3. *The Effect of Solid Viscosity of Surface Layer on the Earthquake Movements.*

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1. The problem of earthquake movements of the superficial layer was studied observingly by Ishimoto,[1] Saita,[2] Minakami[3] and others. The same problem was treated mathematically by Sezawa[4], Takahasi[5] and others. Owing to its increasing importance in connection with earthquake-proof constructions in our country in recent years, I again attacked this problem, particularly extending the solution to case of visco-elastic solid bodies.

In the present paper I attempt to examine the simplest case in which a distortional wave of a purely plane type propagated vertically upwards in an elastic semi-infinite medium is partly transmitted through bottom boundary of the superficial visco-elastic layer and partly reflected at this bottom as well as at the surface boundary of the same layer.

2. In the present investigation, the problem of forced vibrations will be discussed.

Let the axis of x be drawn vertically upwards from the lower boundary of the layer of thickness H and let u_1, ρ_1 μ_1, μ_1' ; u_2, ρ_2, μ_2, μ_2' be the displacements, densities, elastic constants and coefficients of solid viscosity of the subjacent medium and the stratum respectively. In the case of distortional waves, the equations of motion of the two media are expressed by

$$\rho_1 \frac{\partial^2 u_1}{\partial t^2} = \left(\mu_1 + \mu_1' \frac{\partial}{\partial t}\right) \frac{\partial^2 u_1}{\partial x^2}, \qquad \rho_2 \frac{\partial^2 u_2}{\partial t^2} = \left(\mu_2 + \mu_2' \frac{\partial}{\partial t}\right) \frac{\partial^2 u_2}{\partial x^2} \quad \ldots (1),\ (2)$$

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1) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, 10 (1932), 171; 12 (1934), 234; 13 (1935), 592; 14 (1936), 240; 15 (1937), 536.

2) T. SAITA, and S. SUZUKI, *Bull. Earthq. Res. Inst.*, 12 (1934), 517.

3) T. MINAKAMI, and S. SAKUMA, *Bull. Earthq. Res. Inst.*, 26 (1948), 61; *Bull. Inv. Comm.* Hukui Earthq., (1949).

4) K. SEZAWA, *Bull. Earthq. Res. Inst.*, 8 (1930), 1; K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, 13 (1935), 251.

5) R. TAKAHASI and K. HIRANO, *Bull. Earthq. Res. Inst.*, 19 (1941), 534.

In the case of dilatational waves, it is necessary to replace μ_1, μ_1', μ_2, μ_2' by $\lambda_1+2\mu_1$, $\lambda_1'+2\mu_1'$, $\lambda_2+2\mu_2$, $\lambda_2'+2\mu_2'$.

If the incident waves in the lower medium be of the type

$$u_0=\mathfrak{A}e^{i(pt-f_1x)} \qquad \text{.......(3)}$$

the resulting displacement of the subjacent medium and the stratum are expressed by

$$u_1=u_0+u_0'=\mathfrak{A}e^{i(pt-f_1x)}+Ae^{i(pt+f_1x)}, \qquad \text{.......(4)}$$

$$u_2=Be^{i(pt-f_2x)}+Ce^{i(pt+f_2x)}, \qquad \text{.......(5)}$$

where

$$p=2\pi/T, \qquad T=\text{the period of transmission}, \qquad \text{.......(6)}$$

$$f_1^2=\rho_1p^2/(\mu_1+i\mu_1'p), \quad f_2^2=\rho_2p^2/(\mu_2+i\mu_2'p), \qquad \text{.......(7)}$$

and A, B, C are the arbitrary constants determined by the boundary conditions.

The boundary conditions at $x=0$ are

$$u_1=u_2, \quad \left(\mu_1+\mu_1'\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial x}=\left(\mu_2+\mu_2'\frac{\partial}{\partial t}\right)\frac{\partial u_2}{\partial x}, \qquad \text{.......(8), (9)}$$

while at the free surface, $x=H$ is

$$\left(\mu_2+\mu_2'\frac{\partial}{\partial t}\right)\frac{\partial u_2}{\partial x}=0. \qquad \text{.......(10)}$$

Assuming, for simplicity, $\mu_1'\to 0$, and substituting (3)~(5) in (8)~(10), we get

$$\left.\begin{aligned}\frac{A\Phi}{\mathfrak{A}}&=\cos f_2H-iM\sin f_2H, \quad \frac{B\Phi}{\mathfrak{A}}=\cos f_2H+i\sin f_2H,\\ \frac{C\Phi}{\mathfrak{A}}&=\cos f_2H-i\sin f_2H,\end{aligned}\right\} \qquad \text{......(11)}$$

where

$$\Phi=\cos f_2H+iM\sin f_2H, \quad M=\frac{f_2(\mu_2+ip\mu_2')}{f_1\mu_1}. \qquad \text{.......(12)}$$

In order to avoid complexity in numerical calculations, we treated a particular case in which μ_2' is very small, then, if small quantities higher than the second order in $\mu_2'p/\mu_2$ are omitted, a characteristic equation of superficial layer becomes

$$\frac{|u_{2x=H}|}{|u_0|}=\frac{2}{\sqrt{\Phi_1^2+\Phi_2^2}}, \qquad \text{.......(13)}$$

where $\Phi_1=\cos P\cosh Q+\sqrt{\frac{\mu_2\rho_2}{\mu_1\rho_1}}\,(R\cos P\sinh Q-S\sin P\cosh Q)$,

$$\Phi_2=\sin P\sinh Q+\sqrt{\frac{\mu_2\rho_2}{\mu_1\rho_1}}\,(R\sin P\cosh Q+S\cos P\sinh Q),$$

$$P=\frac{pH\sqrt{\frac{\rho_2}{\mu_2}}}{\left\{1+\left(\frac{\mu_2'p}{\mu_2}\right)^2\right\}^{\frac{1}{4}}}\cos\left(\frac{1}{2}\tan^{-1}\frac{\mu_2'p}{\mu_2}\right),$$

$$Q=\frac{pH\sqrt{\frac{\rho_2}{\mu_2}}}{\left\{1+\left(\frac{\mu_2'p}{\mu_2}\right)^2\right\}^{\frac{1}{4}}}\sin\left(\frac{1}{2}\tan^{-1}\frac{\mu_2'p}{\mu_2}\right),$$

$$R=\frac{\cos\left(\frac{1}{2}\tan^{-1}\frac{\mu_2'p}{\mu_2}+\right)\frac{\mu_2'p}{\mu_2}\sin\left(\frac{1}{2}\tan^{-1}\frac{\mu_2'p}{\mu_2}\right)}{\left\{1+\left(\frac{\mu_2'p}{\mu_2}\right)^2\right\}^{\frac{1}{4}}},$$

$$S=\frac{\frac{\mu_2'p}{\mu_2}\cos\left(\frac{1}{2}\tan^{-1}\frac{\mu_2'p}{\mu_2}\right)-\sin\left(\frac{1}{2}\tan^{-1}\frac{\mu_2'p}{\mu_2}\right)}{\left\{1+\left(\frac{\mu_2'p}{\mu_2}\right)^2\right\}^{\frac{1}{4}}}. \quad \text{............(14)}$$

Using (13), (14), some special cases are plotted in the following drawings. In Figs. 2 and 3 the cases of $\mu_2'=10^6$ and $\mu_2'=10^7$ C. G. S. besides the conditions $v_1=633$m/s, $v_2=200$m/s, $\mu_1=9\times10^9$, $\mu_2=6\times10^8$ C. G. S., $\rho_1=2.25$, $\rho_2=1.5$. Figs. 4 and 5 gives us the cases of $\mu_2'=10^6$ and $\mu_2'=10^7$ C. G. S. provided the conditions $v_1=200$ m/s, $v_2=100$ m/s, $\mu_1=6\times10^8$, $\mu_2=1.5\times10^8$ C. G. S., $\rho_1=\rho_2=1.5$. In the results of the experimental investigations,[6] the assumed values of coefficient of solid viscosity proved not to be improbable values in the superficial layer of the earth.

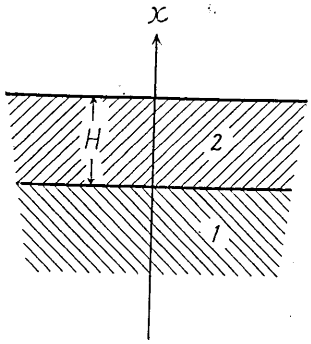

Fig. 1.

The principal results obtained in this investigation are enumerated as follows:

6) K. KANAI, Meeting, Earthq. Res. Inst., Jan. 17, 1950.

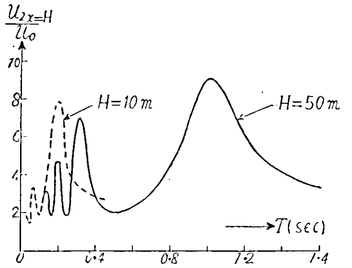

Fig. 2. $\mu_2' = 10^6$ C. G. S., $v_1 = 633$ m/s, $v_2 = 200$ m/s, $\mu_1 = 9 \times 10^9$, $\mu_2 = 6 \times 10^8$ C. G. S., $\rho_1 = 2.25$, $\rho_2 = 1.5$.

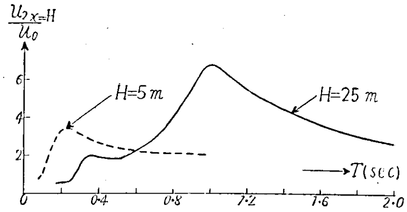

Fig. 3. $\mu_2' = 10^7$ C. G. S., other constants are the same of the case of Fig. 2.

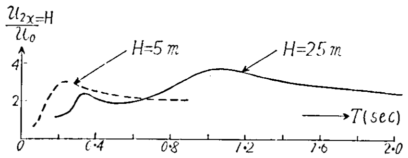

Fig. 4. $\mu_2' = 10^6$ C. G. S., $v_1 = 200$ m/s, $v_2 = 100$ m/s, $\mu_1 = 6 \times 10^8$, $\mu_2 = 1.5 \times 10^8$ C. G. S., $\rho_1 = \rho_2 = 1.5$.

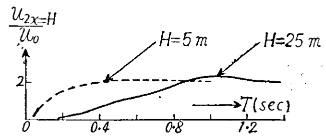

Fig. 5. $\mu_2' = 10^7$ C. G. S., other constants are the same of the case of Fig. 4.

(i) The case of large solid viscosity in superficial layer.

Fig. 3 shows that, when the elasticity ratio of two media μ_2/μ_1 is small, a peak corresponding to fundamental synchronous period appears more clearly. While, as has been shown in Fig. 5, in case the elasticity ratio of two media is relatively not small, the frequency-amplitude curve is flat and then the vibration amplitudes corresponding to ordinary synchronous periods do not grow to large value.

(ii) The case of small solid viscosity in superficial layer.

It will be seen from Fig. 2 that, when the elasticity ratio of two media is small, the synchronous conditions of higher order occur successively. However, if the elasticity ratio of the two media is relatively not small, such phenomenon is not so clearly noticed as shown in Fig. 4.

At all events, the diagrammatical features of the results of the present investigations closely resemble the features[7] of the spectrogramic diagramms

7) M. ISHIMOTO, *loc. cit.* 1).

of earthquake movements of the ground which were obtained by the accelerometer at several places in Tokyo and Yokohama.

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3. 地表層の固体粘性が地震動の振幅に及ぼす影響

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地表層に固体粘性がある場合の地震動の振幅の性質をしらべたところ，次のようなことがわかつた．

(1) 地表層の固体粘性が大きい場合

表面層の彈性が下層のそれに比べて相當に小さい場合には，基本固有周期のところの振幅が明かに卓越するが（第3圖參照），この比が余り小さくない場合には，固有周期のところの振幅の特性をほとんど現はさない（第5圖參照）．

(2) 地表層の固體粘性が余り大きくない場合

兩層の彈性比（μ_2/μ_1）が相當に小さい場合には基本固有周期にあたる附近の振幅が無論卓越するが，高次の固有周期附近でも振幅の増大がおこる．しかし，この彈性比が余り小さくない場合には，これらの性質はそれ程はつきりしない．

以上の事柄は，一般の性質としては波動問題として容易に想像されるところではあるが，實際の場合に近い常數を入れて數値計算を行つた結果，種々の地盤に特有な地震動の性質が，この種の研究で相當程度説明できそうなことがわかつた譯である．
