3. *A Note on the Temperature-Distribution within the Earth.*

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In spite of the recent development of researches on the earth's interior, few studies have been carried out concerning the temperature-distribution within the earth. The main difficulty of this problem is due to the fact that we cannot get sufficient data by which we presume the distribution from the physical standpoint. L. H. Adams[1] and H. Jeffreys[2] discussed the cooling of the earth under some assumptions. On account of the ambiguities of various quantities included, however, their results seem to be probable solutions for this problem. E. C. Bullard[3] also made a similar study along the line of Jeffreys' treatment. Meanwhile, B. Gutenberg[4] proposed a different distribution.

Recently, H. P. Coster[5] measured changes in electrical conductivity of rocks at high temperatures and, with the aid of the theory of lattice defect of ionic crystal, presumed considerable increase in electrical conductivity at the depth of several hundred kilometers in the earth. On the other hand, the writer[6] obtained an approximate distribution of the electrical conductivity in the earth's interior from the standpoint of electromagnetic induction theory which has been developed by S. Chapman,[7] A. T. Price,[8] B. N. Lahiri[9] and others. Studying electromagnetic induction by S_D (solar daily disturbance variation), bay-type disturbance, variation connected with solar eruption and sudden commencement of magnetic storm

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1) L. H. ADAMS, *Washington Acad. Jour.*, **14** (1924) 459.

2) H. JEFFREYS, *The Earth.* Cambridge (1924), p. 79.

3) E. C. BULLARD, *M. N. R. A. S. Geophys. Suppl.*, **4** (1939) 534.

4) B. GUTENBERG, *Physics of the Earth* VII. *Internal Constitution of the Earth.* 1st. ed. p. 153.

5) H. P. COSTER, *M. N. R. A. S. Geophys. Suppl.*, **5** (1949) 193.

6) T. RIKITAKE, Reported to *Intern. Ass. Terr. Mag. Electr. Oslo Assembly*, Aug. 1948.

7) S. CHAPMAN, *Trans. Roy. Soc. London* A **218** (1919) 1.

8) S. CHAPMAN and A. T. PRICE, *Trans. Roy. Soc. London*, A, **229** (1930) 427.

9) B. N. LAHIRI and A. T. PRICE, *Trans. Roy. Soc. London*, A, **237** (1939) 509.

together with S_q (solar diurnal variation on quiet days) and D_{st} (average main phase of magnetic storm), the writer concluded, as shown in Fig. 1, that the conductivity increases discontinuously below the depth of 400 *km* amounting to the order of 10^{-12} *e. m. u.* while the outer layer seems to be almost non-conducting, its conductivity amounting to only 10^{-15} *e. m. u.* near the earth's surface. On the basis of the conductivity-distribution thus obtained, the writer will presume the temperature-distribution in the earth in a similar way with Coster's study.

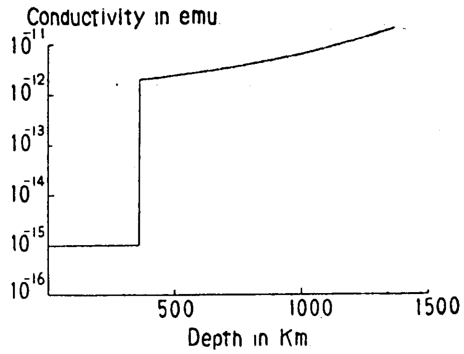

Fig. 1.

According to T. Nagata[10], the electrical conductivity of igneous rocks at high temperature satisfies approximately the equation

$$\sigma = Ae^{-B/T} \quad \text{.................................} (1)$$

as well as obtained by Coster, where A and B are constant. T denotes the absolute temperature. According to the theory of electric conduction in ionic crystal[11], B is interpreted as

$$B = \epsilon/k = \frac{1}{k}\left(\frac{1}{2}w_0 + v_0\right) \quad \text{.............................} (2)$$

where k, w_0 and v_0 denote respectively Boltzmann's constant, energy required to make a lattice defect at zero temperature and activation energy at zero temperature. Thus we may regard the main part of electric conduction in rocks as due to the motion of ions in ionic crystals.

On logarithmically differentiating, we get from (1)

$$\frac{\partial \log \sigma}{\partial p} = -\frac{1}{kT}\frac{\partial \epsilon}{\partial p} + \frac{\partial \log A}{\partial p} \quad \text{........................} (3)$$

where the second term of the right may be neglected except at very high temperature. Further, we write

$$\frac{\partial \epsilon}{\partial p} = -\epsilon \chi \frac{\partial \log \epsilon}{\partial \log V} \quad \text{..................................} (4)$$

where χ and V denote respectively the compressibility and the volume of unit mass.

10) T. NAGATA, *Bull. Earthq. Res. Inst.*, **15** (1937) 663.
11) N. F. MOTT and R. W. GURNEY, *Electronic Processes in Ionic Crystals.* Oxford, 1940.

Hence, we have

$$\frac{\partial \log \sigma}{\partial p}=\frac{\chi\epsilon}{kT}\frac{\partial \log \epsilon}{\partial \log V} \qquad \cdots\cdots\cdots(5)$$

$\partial \log \epsilon/\partial \log V$ is thought to be about the same for all substances and

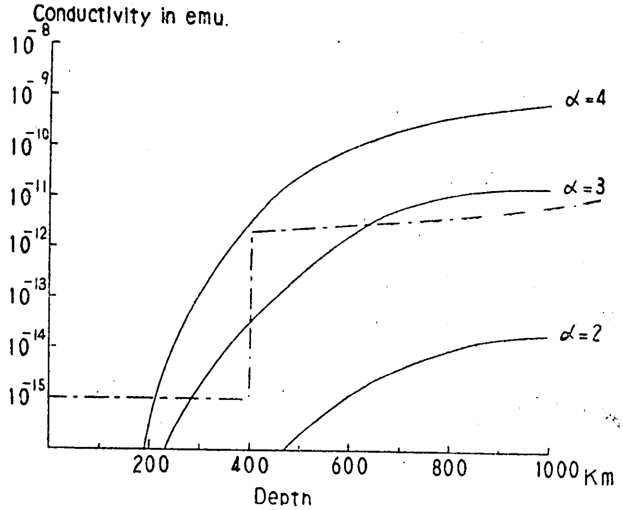

Fig. 2.

takes a value of about -2 from the standpoint of theoretical physics. From (4). we get

$$\epsilon=\epsilon_0 e^{-\chi p\frac{\partial \log \epsilon}{\partial \log V}} \qquad \cdots\cdots(6)$$

where ϵ_0 denotes ϵ at zero pressure. Putting $\chi=0.7\times10^{-12}\,cgs$ and $\epsilon_0=2.3\,eV$ as taken by Coster, ϵ is obtainable for given pressure. Strictly speaking, χ depends on pressure. But it may be assumed that χ does not depend on pressure for rough estimation.

Next, giving $10^5\,ohm^{-1}\,cm^{-1}$ to A as obtained from Nagata's and Coster's experiments, we get σ from (1) and (2) for zero pressure and any temperature. As to the temperature-distribution within the earth, the simplest expression

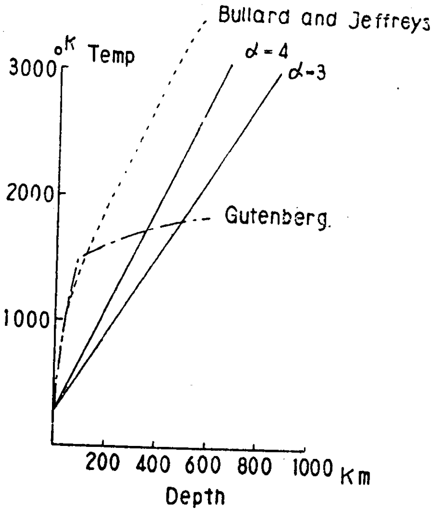

Fig. 3.

$$T = 273 + \alpha x$$

is assumed for the first time, where x denotes the depth and α is a constant. Then, taking into consideration the pressure-distribution obtained by K. E. Bullen[12], σ is reduced to the value that corresponds to the actual pressure. As shown in Fig. 2, σ is calculated for $\alpha=2$, 3 and 4° K/km. Comparing these results to its distribution obtained by the writer, we see $\alpha=3°$ K/km is the most probable. Meanwhile, σ calculated from Jeffreys and Bullards' or Gutenberg's temperature-distribution does not agree well with the writer's distribution.

Comparison of the temperature distribution obtained here to the other ones is shown in Fig. 3. Though the temperature obtained here takes rather lower value than Jeffreys and Bullards' studies, it is of interest that two studies based on quite different standpoints agree with each other in their tendencies. However, the values of the constants included in the present study are not accurate. In order to get more accurate distribution of temperature, we must determine various quantites by experiments under high temperature and pressure.

In conclusion, the writer wishes to express his hearty thanks to Dr. T. Nagata for his criticisms.

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12) K. E. BULLEN, *Bull. Seism. Soc. Amer.*, **30** (1940), 246.