1. *Boundary Conditions in the Problem of Generation of Elastic Waves.*

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The problem of generation of elastic waves attracted the interest of many seismologists in our country, and a number of excellent papers are published[1]. But in these works, boundary conditions are not without some restriction. In this paper we will take up this point and show how to remove this restriction and give the boundary condition in a most general form.

The equation of motion of the elastic solid in spherical polar coordinates (R, θ, φ) are written in the following form......

$$\left\{\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2}&=(\lambda+2\mu)\frac{\partial\Delta}{\partial R}-\frac{2\mu}{R\sin\theta}\frac{\partial}{\partial\theta}(\omega_\varphi\sin\theta)+\frac{2\mu}{R\sin\theta}\frac{\partial\omega_\theta}{\partial\varphi} \\ \rho\frac{\partial^2 v}{\partial t^2}&=(\lambda+2\mu)\frac{1}{R}\frac{\partial\Delta}{\partial\theta}-\frac{2\mu}{R\sin\theta}\frac{\partial\omega_R}{\partial\varphi}+\frac{2\mu}{R}\frac{\partial(R\omega_\varphi)}{\partial R} \\ \rho\frac{\partial^2 w}{\partial t^2}&=(\lambda+2\mu)\frac{1}{R\sin\theta}\frac{\partial\Delta}{\partial\varphi}-\frac{2\mu}{R}\frac{\partial(R\omega_\theta)}{\partial R}+\frac{2\mu}{R}\frac{\partial\omega_R}{\partial\theta} \end{aligned}\right. \qquad \text{(1)}$$

where ρ........density

λ, μ.....Lamé's elastic constants

u, v, w..radial, colatitudinal and azimuthal components of displacement

ω_R, ω_θ, ω_φ...components of rotation

Δ.........volume dilatation

If we denote the displacement vector by $\mathfrak{D}$, we can write

$$\mathfrak{D}={}_0\mathfrak{D}+{}_1\mathfrak{D}+{}_2\mathfrak{D} \qquad \text{(2)}$$

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1) K. SEZAWA, *Bull, Earthq. Res. Inst.*, **2** (1927). **17**; **10** (1932), 299. *Sindôgaku* (1932) 653.

H. KAWASUMI, *Bull. Earthq. Res.*, **11** (1933), 403.

H. KAWASUMI and R. YOSHIYAMA, *Disin*, **7** (1935), 367.

W. INOUE, *Bull. Earthq. Res. Inst.*, **14** (1936), 582; **15** (1937), 90, 674, 686, 956; **16** (1938), 597.

S. SYÔNO, *Geophys. Mag.*, **9** (1935) 285; **12** (1937) 83; **13** (1938) 1.

where
$$\begin{cases} {}_0\mathfrak{D} = \text{grad}\ \Phi \\ {}_1\mathfrak{D} = \text{rot}\ ({}_1\Psi,\ 0,\ 0) \\ {}_2\mathfrak{D} = \text{rot rot}\ ({}_2\Psi,\ 0,\ 0) \end{cases} \qquad (3)$$

Using these notations we can write the solutions of the equations of motion as follows (omitting the time factor $\exp(ipt)$) :—

$$\begin{cases} \Phi = f(hR)\overline{P}_n^m(\cos\theta)(A\cos m\varphi + A'\sin m\varphi) \\ {}_1\Psi = Rg(kR)\overline{P}_n^m(\cos\theta)(B\cos m\varphi + B'\sin m\varphi) \\ {}_2\Psi = Rg(kR)\overline{P}_n^m(\cos\theta)(C\cos m\varphi + C'\sin m\varphi) \end{cases} \qquad (4)$$

where
$$\begin{cases} \overline{P}_n^m(\cos\theta) \ldots\ldots \text{associated Legendre function by Ferrer's definition} \\ f(hR) = (hR)^{-\frac{1}{2}} H_{n+1/2}^{(2)}(hR) \\ g(kR) = (kR)^{-\frac{1}{2}} H_{n+1/2}^{(2)}(kR) \\ h^2 = p^2\rho/(\lambda+2\mu), \quad k^2 = p^2\rho/\mu \end{cases} \qquad (5)$$

From (2), (3) and (4), we get

$$\begin{cases} u = \left\{ A\dfrac{df}{dR} + C\dfrac{n(n+1)}{R}g \right\} \overline{P}_n^m(\cos\theta)\cos m\varphi \\ \quad + \left\{ A'\dfrac{df}{dR} + C'\dfrac{n(n+1)}{R}g \right\} \overline{P}_n^m(\cos\theta)\sin m\varphi \\ v = \left[\left\{ A\dfrac{f}{R} + C\dfrac{1}{R}\dfrac{d(Rg)}{dR} \right\} \dfrac{d}{d\theta}\overline{P}_n^m(\cos\theta) + B'mg\dfrac{1}{\sin\theta}\overline{P}_n^m(\cos\theta) \right] \cos m\varphi \\ \quad + \left[\left\{ A'\dfrac{f}{R} + C'\dfrac{1}{R}\dfrac{d(Rg)}{dR} \right\} \dfrac{d}{d\theta}\overline{P}_n^m(\cos\theta) - Bmg\dfrac{1}{\sin\theta}\overline{P}_n^m(\cos\theta) \right] \sin m\varphi \\ w = \left[m\left\{ A'\dfrac{f}{R} + C'\dfrac{1}{R}\dfrac{d(Rg)}{dR} \right\} \dfrac{1}{\sin\theta}\overline{P}_n^m(\cos\theta) - Bg\dfrac{d}{d\theta}\overline{P}_n^m(\cos\theta) \right] \cos m\varphi \\ \quad + \left[-m\left\{ A\dfrac{f}{R} + C\dfrac{1}{R}\dfrac{d(Rg)}{dR} \right\} \dfrac{1}{\sin\theta}\overline{P}_n^m(\cos\theta) - B'g\dfrac{d}{d\theta}\overline{P}_n^m(\cos\theta) \right] \sin m\varphi \end{cases} \qquad (6)$$

Now, putting

$$\begin{cases} \mathfrak{A} \equiv A\dfrac{df}{dR} + C\dfrac{n(n+1)}{R}g, & \mathfrak{A}' \equiv A'\dfrac{df}{dR} + C'\dfrac{n(n+1)}{R}g \\ \mathfrak{C} \equiv A\dfrac{f}{R} + C\dfrac{1}{R}\dfrac{d(Rg)}{dR}, & \mathfrak{C}' \equiv A'\dfrac{f}{R} + C'\dfrac{1}{R}\dfrac{d(Rg)}{dR} \\ \mathfrak{B} \equiv Bg \quad , & \mathfrak{B}' \equiv B'g \end{cases} \qquad (7)$$

we obtain

$$\begin{cases} u=\mathfrak{A}\, P_n^m \cdot \cos m\varphi + \mathfrak{A}'\, P_n^m \cdot \sin m\varphi \\ v=\left[\mathfrak{C}\dfrac{d}{d\theta}P_n^m \cdot + \mathfrak{B}' m \dfrac{1}{\sin\theta} P_n^m\right]\cos m\varphi \\ \quad +\left[\mathfrak{C}'\dfrac{d}{d\theta}P_n^m - \mathfrak{B}' m \dfrac{1}{\sin\theta} P_n^m\right]\sin m\varphi \\ w=\left[\mathfrak{C}' m\dfrac{1}{\sin\theta}P_n^m - \mathfrak{B}\dfrac{d}{d\theta} P_n^m\right]\cos m\varphi \\ \quad +\left[-\mathfrak{C} m\dfrac{1}{\sin\theta}P_n^m - \mathfrak{B}'\dfrac{d}{d\theta} P_n^m\right]\sin m\varphi \end{cases} \qquad \text{.......................(8)}$$

where P_n^m means $\bar{P}_n^m(\cos\theta)$.

Hitherto, the boundary-value problems, in which the displacements are given on the spherical surface $R=a$, have, on account of the forms of expressions in (8), always been introduced in the following scheme[2].

$$\begin{cases} u=D\cdot \bar{P}_n^m(\cos\theta)\cdot\cos m\varphi \\ v=w=0 \end{cases} \qquad \text{..............................(9)}$$

or

$$\begin{cases} u=0 \\ v=Em\dfrac{1}{\sin\theta}\bar{P}_n^m(\cos\theta)\cdot\cos m\varphi \\ w=-Em\dfrac{d}{d\theta}\bar{P}_n^m(\cos\theta)\cdot\sin m\varphi \end{cases} \qquad \text{....................(10)}$$

But these are very unsatisfactory, because in the expression (9) v and w are both zero, and in (10) v and w must satisfy some definite relation, which made us unable to give v and w both arbitrarily. This fact is clearly pointed out by Mr. S. Honma in his initial-value problem[3]. In this paper we will study how to remove this defect, and give the displacement u, v and w all arbitrary on the spherical surface $R=a$.

Now, we assume

$$u=U(\theta,\ \varphi),\quad v=V(\theta,\ \varphi),\quad w=W(\theta,\ \varphi) \qquad \text{.................(11)}$$

on the surface $R=a$ and will show that the coefficients A, A', B, B', C, C'

2) S. Syôno, *loc. cit.* (1) second paper.
H. Kawasumi and R. Yoshiyama, *loc. cit.*
W. Inoue, *loc. cit.*
S. Honma, *Kensin Ziho*, **12** (1942), 106.

3) *loc, cit.* (2)

are uniquely determined, if certain conditions are satisfied which enable us the following treatment.

First, expand $U(\theta, \varphi)$, $V(\theta, \varphi)$, $W(\theta, \varphi)$ into Fourier series

$$\left\{\begin{aligned} U(\theta,\varphi)&=\sum_m U^m(\cos\theta)\cos m\varphi+\sum_m U^{m\prime}(\cos\theta)\sin m\varphi \\ V(\theta,\varphi)&=\sum V^m(\cos\theta)\cos m\varphi+\sum V^{m\prime}(\cos\theta)\sin m\varphi \\ W(\theta,\varphi)&=\sum W^m(\cos\theta)\cos m\varphi+\sum W^{m\prime}(\cos\theta)\sin m\varphi \end{aligned}\right. \quad \text{.........(12)}$$

Comparing the expressions (12) with the generalized forms of (8) (in which $\mathfrak{A}$, $\mathfrak{A}'$ etc. are respectively replaced by $\sum_m\sum_n \mathfrak{A}_n^m$, $\sum_m\sum_n \mathfrak{A}_n^{m\prime}$, etc. and equating the corresponding right hand sides of the two sets of equations respectively, we have

$$\left\{\begin{aligned} &\sum_n \mathfrak{A}_n^m P_n^m=U^m(\cos\theta) && \text{.....................(13·1)} \\ &\sum \mathfrak{A}_n^{m\prime} P_n^m=U^{m\prime}(\cos\theta) && \text{.....................(13·2)} \\ &\sum\left(\mathfrak{C}_n^m\frac{d}{d\theta}P_n^m+\mathfrak{B}_n^{m\prime}m\frac{1}{\sin\theta}P_n^m\right)=V^m(\cos\theta) && \text{.....................(13·3)} \\ &\sum\left(\mathfrak{C}_n^{m\prime}\frac{d}{d\theta}P_n^m-\mathfrak{B}_n^m m\frac{1}{\sin\theta}P_n^m\right)=V^{m\prime}(\cos\theta) && \text{.................(13·4)} \\ &\sum\left(\mathfrak{C}_n^{m\prime} m\frac{1}{\sin\theta}P_n^m-\mathfrak{B}_n^m\frac{d}{d\theta}P_n^m\right)=W^m(\cos\theta) && \text{.................(13·5)} \\ &\sum\left(-\mathfrak{C}_n^m m\frac{1}{\sin\theta}P_n^m-\mathfrak{B}_n^{m\prime}\frac{d}{d\theta}P_n^m\right)=W^{m\prime}(\cos\theta) && \text{..............(13·6)} \end{aligned}\right.$$

$\mathfrak{A}_n^m$ and $\mathfrak{A}_n^{m\prime}$ are directly obtained from (13.1) and (13.2) by using the orthogonality of P_n^m functions.

$$\left\{\begin{aligned} \mathfrak{A}_n^m&=\frac{2n+1}{2}\frac{(n-m)!}{(n+m)!}\int_{-1}^{1}U^m(\cos\theta)\,\bar{P}_n^m(\cos\theta)\,d(\cos\theta) \\ \mathfrak{A}_n^{m\prime}&=\frac{2n+1}{2}\frac{(n-m)!}{(n+m)!}\int_{-1}^{1}U^{m\prime}(\cos\theta)\,\bar{P}_m^m(\cos\theta)\,d(\cos\theta) \end{aligned}\right. \quad \text{.........(14)}$$

Next, we will consider the other unknowns. $\mathfrak{C}_n^m$ and $\mathfrak{B}_n^{m\prime}$ are involved only in (13.3) and (13.6), and $\mathfrak{C}_n^{m\prime}$ and $\mathfrak{B}_n^m$ in (13.4) and (13.5). So, we will at first take up (13.4) and (13.5).

Multiplying $\sin\theta$ on both sides of (13.5) and then operating $\frac{d}{d\theta}$, we get

$$\sum_n\left(\mathfrak{C}_n^{m\prime}\, m\frac{d}{d\theta}P_n^m-\mathfrak{B}_n^m\frac{d}{d\theta}\left\{\sin\theta\frac{d}{d\theta}P_n^m\right\}\right)=\frac{d}{d\theta}\{\sin\theta\, W^m\} \quad \text{.........(15)}$$

Then multiplying $-m$ on the expression (13.4) and adding to (15)

$$\sum\mathfrak{B}_n^m\left(-\frac{d}{d\theta}\left\{\sin\theta\frac{d}{d\theta}P_n^m\right\}+m^2\frac{1}{\sin\theta}P_n^m\right)=\frac{d}{d\theta}\{\sin\theta\, W^m\}-mV^m$$

or
$$\sum-\mathfrak{B}_n^m\sin\theta\left(\frac{d^2}{d\theta^2}P_n^m+\cot\theta\frac{d}{d\theta}P_n^m-m^2\frac{1}{\sin^2\theta}P_m^n\right)$$

$$=\frac{d}{d\theta}\{\sin\theta\, W^m\}-mV^m \quad \text{.......................(16)}$$

On the other hand, associated Legendre function P_n^m satisfies the following differential equation with respect to θ

$$\frac{d^2}{d\theta^2}P_n^m(\cos\theta)+\cot\theta\frac{d}{d\theta}P_n^m(\cos\theta)+\left\{n(n+1)-\frac{m^2}{\sin^2\theta}\right\}P_n^m(\cos\theta)=0 \quad \text{...(17)}$$

Introducing (17) into (16), and dividing both sides by $\sin\theta$, we get

$$\sum n(n+1)\mathfrak{B}_n^m P_n^m=\frac{1}{\sin\theta}\frac{d}{d\theta}\{\sin\theta W^m\}-mV^{m\prime} \quad \text{..............(18)}$$

From this expression we can easily determine $\mathfrak{B}_n^m$, as we determined $\mathfrak{A}_n^m$ from (13.1). Namely,

$$\mathfrak{B}_n^m=\frac{2n+1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}\int_{-1}^{1}\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\{\sin\theta\, W^m(\cos\theta)\}\right.$$

$$\left.-mV^{m\prime}(\cos\theta)\right]\cdot\bar{P}_n^m(\cos\theta)\,d(\cos\theta) \quad \text{..........................(19)}$$

By the similar manner we may get $\mathfrak{C}_n^{m\prime}$. The result is as follows:

$$\mathfrak{C}_n^{m\prime}=-\frac{2n+1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}\int_{-1}^{1}\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\{\sin\theta V^{m\prime}(\cos\theta)\}\right.$$

$$\left.-mW^m(\cos\theta)\right]\cdot\bar{P}_n^m(\cos\theta)d(\cos\theta) \quad \text{..........................(20)}$$

Using the above solutions we can obtain $\mathfrak{B}_n^{m\prime}, \mathfrak{C}_n^m$ easily; if we substitute $\mathfrak{C}_n^m$, $-\mathfrak{B}_n^{m\prime}$ and V^m, $-W^{m\prime}$ in place of $\mathfrak{C}_n^{m\prime}$, $\mathfrak{B}_n^m$ and $V^{m\prime}$, W^m respectively, then the expression (13.4) changes to (13.3), and (13.5) to (13.6). Therefore the unknowns $\mathfrak{B}_n^{m\prime}$, $\mathfrak{C}_n^m$ are at once obtained from the expressions (19) and (20) by applying the above substitution.

$$\mathfrak{B}_n^{m\prime}=\frac{2n+1}{2n(n+1)}\frac{(n-m)!}{(n+m)!}\int_{-1}^{1}\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\{\sin\theta\, W^{m\prime}(\cos\theta)\}\right.$$

$$\left.+mV^m(\cos\theta)\right]\cdot\bar{P}_n^m(\cos\theta)d(\cos\theta) \quad \text{..........................(21)}$$

$$\mathfrak{C}_n^m = \frac{2n+1}{2n(n+1)} \frac{(n-m)!}{(n+m)!} \int_{-1}^{1} \Big[\frac{1}{\sin\theta} \frac{d}{d\theta} \{\sin\theta\, V^m(\cos\theta)\}$$

$$+ mW^{m\prime}(\cos\theta) \Big] \cdot \bar{P}_n^m(\cos\theta)\, d(\cos\theta) \quad \text{.........................(22)}$$

Thus the quantities $\mathfrak{A}_n^m$, $\mathfrak{A}_n^{m\prime}$ etc. are all solved explicitly. And the next step to be taken is to obtain A_n^m, $A_n^{m\prime}$ etc. But this is very simple.

Solving (7) with respect to A, A' etc. we have at once

$$\begin{cases} A = \left\{ \dfrac{1}{R} \dfrac{d(Rg)}{dR} \mathfrak{A} - \dfrac{n(n+1)}{R} g \mathfrak{C} \right\} \Big/ D \\ A' = \left\{ \dfrac{1}{R} \dfrac{d(Rg)}{dR} \mathfrak{A}' - \dfrac{n(n+1)}{R} g \mathfrak{C}' \right\} \Big/ D \\ C = \left\{ -\dfrac{f}{R} \mathfrak{A} + \dfrac{df}{dR} \mathfrak{C} \right\} \Big/ D \\ C' = \left\{ -\dfrac{f}{R} \mathfrak{A}' + \dfrac{df}{dR} \mathfrak{C}' \right\} \Big/ D \\ B = \dfrac{1}{g} \mathfrak{B} \\ B' = \dfrac{1}{g} \mathfrak{B}' \end{cases} \quad \text{.................................(23)}$$

where $$D = \begin{vmatrix} \dfrac{df}{dR} & \dfrac{n(n+1)}{R} \\ \dfrac{f}{R} & \dfrac{1}{R} \dfrac{d(Rg)}{dR} \end{vmatrix} \quad \text{.............................(24)}$$

and all R_s' involved in the above expressions are substituted by a after the differentiation is performed. (The affixes m and n are abridged for brevity.)

In this way, the values of A, A' etc. are uniquely determined when we give the values of u, v, w on the spherical surface by the expression (11), so long as the operations so far performed are permissible. This fact means that the boundary-value problem given on the spherical surface is completely solved.

Initial-value problem in the theory of elastic waves, studied by Mr. S. Honma, is in its essential part, equivalent to the above problem. If the initial displacements are given by

$$u_0 = U_0(R, \theta, \varphi), \quad v_0 = V_0(R, \theta, \varphi), \quad w_0 = W_0(R, \theta, \varphi) \quad \text{........(25)}$$

and the functions U_0, V_0, W_0 are expressed by the integral form such as

$$U_0(R, \theta, \varphi) = \int F(R; s) U(s; \theta, \varphi)\, ds \quad \text{......................(26)}$$

which are uniformly convergent with respect to R, then we may treat the integrand by nearly equal manner with U^m, $U^{m\prime}$ etc. of (13) and integrate with respect to s.

In the potential theory there are two types of problems; one is that of Dirichlet and the other is that of Neumann. In the former case potential is given on some closed surface, and in the latter, normal derivative is given. In the theory of elastic waves, however, it has little meaning to give displacement potential on the surfaces. In the above theory so far shown, displacement components were given on the spherical surface, that is to say the first derivatives of potential were given. Therefore, this problem corresponds to that of Neumann. But another type of problem is possible; that is to give normal components of stresses on the closed surface. This is a particular subject which has no corresponding one in the potential theory, and because of the important position of "forces" in the field of mechanics this is a subject of great interest.

The stress components are given by

$$\left\{\begin{aligned} T_{RR} &= \lambda\,\Delta\,\mathfrak{D} + 2\mu\frac{\partial u}{\partial R} \\ T_{R\theta} &= \mu\left\{R\frac{\partial}{\partial R}\left(\frac{v}{R}\right) + \frac{1}{R}\frac{\partial v}{\partial\theta}\right\} \\ T_{R\varphi} &= \mu\left\{\frac{1}{R\sin\theta}\frac{\partial u}{\partial\varphi} + R\frac{\partial}{\partial R}\left(\frac{w}{R}\right)\right\} \end{aligned}\right. \qquad \text{.....(27)}$$

From (2), (3) and (5) we get the following relation :—

$$n(n+1)\frac{\Psi}{R^2} = \frac{\partial^2\Psi}{\partial R^2} + k^2\Psi \qquad \text{.....(28)}$$

where Ψ is ${}_1\Psi$ or ${}_2\Psi$ as already defined in (3).
Introducing (4) and (28) into (27), we get

$$\left\{\begin{aligned} T_{RR} &= \mathfrak{D}\,P_n^m\cdot\cos m\varphi + \mathfrak{D}'\,P_n^m\cdot\sin m\varphi \\ T_{R\theta} &= \left[\mathfrak{F}\frac{d}{d\theta}P_n^m + \mathfrak{G}m\frac{1}{\sin\theta}P_n^m\right]\cos m\varphi \\ &\quad + \left[\mathfrak{F}'\frac{d}{d\theta}P_n^m - \mathfrak{G}m\frac{1}{\sin\theta}P_n^m\right]\sin m\varphi \qquad \text{.....(29)} \\ T_{R\varphi} &= \left[\mathfrak{F}'m\frac{1}{\sin\theta}P_n^m - \mathfrak{G}\frac{d}{d\theta}P_n^m\right]\cos m\varphi \\ &\quad + \left[-\mathfrak{F}m\frac{1}{\sin\theta}P_n^m - \mathfrak{G}'\frac{d}{d\theta}P_n^m\right]\sin m\varphi \end{aligned}\right.$$

where
$$\begin{cases} \mathfrak{D} = A\left\{-\lambda h^2 f + 2\mu \dfrac{d^2 f}{dR^2}\right\} + C\cdot 2\mu n(n+1)\dfrac{d}{dR}\left(\dfrac{g}{R}\right) \\ \mathfrak{D}' = A'\{ \quad \text{''} \quad \} + C'\cdot \quad \text{''} \\ \mathfrak{F} = \mu\left\{A\cdot 2\dfrac{d}{dR}\left(\dfrac{f}{R}\right) + C\cdot\left(2\dfrac{d}{dR}\left(\dfrac{1}{R}\dfrac{d(Rg)}{dR}\right) + k^2 g\right)\right\} \\ \mathfrak{F}' = \mu\{A'\cdot \quad \text{''} \quad + C'\cdot \quad \text{''} \quad \} \\ \mathfrak{E} = B\cdot R\dfrac{d}{dR}\left(\dfrac{g}{R}\right) \\ \mathfrak{E}' = B'\cdot \quad \text{''} \end{cases} \qquad \text{......(30)}$$

Now the forms of the equations (29) is completely similar to (8) provided $\mathfrak{A}$, $\mathfrak{A}'$, $\mathfrak{B}$, $\mathfrak{B}'$, $\mathfrak{C}$, $\mathfrak{C}'$ are interchanged by $\mathfrak{D}$, $\mathfrak{D}'$, $\mathfrak{E}$, $\mathfrak{E}'$, $\mathfrak{F}$, $\mathfrak{F}'$ respectively. Therefore the problem, in which the normal components of stress are given on the surface $R=a$, is theoretically equivalent with the former case. The only difference existing in the two cases is that the equations which determine $\mathfrak{A}$, $\mathfrak{A}'$ etc. (equations (7)) and $\mathfrak{D}$, $\mathfrak{D}'$ etc. (equations (30)) have not the same coefficients. After getting $\mathfrak{D}$, $\mathfrak{D}'$ etc. by the application of expressions (14), (19), (20), (21) and (22) we may solve the linear equations (30), and obtain A, A' etc. as follows :—

$$\begin{cases} A = \mu\left\{\mathfrak{D}\left(2\dfrac{d}{dR}\left(\dfrac{1}{R}\dfrac{d(Rg)}{dR}\right) + k^2 g\right) - \mathfrak{F}\cdot 2n(n+1)\dfrac{d}{dR}\left(\dfrac{g}{R}\right)\right\}\Big/E \\ A' = \mu\{\mathfrak{D}'(\quad \text{''} \quad) - \mathfrak{F}'\cdot \quad \text{''} \quad \}/E \\ C = \left\{-\mathfrak{D}\cdot\mu\dfrac{d}{dR}\left(\dfrac{f}{R}\right) + \mathfrak{F}\left(-\lambda h^2 f + 2\mu\dfrac{d^2 f}{dR^2}\right)\right\}\Big/E \\ C' = \{-\mathfrak{D}'\cdot \quad \text{''} \quad + \mathfrak{F}'(\quad \text{''} \quad)\}/E \\ B = \mathfrak{E}/R\dfrac{d}{dR}\left(\dfrac{R}{g}\right) \\ B' = \mathfrak{E}'/ \quad \text{''} \end{cases} \qquad \text{.................(31)}$$

where
$$E = \begin{vmatrix} -\lambda h^2 f + 2\mu\dfrac{d^2 f}{dR^2} & 2\mu n(n+1)\dfrac{d}{dR}\left(\dfrac{g}{R}\right) \\ \mu 2\dfrac{d}{dR}\left(\dfrac{f}{R}\right) & \mu\left(2\dfrac{d}{dR}\left(\dfrac{1}{R}\dfrac{d(Rg)}{dR}\right) + k^2 g\right) \end{vmatrix} \qquad \text{.........(32)}$$

and after carrying out the differentiation, all $R's$ are replaced by a.

In this paper we have studied the problem of the generation of elastic waves, in which the boundary conditions on the spherical surface are given, in most general forms; and showed the solutions explicitly. Thus the previous defect pointed out by Mr. S. Honma, in boundary—or initial—value problem, is excluded altogether.

In concluding this paper, the writer wishes to express sincere thanks to Dr. H. Kawasumi and Mr. S. Honma for the valuable advices and suggestions throughout this study.