15. *A Short Note on Kanai's Formula.*

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When a building with rigid floors is vibrating with a period T, we have, concentrating the mass of pillars at the floors,

$$\left.\begin{aligned} m_n\ddot{y}_n &= E_nI_ny''', \\ m_{n-1}\ddot{y}_{n-1} &= E_{n-1}I_{n-1}y''' - E_nI_ny''', \\ &\cdots\cdots\cdots\cdots \\ m_s\ddot{y}_s &= E_sI_sy''' - E_{s+1}I_{s+1}y''', \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (1)$$

where $\ddot{y}_s$'s are accelerations, and E_sI_sy''' the shearing force in the pillar between m_s and m_{s-1}, of which the length is l_s.

Summing up (1), we have

$$\sum_{r=s}^{n} m_r\ddot{y}_r = E_sI_sy'''. \quad \cdots\cdots\cdots \quad (2)$$

Since $\ddot{y}_r = -4\pi^2 y_r/T^2$, we can write (2)

$$-y''' = 4\pi^2 \sum_{r=s}^{n} m_r y_r / E_sI_sT^2 \equiv S\,, \quad \cdots\cdots\cdots \quad (3)$$

which assumes a constant value, say S, for the pillar between m_s and m_{s-1}.

We assume for the same pillar,

$$\left.\begin{aligned} l_sy'' &= -ky' && \text{at} \quad x = l_s\,, \\ l_sy'' &= hy' && \text{at} \quad x = 0\,, \\ y &= y_s - y_{s-1} && \text{at} \quad x = l_s\,, \\ y &= 0 && \text{at} \quad x = 0\,, \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (4)$$

taking the origin temporarily at m_{s-1}. Then from (3) and (4) we obtain after a simple calculation

$$S = \frac{12\,(h+k+hk)}{(12+4h+4k+hk)} \cdot \frac{y_s - y_{s-1}}{l_s^3}\,. \quad \cdots\cdots\cdots \quad (5)$$

Eliminating S from (3) and (5), we get finally

$$E_sI_s=\frac{12+4h+4k+hk}{3\,(h+k+hk)}\cdot\frac{\pi^2 l_s^3 \sum_{r=s}^{n} m_r y_r}{T^2\,(y_s-y_{s-1})}\text{, } \dots\dots\dots\dots \quad (6)$$

which is Kanai's formula generalized.[1]

For the clamped end of pillars, put h or k in the formula equal to ∞, and for the hinged end h or $k=0$.

Values of $\frac{12+4h+4k+hk}{3\,(h+k+hk)}$

		Upper End of Pillar		
		Clamped	Elastic	Hinged
Lower End of Pillar.	Clamped	1/3	$(4+k)/3(1+k)$	4/3
	Elastic	$(4+h)/3(1+h)$	$\frac{(12+4h+4k+hk)}{3(h+k+hk)}$	$4(3+h)/3h$
	Hinged	4/3	$4(3+k)/3k$	∞

1) Kiyoshi KANAI, *Monthly Coll. Earthq. Res. Inst., Tokyo Univ.* Nov. 15, 1949.

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建物の振動實驗の結果から各階の柱の剛性を求める金井清氏の式の擴張として

$$E_sI_s=\frac{12+4h+4k+hk}{3(h+k+hk)}\cdot\frac{\pi^2 l_s{}^3 \sum_{s=s}^{n} m_r y_r}{T^2(y_s-y_{s-1})}$$

なる式を得た．此處に k 及び h は夫々柱の上及び下の節點の條件による常數で，剛節の場合 ∞ に，鉸節の場合 0 に取るべきものである．他の記號は金井式の場合と同樣である．