldots\ldots(30\cdot9)$$

ii) The potential energy V'' in the first layer will be

$$V'' = \int_{H'}^{H''}\frac{1}{2}\mu''\left(\frac{\partial u''}{\partial x}\right)^2dx$$

$$= \frac{1}{2}\mu''\lambda^2k^2\sin^2pt\int_{H'}^{H''}\sin^2\frac{2\pi}{H''}x\cdot dx$$

$$= \frac{1}{4}\mu''\lambda^2k^2\sin^2pt\left\{(H''-H')+\frac{H'}{4\pi}\sin\frac{4\pi H'}{H''}\right\}. \qquad (30\cdot10)$$

The total potential energy V_0 of the entire vibrating system will be given from (30·9) and (30·10) by,

$$V_0 = V'+V''$$

$$= \frac{1}{2}k^2H''\sin^2pt\left\{\delta\mu'+\frac{1}{2}\mu''\lambda^2\left(1-\delta+\frac{1}{4\pi}\sin 4\pi\delta\right)\right\}\ldots(30\cdot11)$$

where

$$\delta = \frac{H'}{H''},\quad \lambda = \frac{\mu'}{\mu''}\Big/\sin 2\pi\delta\,.$$

If we equate the maximum values of the kinetic and potential energies, the frequency equation will be obtained.

$$(L_0)_{\max} = (V_0)_{\max}$$

i.e.

$$p^2H''^2\left[\frac{1}{3}\rho'\delta^3 + \rho''\left\{\frac{\lambda^2}{8\pi^2} + \left(\delta + \frac{\lambda}{2\pi}\cos 2\pi\delta\right)^2\right\}(1-\delta) + \rho''\frac{\lambda}{2\pi^2}\left\{\delta + \frac{\lambda}{2\pi}\left(\cos 2\pi\delta - \frac{1}{8}\right)\right\}\sin 2\pi\delta\right]$$

$$= \delta\mu' + \frac{1}{2}\mu''\lambda^2\left(1-\delta+\frac{1}{4\pi}\sin 4\pi\delta\right). \qquad \text{.........(30·12)}$$

The period T_1 will be obtained

$$T_1 = \frac{2\pi}{p} = \frac{2\pi H''}{V_2}\sqrt{\frac{R_1}{S_1}}, \qquad \text{..................(30·13)}$$

where

$$R_1 = \frac{1}{3}\delta^3 + \frac{\rho''}{\rho'}\left\{\frac{\lambda^2}{8\pi^2} + \left(\delta+\frac{\lambda}{2\pi}\cos 2\pi\delta\right)^2\right\}(1-\delta) + \frac{\lambda}{2\pi^2}\frac{\rho''}{\rho'}\left\{\delta+\frac{\lambda}{2\pi}\left(\cos 2\pi\delta - \frac{1}{8}\right)\right\}\sin 2\pi\delta,$$

$$S_1 = \delta + \frac{1}{2}\frac{\mu''}{\mu'}\lambda^2(1-\delta+\frac{1}{4\pi}\sin 4\pi\delta),$$

$$\lambda = \frac{H'}{H''}\frac{1}{\sin 2\pi\delta},$$

$$\delta = \frac{H'}{H''}.$$

Table IV.

Physical constants of surface layers.

Surface layer	V_S	μ	ρ
1st layer	km/sec. 0.24	C.G.S. 8.9×10^8	gr/cm³ 1.59
2nd layer	1.14	2.26×10^{10}	2.2

Next we will assume that the lower boundary of the second layer is at the depth of 4 km as calculated by T. Matuzawa, and that the numerical constants determining the physical qualities of these first and second layers are such as will be seen in Table IV, after T. Matuzawa and Z. Kinosita.

Then the period of the stationary wave calculated from the equation (30·13) will be shown in Fig. 2 for different values of the ratios of the thicknesses

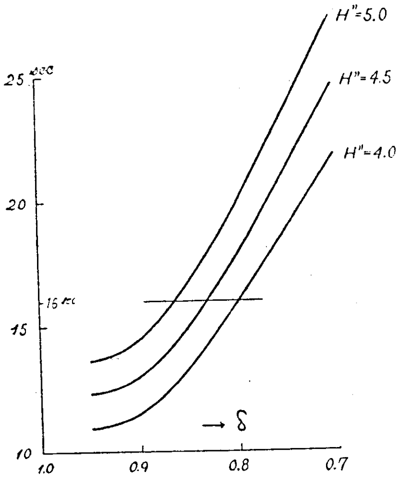


Fig. 2. Free oscillation periods of surface layers (in case of two layers). $\delta = H'/H''$.

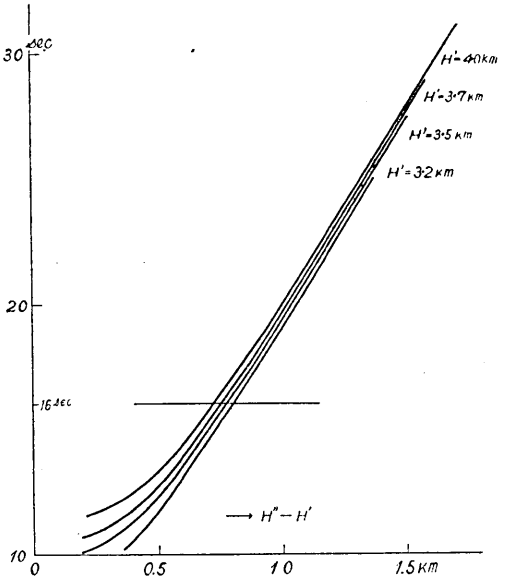


Fig. 3. Free oscillation period of surface layer (in case of two layers). Abscissa is the thickness of the upper layer; the parameter of the family of curves is the thickness of the lower layer.

Table V.

Thickness of surface layer within which the free oscillations of the period of 16.0 sec. and 16.4 sec. can take place.

d'	d'' ($T = 16.0$ sec.)	d'' ($T = 16.4$ sec.)
km	km	km
4.0	0.730	0.762
3.7	0.761	0.792
3.5	0.778	0.806
3.2	0.803	0.829

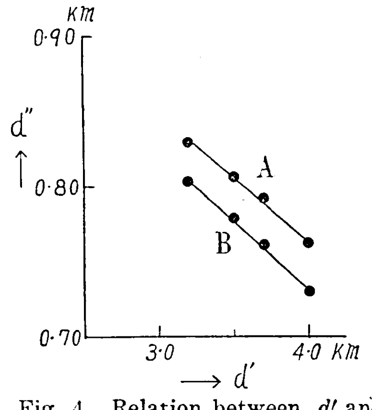


Fig. 4. Relation between d' and d'' when the free oscillation period T is T=16.4 sec. (A), and T=16.0 sec. (B).

of the first and second layers. In this figure, the parameter is the depth of the lower boundary of the second layer. It will be seen from this figure that the value of T_I will be 16 sec. when $H''=4.5$ km and $\delta = 8.3$, or when $H'' = 5.0$ km and $\delta = 8.6$. For the purpose of making clear the relation between the period of the stationary waves and the thicknesses of the respective layers, the values of T_I are given in Fig. 3 for different values of d'', the thickness of the first layer, with d', the thickness of the second layer, as the parameter. From this the periods of free oscillations will easily be known for any desired combination of thicknesses of the first and the second layers. For instance, the values of d' and d'' that make $T_I=16.0$ sec. and $T_I = 16.4$ sec. respectively will be given by Table V and Fig. 4.

As the relation between d' and d'' in Fig. 4 is approximately linear, we can put

$$d'' = 1.10-0.0909\, d'$$

for the period of 16.0 sec., and

$$d'' = 1.10-0.0929\, d'$$

for that of 16.4 sec. The thickness d'' of the uppermost layer was found to be 400 m by Z. Kinosita, but, in our case d'' is closely related to d' and it will be impossible to assume an extraordinarily large value of d', so that when the existence of the periods of 16.0 and 16.4 sec. is granted, the thicknesses of the first and the second layers will be estimated as,

$$d' = 3.4 \text{ km},\quad d'' = 0.79 \text{ km} \quad \text{for} \quad T_I = 16.0 \text{ sec.}$$

$$d' = 3.5 \text{ km},\quad d'' = 0.80 \text{ km} \quad \text{for} \quad T_I = 16.4 \text{ sec.}$$

§ 31. Free Oscillation Periods of the Surface Layers Calculated by the Rayleigh-Ritz's Method. II. (In Case of Three Layers.)

In the preceding chapter, on the basis of rough calculations, we expected that, if a stationary oscillation due to transversal waves was to be generated within Matuzawa's upper layer, its period would be 16 sec. Now we have had a proof that the existence of a stationary oscillation of 16 sec. is possible when surface layers of certain thicknesses are arranged as assumed above.

Table VI.
Notation of the surface layers.

Surface layer	Displacement	Velocity	Rigidity	Density	Thickness
1	u''	V_3	μ''	ρ''	$d''=H''-H'$
2	u'	V_2	μ'	ρ'	$d'=H'-H$
3	u	V_1	μ	ρ	$d=H$

In the second place let us consider the waves of 20 sec. Already in Chapter 6 we saw that a stationary wave of 20 sec. period was to be expected in layers that have the lower boundary at the depth of about 20 km, and that this corresponded to the depth of the second layer which T. Matuzawa had assumed. We will now proceed to consider a third layer lying beneath the two layers mentioned above, and assume that a stationary oscillation takes place in these three layers with its node at the lower boundary of the third layer and a loop at the free surface. If we use the notation shown in Table VI and Fig. 5, the displacements of the waves in the respective layers will be given by

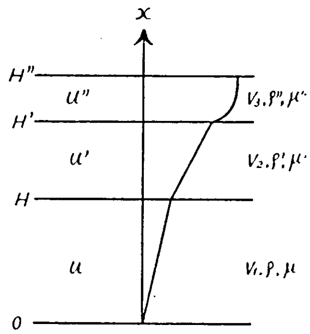


Fig. 5.

$$\left.\begin{aligned} u'' &= A\cos\frac{2\pi}{H''}x+B\,,\\ u' &= Cx+D\,,\\ u &= kx\,,\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots(31\cdot1)$$

where A, B, C, D and k are arbitrary constants to be determined from the following boundary conditions,

$$\left.\begin{aligned} &x=0\,; && u=0 && \\ &x=H\,; && u=u', && \mu\frac{\partial u}{\partial x}=\mu'\frac{\partial u'}{\partial x}\\ &x=H'\,; && u'=u'', && \mu'\frac{\partial u'}{\partial x}=\mu''\frac{\partial u''}{\partial x}\\ &x=H''\,; && && \mu''\frac{\partial u''}{\partial x}=0\,. \end{aligned}\right\}\ldots\ldots(31\cdot2)$$

From these conditions the wave forms in the respective layers will be expressed by

$$\left.\begin{aligned} u &= kx\sin pt\,,\\ u' &= \left\{\frac{\mu}{\mu'}x+H\left(1-\frac{\mu}{\mu'}\right)\right\}k\sin pt\,,\\ u'' &= \left\{\frac{\mu}{\mu'}H'+\left(1-\frac{\mu}{\mu'}\right)H+\frac{H''}{2\pi}\alpha\cos\frac{2\pi H'}{H''}-\frac{H''}{2\pi}\alpha\cos\frac{2\pi}{H''}x\right\}k\sin pt\,,\end{aligned}\right\}(31\cdot3)$$

where

$$a = \frac{\mu}{\mu''}\frac{1}{\sin\dfrac{2\pi H'}{H''}}.$$

The kinetic and the potential energies of such a wave will be calculated as follows.

1) Kinetic energy.

i) The kinetic energy L in the third layer will be

$$L = \int_0^H \frac{1}{2}\rho\left(\frac{\partial u}{\partial t}\right)^2 dx$$

$$= \frac{1}{6}\rho k^2 p^2 H^3 \cos^2 pt. \quad \text{(31·4a)}$$

ii) The kinetic energy L' in the second layer will be

$$L' = \int_H^{H'} \frac{1}{2}\rho'\left(\frac{\partial u'}{\partial t}\right)^2 dx$$

$$= \frac{1}{2}\rho' k^2 p^2 \cos^2 pt\left\{\frac{1}{3}\frac{\mu^2}{\mu'^2}(H'^3 - H^3) + \frac{\mu}{\mu'}\left(1-\frac{\mu}{\mu'}\right)H(H'^2 - H^2)\right.$$

$$\left. + \left(1-\frac{\mu}{\mu'}\right)^2 H^2(H'-H)\right\}. \quad \text{(31·4b)}$$

iii) The kinetic energy L'' in the first layer will be

$$L'' = \int_{H'}^{H''} \frac{1}{2}\rho''\left(\frac{\partial u''}{\partial t}\right)^2 dx$$

$$= \frac{1}{4\pi^2}\rho'' k^2 p^2 H''^2 \cos^2 pt\left\{\left(\frac{a}{4} + \frac{2\pi^2}{H''^2}M^2\right)(H''-H')\right.$$

$$\left. - \frac{1}{16\pi}a^2 H'' \sin\frac{4\pi H'}{H''} + aM\sin\frac{2\pi H'}{H''}\right\}, \quad \text{(31·4c)}$$

where

$$M = \frac{\mu}{\mu'}H' + \left(1-\frac{\mu}{\mu'}\right)H + \frac{H''}{2\pi}a\cos\frac{2\pi H'}{H''}.$$

The total kinetic energy L_0 of the entire vibrating system will be given by,

$$L_0 = L + L' + L''$$

$$= \frac{1}{2}k^2 p^2 H''^3 \cos^2 pt\left[\frac{1}{3}\rho\frac{H^3}{H''^3} + \rho'\left\{\frac{1}{3}\frac{\mu^2}{\mu'^2}\left(\frac{H'^3}{H''^3} - \frac{H^3}{H''^3}\right)\right.\right.$$

$$\left. + \frac{\mu}{\mu'}\left(1-\frac{\mu}{\mu'}\right)\frac{H}{H''}\left(\frac{H'^2}{H''^2} - \frac{H^2}{H''^2}\right) + \left(1-\frac{\mu}{\mu'}\right)^2\frac{H^2}{H''^2}\left(\frac{H'}{H''} - \frac{H}{H''}\right)\right\}$$

$$\left. + \frac{1}{2\pi^2}\rho''\left\{\left(\frac{a^2}{4} + 2\pi^2 N^2\right)\left(1-\frac{H'}{H''}\right) - \frac{1}{16\pi}a^2\sin\frac{4\pi H'}{H''} + aN\sin\frac{2\pi H'}{H''}\right\}\right] \quad \text{(31·5)}$$

where $$N = \frac{1}{H''} M .$$

2) Potential energy.

i) The potential energy V in the third layer will be

$$V = \int_0^H \frac{1}{2} \mu \left(\frac{\partial u}{\partial x} \right)^2 dx$$

$$= \frac{1}{2} \mu k^2 H \cdot \sin^2 pt . \qquad \ldots\ldots\ldots\ldots\ldots\ldots (31{\cdot}6a)$$

ii) The potential energy V' in the second layer will be

$$V' = \int_H^{H'} \frac{1}{2} \mu' \left(\frac{\partial u'}{\partial x} \right)^2 dx$$

$$= \frac{1}{2} \frac{\mu^2}{\mu'} k^2 (H'-H) \sin^2 pt . \qquad \ldots\ldots\ldots\ldots (31{\cdot}6b)$$

iii) The potential energy V'' in the first layer will be

$$V'' = \int_{H'}^{H''} \frac{1}{2} \mu'' \left(\frac{\partial u''}{\partial x} \right)^2 dx$$

$$= \frac{1}{4} \mu'' \alpha^2 k^2 \sin^2 pt \left\{ (H''-H') + \frac{H''}{4\pi} \sin \frac{4\pi H'}{H''} \right\} . \qquad \ldots\ldots (31{\cdot}6c)$$

The total potential energy V_0 of the entire vibrating system will be given by,

$$V_0 = V + V' + V''$$

$$= \frac{1}{2} k^2 \sin^2 pt \left\{ \mu H + \frac{\mu^2}{\mu'} (H'-H) + \frac{1}{2} \frac{\mu^2}{\mu''} \frac{H''-H'}{\sin^2 \frac{2\pi H'}{H''}} \right.$$

$$\left. + \frac{1}{4\pi} \frac{\mu^2}{\mu''} H'' \cot \frac{2\pi H'}{H''} \right\} . \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31{\cdot}7)$$

If we equate the maximum values of the kinetic and potential energies in (31·5) and (31·7), we get a frequency equation,

$$(L_0)\max = (V_0)\max$$

i.e.

$$p^2 H''^2 \left[\frac{1}{3} \rho \delta^3 + \rho' \left\{ \frac{1}{3} \frac{\mu^2}{\mu'^2} (\zeta^3 - \delta^3) + \frac{\mu}{\mu'} \left(1 - \frac{\mu}{\mu'} \right) \delta (\zeta^2 - \delta^2) + \left(1 - \frac{\mu}{\mu'} \right)^2 \delta^2 (\zeta - \delta) \right\} \right.$$

$$\left. + \frac{1}{2\pi^2} \rho'' \left\{ \left(\frac{\alpha^2}{4} + 2\pi^2 N^2 \right) (1-\zeta) - \frac{1}{16\pi} \alpha^2 \sin 4\pi\zeta + \alpha N \sin 2\pi\zeta \right\} \right]$$

$$= \mu\delta + \frac{\mu^2}{\mu'} (\zeta - \delta) + \frac{1}{2} \frac{\mu^2}{\mu''} (1-\zeta) \frac{1}{\sin^2 2\pi\zeta} + \frac{1}{4\pi} \frac{\mu^2}{\mu''} \cot 2\pi\zeta . \qquad \ldots\ldots (31{\cdot}8)$$

where
$$\delta = \frac{H}{H''}, \qquad \zeta = \frac{H'}{H''}.$$

The period $T_{\rm II}$ will be given by

$$T_{\rm II} = 2\pi/p$$
$$= \frac{2\pi H''}{V_1}\sqrt{\frac{R_{\rm II}}{S_{\rm II}}}, \qquad \text{..................(31·9)}$$

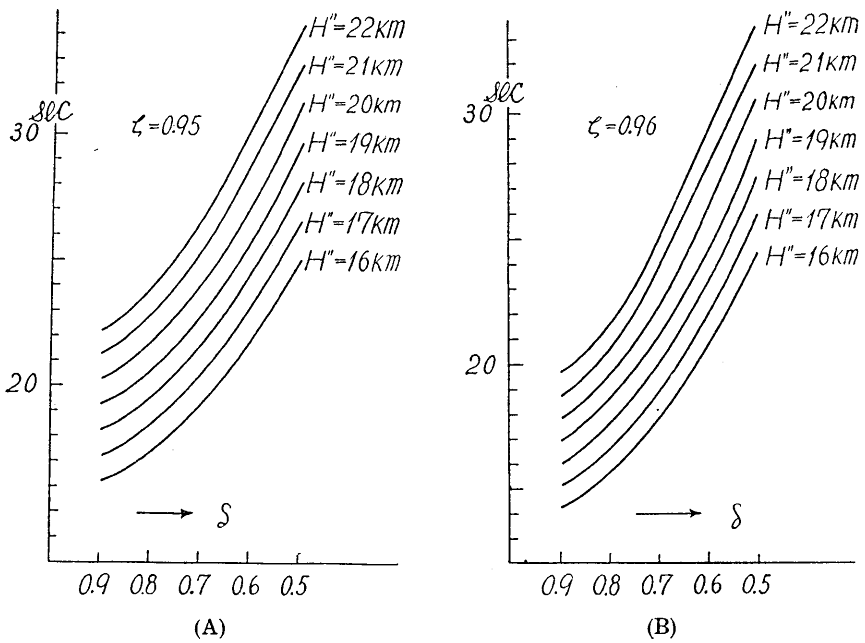
where

$$R_{\rm II} = \frac{1}{3}\delta^3 + \frac{\rho'}{\rho}\left\{\frac{1}{3}\frac{\mu^2}{\mu'^2}(\zeta^3-\delta^3) + \frac{\mu}{\mu'}\left(1-\frac{\mu}{\mu'}\right)\delta(\zeta^2-\delta^2) + \left(1-\frac{\mu}{\mu'}\right)^2\delta^2(\zeta-\delta)\right\}$$
$$+\frac{1}{2\pi^2}\frac{\rho''}{\rho}\left\{\left(\frac{\alpha^2}{4}+2\pi^2N^2\right)(1-\zeta) - \frac{1}{16\pi}\alpha^2\sin 4\pi\zeta + \alpha N \sin 2\pi\zeta\right\}$$

$$S_{\rm II} = \delta + \frac{\mu}{\mu'}(\zeta-\delta) + \frac{1}{2}\frac{\mu}{\mu''}(1-\zeta)\frac{1}{\sin^2 2\pi\zeta} + \frac{1}{4\pi}\frac{\mu}{\mu''}\cot 2\pi\zeta$$

$$\alpha = \frac{\mu}{\mu''}\frac{1}{\sin 2\pi\zeta}$$

$$N = \frac{\mu}{\mu'}\zeta + \left(1-\frac{\mu}{\mu'}\right)\delta + \frac{\alpha}{2\pi}\cos 2\pi\zeta$$

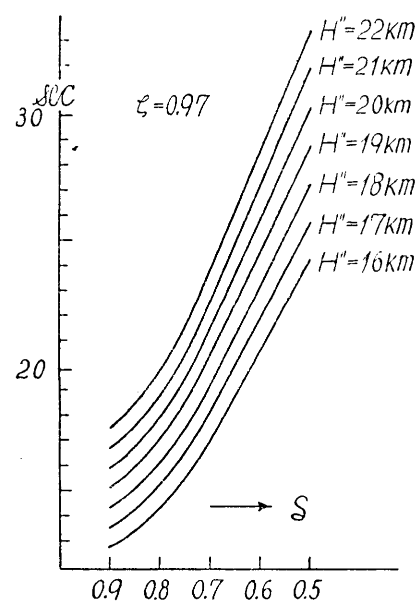


(A) (B)

Fig. 6. Free oscillation period of surface layer (in case of three layers). Abscissa δ is H/H'', parameter H'' is the total thickness of all layers. (A) $\zeta=0.95$; (B), $\zeta=0.96$; (C) $\zeta=0.97$

$$\delta = \frac{H}{H''}, \quad \zeta = \frac{H'}{H''},$$

The numerical values of the physical constants of the first and the second layers are already given in Table IV, and that of the third layer will be given by $V_3 = 3.7$ km/sec., $\mu = 26.8 \times 10^{10}$ *C.G.S.*, and $\rho = 2.7$ gr/cm^3. By means of these values, the period T_{II} of the stationary waves will be calculated from the equation (31·9), and the results will be shown in Fig. 6, the parameter of the figure being the depth of the lower boundary of the third layer. It is necessary from the study in the previous paragraph that the thickness of the uppermost layer should be about 0.7 km, whence it follows that the value of ζ (which is equals to H'/H'') must be about 0.95~0.98 with respect to the assumed value of H'' (about 20 km). Fig. 6, (A), (B) and (C) are prepared for three different values of ζ, $\zeta = 0.95$, 0.96 and 0.97, respectively.



(C)
Fig. 6.

For the purpose of showing more clearly the relation between the period of the stationary wave and the thicknesses of the respective layers, Fig. 7 has been prepared with the thickness of the second layer as abscissa. On the basis of this figure the combinations of the thicknesses of the respective layers that allow the period of the stationary waves to be 20 sec. will be supposed to be as shown in Table VII, where d'' and d' are the thicknesses of the first and second layers and d that of the third layer. The relations between d'' and d' are shown by curves A, B and C in Fig. 8 for the different values of ζ. The values of free oscillations in Table V that have been obtained in the case of the period of 16 sec. are also shown in the same Figure by a broken line D. Now, if we assume that there are three layers existing near the surface, and that a stationary oscillation of 16 sec. is taking place in the first and second layers and an oscillation of 20 sec. in the first, second and third layers, it will follow that the thicknesses of d', d'' and d that have been obtained independently in Table V and Table VII have to be such as will not contradict each other. If we adopt the values of d' and d'' at the point where the two curves B and D cross each other in Fig. 8, the above conditions will at least be fulfilled.

These values are

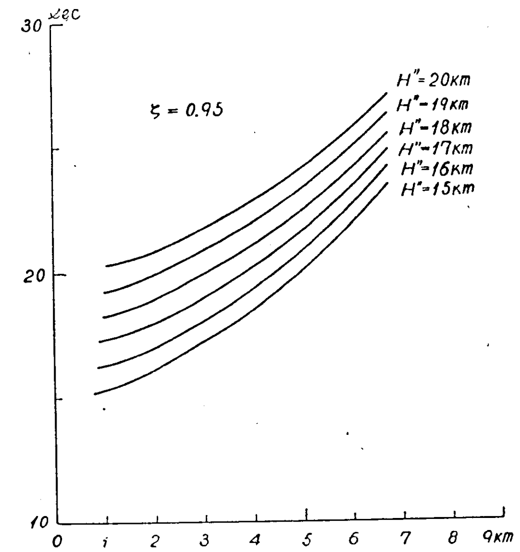
$$d' = 2.95 \text{ km}, \quad d'' = 0.83 \text{ km},$$

and, ζ being equal to 0.97 for curve B, d will be found to be

$$d = 17.1 \text{ km}.$$

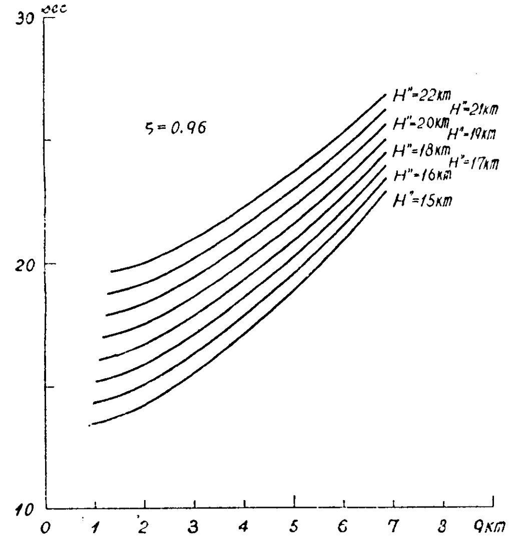
By these steps we have been able to find out the thicknesses of the respective layers, but the value of the thickness of the first layer here obtained is somewhat greater than the value determined by Kinosita. But the depth to the lower boundary of the second layer is 3.7 km which agrees quite well with Matuzawa's value of 4 km, and the thickness of the third layer which we find to be 17.1 km is also in good accord with the thickness 16 km of Matuzawa's second layer.

Thus we have been able to ascertain that, when the two constant periods found in the coda oscillations of earthquake motions are assumed to represent the free oscillation periods of the surface layers, the calculated values of the thickness of the respective surface layers show a good accord with the corresponding values obtained



(A)

Fig. 7. Free oscillation period of surface layer (in case of three layers). Abscissa is the thickness of middle layer, parameter is the total thickness of all layers.
(A), $\zeta=0.95$; (B), $\zeta=0.96$; (C), $\zeta=0.97$

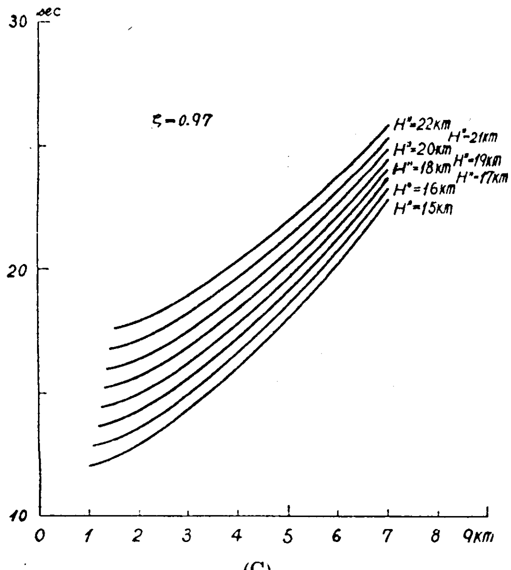


(B)

Fig. 7.

Table VII.
Thickness of surface layers within which the free oscillation of the period of 20 seconds can take place.

H''	$\zeta = 0.95$		$\zeta = 0.96$		$\zeta = 0.97$	
	d'	d''	d'	d''	d'	d''
km	km	km	km	km	km	km
22	—	—	1.95	0.88	3.88	0.66
21	—	—	2.80	0.84	4.29	0.63
20	—	—	3.54	0.80	4.63	0.60
19	2.12	0.95	4.19	0.76	4.94	0.57
18	3.08	0.90	4.55	0.72	5.24	0.54
17	3.82	0.85	4.95	0.68	5.49	0.51
16	4.42	0.80	5.27	0.64	5.68	0.48
15	4.89	0.75	5.54	0.60	5.88	0.45



(C)
Fig. 7.

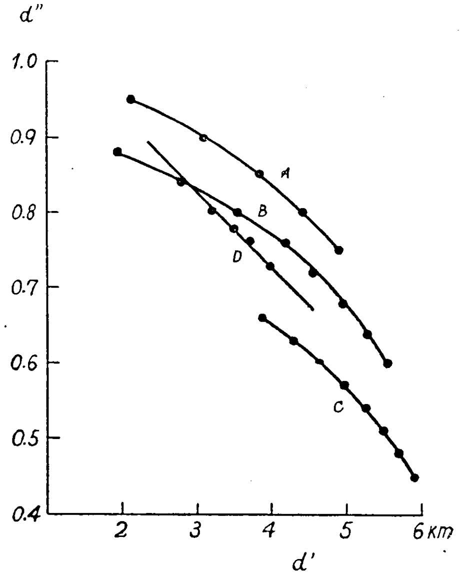


Fig. 8. Relation between d' and d'', when the free oscillation period T of surface layers (in case of three layers) is 20 seconds. The parameter of the curves A, B and C is ζ, curve D represent the relation between d' and d'' when the free oscillation of 16 seconds pericd takes place in the two surface layers.

from the seismometerical point of view.

In the preceding chapters we have pointed out that waves with the period of 20 sec. are seen in the first coda and those with the period of 16 sec. in the second coda, but closer examination shows, with respect to some earthquakes, that waves of 16 sec. are also seen existing in the first coda oscillations, and that this occurs only when the T_1 period of such coda oscillations is about 14~15 sec., namely very near to the period of 16 sec. On the other hand, when the coda periods are about 18~19 sec., free oscillation of the period of 20 sec. are seen to exist. Generally speaking it appears that, in accordance with the period of the external forces, that type of free oscillation whose period is nearer the period of the said forces is generated.

Now we come to the conclusion that the stationary oscillations in the surface layers whose thicknesses are 3.9 km and 21 km have periods of 16 and 20 sec. respectively. In the preceding chapter we assumed that the two sorts of periods, $T_2=20$ sec. and $T_3=16$ sec., that appear in the coda oscillations of earthquake motions may represent the free oscillation periods of the surface layers, as they display such constant values regardless of the differences in their epicentral distances.

The results of the calculations carried out in the present Chapter seem to give an additional support to this assumption.

CHAPTER 8. ON THE OSCILLATION PERIODS OF PROPAGATIVE SEISMIC WAVES SEEN IN THE CODA OSCILLATIONS OF EARTHQUAKE MOTIONS.

§ 32. Waves of T_1 Period.

In Paragraph 27 we mentioned that three types of different periods T_1, T_2 and T_3 are seen in the coda waves. Two of these three periods, namely T_2 and T_3, were studied in detail in the preceding chapter, where it was concluded that they represent the free oscillation periods of the stationary waves that take place in the surface layers. In the present chapter we will go on to study the T_1 period.

The T_1 period obtained by Takahasi-Husimi's method in Chapter 6 are compared in Table VIII with the mean periods T_m obtained in Chapters 1~4, with respect to each corresponding portion. The predominant periods T_A obtained by the method of periodgram analysis are also tabulated in the third column of the same Table.

It will be noticed in the Table that T_1 and T_m of distant earthquakes whose epicentral distances are greater than 10,000 km, have different values from time to time, a fact ascribable to the effect of the waves of T_2 period that coexisted with those of T_1 period. On the contrary, T_m of near earthquakes that has relatively small values is not much affected by T_2, so that

T_1 and T_m of the corresponding portions of near earthquakes are seen to agree quite well. This tendency will be noticed still more clearly in Table IX, in which the predominant period T_M of the respective earthquakes—which is the mean of T_m with regard to each earthquake—and the mean of T_1 of the corresponding earthquakes are compared. In particular the mean value of T_1 period and the mean value of T_m are seen to have the same values in earthquakes whose epicentral distances are smaller than 10,000 km. T_A also agrees very well with T_1 and T_M.

Table VIII.

T_1; Period determined by means of Takahasi-Husimi's method.
T_m; Means of the periods of respective one minute of the coda oscillations.
T_A; Period determined by periodgram analysis.

Earthq. No.	Portion	T_1	T_m			T_A	Earthq. No.	Portion	T_1	T_m			T_A
	min min	sec	sec	sec	sec	sec		min min	sec	sec	sec	sec	sec
16	10∼12	7.0	6.3	6.2	6.4	6.5	49	90∼92	17.3	17.0	17.6	16.3	
〃	12∼14	7.3	6.4	6.3	6.6		〃	92∼94	17.5	16.3	19.3	15.2	
〃	16∼17	7.1	6.8	6.6			〃	101∼103	17.0	19.0	16.1	18.3	
19	14∼15	6.4	6.5	7.6			51	56∼58	17.6	18.8	18.0	17.4	
20	15∼17	6.8	7.0	6.9	8.0		〃	58∼60	17.8	17.4	17.4	17.8	
〃	11∼13	6.9	7.4	6.6	7.2	7.0	〃	71∼73	17.3	16.1	17.3	15.6	18.0
〃	14∼16	5.9	6.3	6.9	7.1		〃	77∼79	17.5	17.5	16.5	16.5	
34	40∼42	13.1	12.7	14.5	14.5		〃	79∼81	17.4	16.5	16.2	16.9	
36	43∼44	15.9	17.7	15.7			〃	59∼61	17.8	18.5	19.2	17.2	
〃	50∼51	15.1	16.8	14.5			〃	70∼72	17.8	18.8	16.9	17.9	
〃	51∼52	15.2	14.5	13.6		14.0	〃	72∼74	17.6	17.9	17.9	18.0	
〃	60∼62	15.6	17.5	15.8	17.5		〃	82∼84	17.4	15.8	17.5	16.4	
〃	71∼73	15.3	18.9	13.5	17.0		52	66∼68	18.3	17.9	18.4	19.8	18.0
37	71∼72	14.9	14.5	13.8			〃	68∼70	18.3	19.8	17.5	17.3	
〃	82∼84	14.9	15.4	12.4	16.9	14.0	〃	81∼82	18.7	16.6	16.0	17.3	19.0
〃	93∼95	15.9	14.6	17.4	19.6		〃	83∼85	18.0	17.3	19.1	26.2	
〃	100∼102	15.6	19.8	15.4	16.1		〃	105∼107	18.2	15.8	18.1	15.8	
〃	102∼104	15.5	16.1	16.2	15.8	17.0	〃	62∼63	18.3	18.2	21.9		
49	69∼70	17.1	17.1	16.7			〃	63∼64	18.2	21.9	19.9		
〃	71∼72	17.9	17.3	17.1			〃	66∼67	18.3	16.9	18.3		
〃	82∼84	17.9	16.7	15.8	16.3	16.0	〃	83∼84	18.0	16.1	18.0		
〃	84∼85	16.6	16.3	17.8									

In Chapters 1∼4, where we calculated the predominant period from the mean periods of the successive one-minute means, there had been left some doubts as to whether this predominant period calculated in such a way represented a period really existing in the coda oscillations. Now, from Table IX we are convinced that the predominant period T_M determined from the mean period represents a period really existing in the coda oscil-

Table IX.

Comparison of T_1, T_M and T_A.

T_1; Predominant period by means of Takahasi-Husimi's method.
T_M; Predominant period determined from the curve of period of successive one minute means of the coda oscillations.
T_A; Predominant period determined by periodgram analysis.

Earthq. No.	T_1	T_M	T_A
	sec.	sec.	sec.
16	7.1	6.5	6.5
19	6.4	6.8	—
20	6.5	7.2	7
54	10.7	—	10
34	13.1	13.4	—
36	15.4	15.9	14
37	15.4	15.3	15
49	17.3	17.5	16
51	17.6	18.1	18
52	18.3	17.7	18

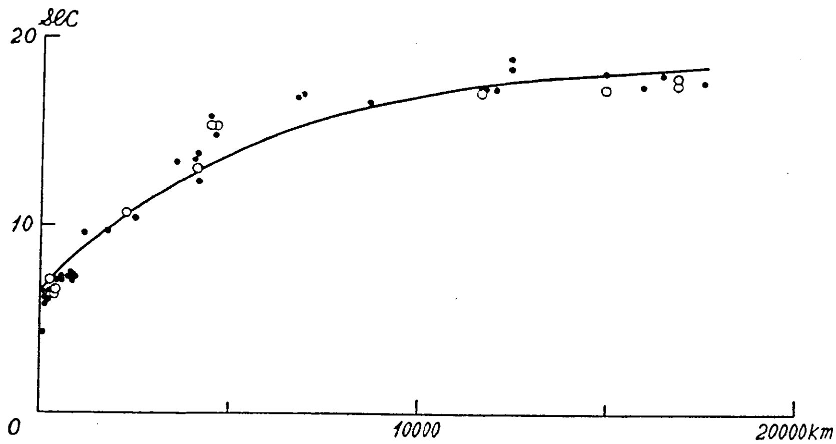


Fig. 9. Relation between T_1 period and epicentral distances.

lations of the respective earthquakes. As we have already noted, in this study of coda oscillations, these analyses by Takahasi-Husimi's method and periodgram analysis were carried out with only eight earthquakes selected out of 54 which were studied by the mean period method in Chapters 1∼4. The results here obtained, however, lead us to the conjecture that the predominant coda periods of the remaining 46 earthquakes to which neither Takahasi-Husimi's method of analysis nor periodgram analysis has been applied will represent periods really existing in their coda oscillations, and further that these predominant periods are ones that belong to the T_1 group.

If it be allowed to make such a conjucture, we have now come to know the T_1 period of the coda oscillations of 54 earthquakes whose epicentral distances range from 30 km to almost 18,000 km. The relation between the period T_1 and the epicentral distance Δ of the respective earthquakes will be found from Table IV in Chapter 1, Table IV in Chapter 3, Table VI in Chapter 4 and Table VII in Chapter 6, the result being given in Fig. 9, in which it will be clearly observed that as the epicentral distance increases T_1 also increases.

§ 33. The Relation Between T_1 Period and the Epicentral Distance.

On the basis of seismometerical observations it has been pointed out by many investigators that the period of seismic waves grows larger as they are propagated to long distances. An exhaustive study of this problem, however, is only possible after observations of earthquake motions have been carried out on a worldwide scale. Owing to this difficulty, coupled by numerous others, no sufficient data on this problem have yet been collected until now. In that sense, the curve in Fig. 9 will be one of the important data concerning the propagation of seismic waves. Now, using these data and consulting the method introduced by B. Gutenberg, the present writer wants to evolve a way of interpretation cencerning the phenomenon of the period of waves increasing with the epicentral distance.

Assuming that, by the effect of the friction within the earth's crust, the period of seismic waves becomes longer and longer with the increase of epicentral distance, B. Gutenberg[14], using the equation obtained by K. Sezawa[15], put

$$\frac{\partial^2 \Theta}{\partial t^2} = \frac{\lambda+2\mu}{\rho} \nabla^2 \Theta + \frac{\lambda'+2\mu'}{\rho} \frac{\partial \nabla^2 \Theta}{\partial t} \qquad (33\cdot 1)$$

$$\frac{\partial^2 \omega}{\partial t^2} = \frac{\mu}{\rho} \nabla^2 \omega + \frac{\mu'}{\rho} \frac{\partial \nabla^2 \omega}{\partial t} \qquad (33\cdot 2)$$

14) B. GUTENBERG, Hand. d. Geophys., IV (1932). 16.

15) K. SEZAWA, *Bull. Earthq. Res. Inst.*, **2** (1927), 13.

where Θ and ω give the dilatation and rotation of the body, λ and μ are Lamé's constants, and λ' and μ' the voluminal and equivoluminal viscosities. Then, for instance, the displacement near the origin will be given by $t=0$, $u=\frac{B}{b}e^{-x^2/b^2}$; and the value of u at any time and place, by

$$u=\frac{B}{2\sqrt{4Wt+b^2}}\left[e^{-\frac{x+Vt}{4Wt+b^2}}+e^{-\frac{x-Vt}{4Wt+b^2}}\right]\ldots\ldots\ldots\ldots(33\cdot3)$$

This shows that the wave form becomes longer and longer with the increase of the epicentral distance. In this, V is the velocity of the wave, W is a term representing the effect of viscosity and is given by $W=(\lambda'+2\mu')/2\rho$. Now, with regard to the waves determined by equation (33·3), we will define the wave-length (which we will denote by L) as twice the length of the distance between the two points, a point where u is equal to half the amplitude of a maximum, and another where u is equal to half the amplitude of the next minimum.

Now, L will be given by,

$$L=(2\cdot86\,b^2+11.2\;Wt)^{1/2}=(L_0+11.2\;Wt)^{1/2}\ldots\ldots(33\cdot4)$$

Taking the period of the wave as T, the epicentral distance as Δ, we get, since $L=VT$ and $t=\Delta/V$, an equation that combines T with Δ,

$$T=\left(T_0^2+\alpha\frac{\Delta}{V^3}\right)^{1/2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33\cdot5)$$

This shows that the period increases hyperbolically with the epicentral distance. Since this relation seems to be perceptible in Fig. 9, we calculated the values of T_0 and α by means of the least square method, using the

Table X.
Values of T_0, α and η.

T_0	V	α	η (C.G.S.)
sec.	km/sec.	cm²/sec.	
7.6	3.2	0.6×10^{10}	0.3×10^{10}
	3.5	0.8×10^{10}	0.4×10^{10}
	4.0	1.2×10^{10}	0.5×10^{10}

observed values of T_1 and Δ. In carrying out this calculation two times of weight has been put upon T_1 period obtained by Takahasi-Husimi's method, as T_1 was considered more trustworthy than the T_M period obtained

Table XI.

Propagative velocity of the wave of T_1 period.

Earthq. No.	Epicentral distance	Commencement of T_1 wave	V
	km	about min	km/sec.
36	4,470	21	3.5
37	4,550	22	3.4
49	11,600	58	3.3
51	14,900	56	4.4
52	16,800	66	4.2
53	17,500	80	3.6

by the mean period method. The values of α and T_0 thus calculated are given in Table X. In computing the velocity V used in the above calculation, the commencement time of waves with the T_1 period has been considered the time of arrival of these waves. The values of V are tabulated in Table XI, and referring to this Table η was calculated in Table X for the values of V, $V=3.2$, 3.5 and 4.0 km/sec.

§ 34. The Coefficient η of the Internal Friction of the Earth's Crust.

Now, after Gutenberg[16] λ' and μ' are connected to η as $\mu'=\eta$, $\lambda'=-\frac{2}{3}\mu'$ and in Table X are given the values of α, and from (33·4) and (33·5) it is to be derived the relation $\alpha \doteqdot 10\,W$, so that the order of magnitude of the values of η will be given[17] by

$$\eta \doteqdot \frac{1}{2}\alpha.$$

From this the values of η will be calculated, the result being as shown in the last column of Table X. The values of η here obtained by the present writer is somewhat smaller than the values obtained by B. Gutenberg.

From these considerations it may be safe to conclude that the coda waves of earthquake motions having their oscillation period equal to T_1 are waves of a propagative type, and that their nature is such that as they are propagated further and further their periods become longer and longer owing to the effect of the inner friction of the earth's crust. The coefficient of the inner friction calculated from the rate of elongation of the period will be $\eta = 0.4\times 10^{10}$ C.G.S.

16) B. GUTENBERG, Hand. d. Geophys., II (1931), 533.
17) H. TAKEUCHI, Read at the meeting of *Earthq. Res. Inst.*, on July, 1, 1944.

Summary and Conclusion.

The analysis of the coda oscillations of earthquake motions has been carried out in Chapters 1 to 8 with respect to 54 earthquakes observed at Hongo, Tokyo, whose epicentral distances ranged from 33 km to almost 18,000 km.

Some examples of seismograms used for the analysis of the coda oscillations will be seen in Fig. 10.

In the first place, (1) frequency distribution diagrams of all the periods of coda waves found in every successive one minute were prepared. The peaks of such diagrams were not sharp enough to enable one to determine the predominant periods of coda oscillations.

In the second place (2) the mean period of every one minute has been calculated. The mean period of each successive one minute was, so far as a single earthquake was concerned, fairely uniform throughout the whole duration of the coda oscillations, and this mean period agreed well with the predominant period determined from the frequency diagrams of the same earthquake. In the case of extremely distant earthquakes the coda oscillations were found to continue even after the large surface waves propagated along the major arc of the earth's surface had died away. This part of the coda waves has been named "the second coda waves", and in contrast to it the coda waves that precede the large waves propagated along the earth's surface have been called "the first coda waves." The predominant periods of the second coda waves were seen to be constantly 16 sec. with all the earthquakes. These were described in Chapters 1~4.

Then (3) in Chapter 5, with a view to establishing more definitely the periodicity of coda oscillations, a method of periodgram analysis has been resorted to with eight selected earthquakes having different epicentral distances. The periodgrams of the second coda waves showed a single very high peak at about $T_A = 16$ sec. Those of the first coda waves showed two high peaks, one of which always appeared at a period that agreed well with the predominant period of the same earthquake determined by the method described in (1) and (2), while the other peak was seen at the period of about 20 sec.

Lastly (4) in Chapter 6, for the purpose of making clear the oscillation period of coda waves as accurately as possible, an expanded Takahasi-Husimi's method of analysis has been applied for the study of the period of coda oscillations of ten selected earthquakes. We obtained an oscillation curve for each portion of the respective earthquakes—a composite curve made up of two sine curves of a damping nature. After elaborate calculations we decomposed each sine curve and thus were able to find out separately the period of each of these two constituent curves.

From the studies described in (3) and (4), it has become clear that three types of waves of different periods T_1, T_2 and T_3 are existent in the first coda waves of earthquake motions. T_1 agreed well with the predominant period decided by (1) and (2), while T_2 and T_3 were constant, being 20 sec. and 16 sec. respectively. Moreover, it was remarkable to notice that T_1, when it was larger than 19 sec., was accompanied by T_2, and when much smaller than 19 sec., by T_3. In the second coda waves, however, we found only one period, i.e. T_3 ($T_3=16$ sec.), and this agreed well with the result obtained in (1) and (2).

These studies made it clear that three types of waves of different periods exist in the coda oscillations of earthquake motions. T_1 increases with the epicentral distance, while T_2 and T_3 have constant values with all the earthquakes irrespective of their difference in the epicentral distances. In Chapter 8 it was concluded that T_1 represents a period of waves propagated from the origin, and that the length of the period of those waves increases with the epicentral distances Δ, the relation between T_1 and Δ being given by $T_1 = \left(T_0^2 + \alpha \frac{\Delta}{V^3}\right)^{1/2}$, where α is a constant connected with the coefficient η of the internal friction of the earth's crust. From the observed values of T_1 and Δ we get $\eta = 0.4 \times 10^{10}$ C.G.S.

Unlike T_1, the periods of T_2 and T_3 were considered to represent the periods proper to the vibrating systems upon which the observations of earthquake motions have been made. As for the vibrating systems it will be most reasonable to think of the earth's surface layers, and it was assumed that these periods T_2 and T_3 might represent the free oscillation periods of the standing shear waves that take place in the surface layers. For the purpose of corroborating this assumption, in Chapter 7, the periods of the standing waves that take place in surface layers have been calculated by Rayleigh-Ritz's method. On the basis of the most reasonable numerical constants for the physical properties of the surface layers, it was calculated that, in order that the period of the standing shear waves taking place in the first and second layers might be equal to 16.0 sec. and that in all the three layers equal to 20 sec., the thicknesses of the three layers must of necessity be 0.83 km, 3.95 km and 17.1 km respectively. These figures agree very well with the result obtained from seismological investigations. From these facts it may be reasonably concluded that two predominant periods

T_2 and T_3 that appear in the coda oscillations of earthquake motions represent the free oscillation periods of the earth's surface layers.

Acknowledgment.

In concluding this article the present writer wishes to express his cordial thanks to Prof. H. Tsuya, the director of our Institute, for his constant encouragement. His hearty thanks are also due to Prof. T. Hagiwara for his kind guidance and apt suggestions throughout the course of this study, to Prof. C. Tsuboi for his valuable advices and encouragement, and to Prof. H. Kawasumi for his enlightening criticisms. He also heartily appreciates the collaboration of Miss K. Kobayashi, Miss M. Makino and Miss C. Kawakami, members of the Komoro Branch of the Institute, who assisted untiringly him in preparing this paper. A part of the expense of this study was defrayed out of the Scientific Research Fund from the Department of Education.

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10. 地震動の尾部について (其の6)

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第 7 章 第 1~6 章の解析により地震動の尾部の震動の中には震央距離に拘はらず 16 秒及び 20 秒の一定の週期をもつ振動があらはれることが認められた. 東京附近の地殼上層の地層の配列及それらの層の物理的性質は松澤教授其の他多くの研究者の研究の結果可なり明瞭に知られてゐるので, それらの層の中に各層の下端を節として地表面を腹とするような定常振動が生じたと考へた場合そのような定常振動の振動週期を計算してみた. この場合振動の mode は必ずしも明らかではないが振動週期だけを問題とする限りでは Rayleigh-Ritz の方法により週期の算出を行うかぎりでは, 假定した mode が多少誤つていても週期の値にはあまり差異を示さないといふ利點があるのでこの方法を用ひて週期を計算した. 地層配列のモデルとしては松澤教授の見出された深さ 4 km の層の上に木下學士が求めた 400 m の薄い層があるとして, 之等の中に 16 秒の定常振動が生じると假定し, 次に之等の層の下にあつさ 16 km の松澤教授の層があるとしてそれら三層の中に 20 秒の週期をもつ定常振動が生じると假定した.

第 8 章 第 1~6 章の週期解析の結果地震動の尾部の振動週期の中には震央距離と共に次第に長くなるものがあることが見出された. Gutenberg の考へ方に從つて震源から出發した地震波は地殼の粘性の影響により週期がのびると考へれば震央距離 Δ の所で觀測される週期 T は略々

$$T = \left\{ T_0^2 + \alpha \frac{\Delta}{V^3} \right\}^{1/2}$$

の關係をみたしてゐる. 玆に T_0 は震源での波の週期, V は地震波の傳播速度, α は $W = (\lambda' + 2\mu')/2\rho$

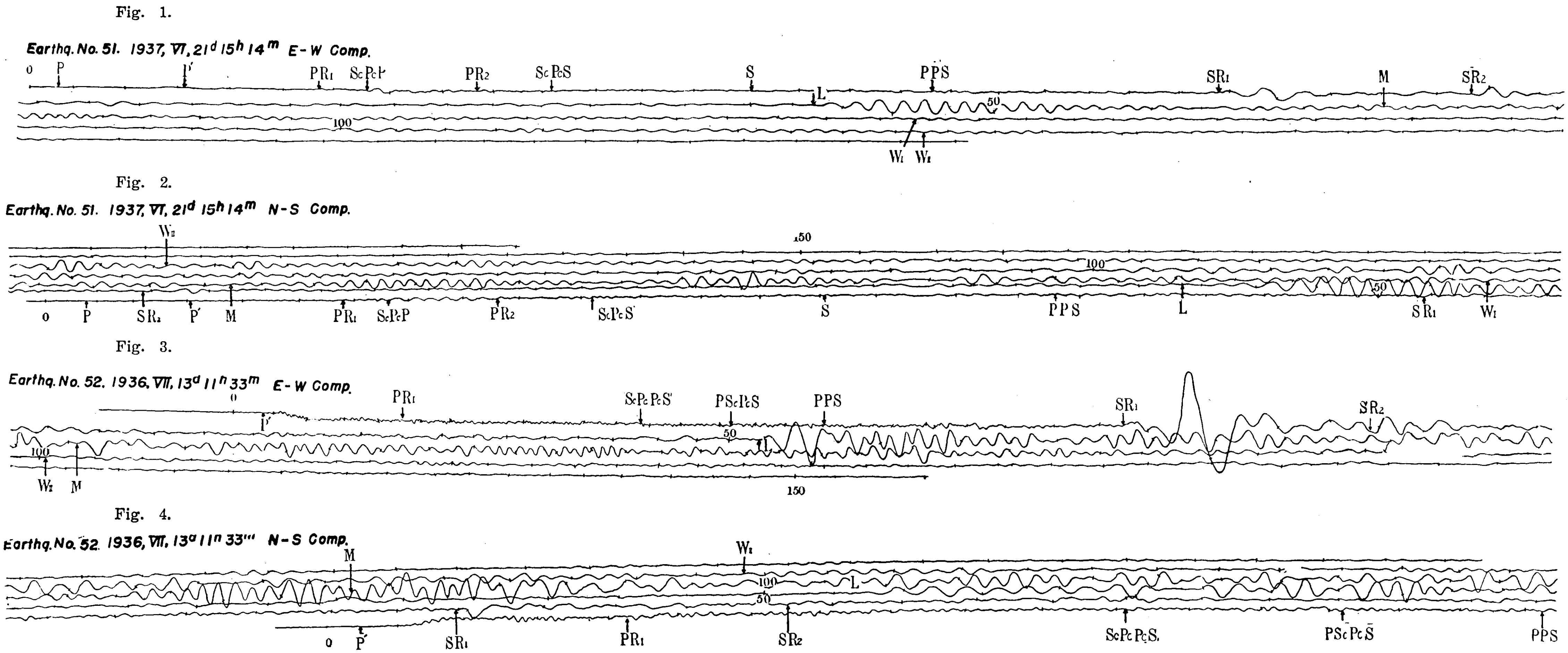

Fig. 10. Examples of seismograms.

(λ', μ' は粘性, ρ 密度) であらはされる粘性の項 W と $\alpha \doteqdot 10W$ の關係をもつ値である. これより T_0 及 α を求めれば $T_0 = 7.6$ sec., $\alpha \doteqdot 1 \times 10^{10}$ cm^2/sec. となる. 之より地殼の内部摩擦係數を求めれば $\eta \doteqdot 0.4 \times 10^{10}$ C.G.S. となる.

結　語　以上第 1 章より第 8 章にわたつて震央距離 33 km の近地地震から殆ど 18000 km に及ぶ遠地地震に至るまで 54 個の地震について東京本郷に於て觀測された記録にもとづいて尾部の振動週期の解析を行つた.

(1) 尾部の振動について 1 分毎に週期頻度圖をつくつて卓越週期を求めようとしたがその peak はあまり明瞭ではなかつた (第 1～4 章).

(2) 次に各分毎の平均週期を求めた. 1 つの地震については各分毎の平均週期はほぼ一定の値を示し, 各地震毎に尾部の卓越週期を求めることが出來た. 震央距離の極めて大きい地震については地球の劣弧のみならず優弧に沿ふ表面波が到着した後までも尾部の振動がつづくことが認められこの部分を“第2尾部”となづけた. 之 對してそれより前の部分を“第1尾部”となづけた. 第 2 尾部の振動週期はいづれの地震についても 16 秒であることが見出された. (第1～4 章)

(3) 第 5 章に於ては periodgram analysis を行つた. その結果第 2 尾部では常に 16 秒の週期が見られたのに對し第 1 尾部では 2 つの peak が見られるものが多く, この 2 つの peak の中 1 つは夫々の地震について平均週期で見出された週期とほぼ一致する値を示し他は 16 秒又は 20 秒の週期を示すものであつた.

(4) 第 6 章に於ては高橋-伏見の方法により地震動の尾部の各部分の振動週期の解析を行ひ振動曲線をもとめた. 之等の振動曲線のあるものは單純な正弦曲線であつたが他のものは 2 つの正弦曲線が重ね合せられてゐるものであつた.

以上第 1～6 章の解析の結果は互によく一致した値を與へ地震動の尾部には 3 種類の振動週期をもつ波があることが明らかとなり其等の週期を T_1, T_2, T_3 と區別した. T_1 は震央距離と共に次第に長くなる週期であり, 之に反して T_3 及び T_2 はいづれの地震についても一定の値をもち $T_2=20$ 秒, $T_3 = 16$ 秒となつた.

第 8 章に於て T_1 は震源から傳播してきた波の週期をあらはすものであることが結論せられ震央距離に對して週期の增加する割合から地殼の内部摩擦係數を計算した結果その値は $\eta = 0.4 \times 10^{10}$ C.G.S. と求められた.

第 7 章に於ては T_2 及び T_3 が一定の値をもつといふ點より考へ之が地殼表層の中に生ずる自由振動週期を現はすと假定して, そのような定常振動に對し適當なモードを假定し Rayleigh-Ritz の方法より週期を計算し, その第 1, 2 の層の中の定常振動に對しては丁度 16 秒, 第 1, 2, 3 層の中の定常振動に對しては丁度 20 秒となるためには第 1 層の厚さ 0.83 km. 第 2 層の厚さ 3.95 km 及び第 3 層の厚さ 17.1 km と求めることが出來た. これらの値は松澤教授及び木下學士等によつて既に求められてゐる値と極めてよく一致し驗震學的に求められてゐる地層配列と矛盾しない値が得られたこととなる.