18. *Field Determination of the Elastic Property of Soil Layers (1).*

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Through the measurement of the artificical earthquakes such as caused by blast, vibrating machine and falling body, we can determine the velocities of elastic waves which are propagated in the superficial soil layer, and from these velocities the elastic constants of the material constituting the ground can be deduced. For the longitudinal wave, the velocity may be expressed as

$$V_1=\sqrt{\frac{E}{\rho}\left(\frac{1-\sigma}{1-\sigma-2\sigma^2}\right)}, \quad \text{..........................(1)}$$

and for the transverse wave,

$$V_2=\sqrt{\frac{n}{\rho}}=\sqrt{\frac{E}{\rho}\frac{1}{2(1+\sigma)}}, \quad \text{..........................(2)}$$

where E denotes the Young's modulus, n the shear modulus, ρ the density and σ the Poisson's ratio of the wave-transmitting medium. The velocity, however, is not always constant, but it varies with depth, especially, there is a remarkable change near the surface of the ground. Consequently, the travel-time curve of the wave becomes concave downwards when the distance $\triangle$ is taken on the abscissa, such as, for example, the curve which may be expressed as

$$\Delta=at+bt^2, \quad \text{..................................(3)}$$

wherh t is the travel-time of the wave.

For various superficial layers, the time-distance curves and the subsurface distributions of velocity have been already given in my previous paper[1]. In general, the actual velocities at the ground surface are far smaller than those reported by many authors heretofore for various kinds of soil, because these reported values should hold at a certain depth from the surface where the soils are more or less compacted or hardened under the weight of the overlying parts of the deposits.

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1) N. NASU, *Bull.Earthq. Res. Inst.*, 18 (1940), 289.

The elastic properties of the soil layers may be determined statically by another method, i, e., the load test. However, the result of this test is available only for the materials lying near the ground surface, unless the loading plate of a large area is used, because the distribution of the load intensity (load per unit area) in the interior of the earth shows a rapid decrease with increasing distance from the loading plate. Under these circumstances, we often failed in understanding the results of this kind of test when they were compared with those obtained from the velocities of elastic waves. To make this point clearer, a comparison will be made in the following between the results obtained by the two different methods above mentioned.

Let us take up two examples; one was carried out by the Railway Technical Research Institute in 1940, and the other recently by the writer. The both tests were made on the loam in the up-town of Tokyo. By the weight of the water filled in the tanks the ground was loaded and the amount of settlement was measured by the dial gauges.

The deduction of E was made as follows[2]:

As a first approximation, let E be constant. Then, the settlement of the loading plate s may be expressed as

$$s=\frac{1}{E}\int_0^\infty \sigma_z dz, \qquad \cdots\cdots(4)$$

where σ_z denotes the load intensity at depth z and its distribution in the interior of the soil is assumed to be in the form

$$\sigma_z=p(1-\cos^3\alpha), \qquad \cdots\cdots(5)$$

where p is the load intensity applied on the loading plate, and $\cot\alpha=z/r_0$, r_0 is the radius of the circular loading plate.

After integration, we have

$$s=\frac{2r_0p}{E}\cdots\cdots(6)$$

From the observed values of p and s, E may be calculated. It should be remembered, however, that a certain amount of error is inavoidable in the value of E deduced in this way, because the assumptions above given may not always be qualified for fixing the elastic condition of the subsurface layers.

(a) The first load test was made at Toyamagahara in the N. W. part of the old city of Tokyo. The area of the loading plate was 2,000 cm^2

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[2] For example, see A. Ramspeck, *VDI-Zeitschrift*, 83 (April, 1939), Nr. 17.

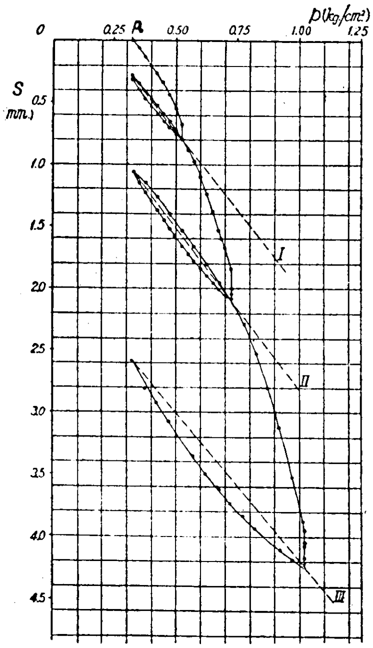

Fig. 1. Load test at Toyamagahara (loam). Area of loading plate$=2{,}000\,\mathrm{cm}^2$, s settlement, $p=$load per unit area of the plate, p_0 that due to the dead weight of the testing apparatus$=0.325\,\mathrm{kg./cm^2}$.

and the load intensity was raised to more than 1 kg./cm². in the maximum. Under such a load, the deformation of the ground already exceeded the elastic limit. Indeed, the permanent settlement could be seen, though slightly, when the intensity became larger than 0.5 kg./cm². By gradually increasing and decreasing the load, the hysteresis phenomenon was observed (Fig 1). The speed of loading, the ultimate load intensity (the dead load of testing apparatus is inclusive) and the values of E calculated for the three loops of the curves in this figure are show in Table I.

In the third loop, an increase of E of about 7 % could be seen. The test however, was made on the ground surface somewhat drug down or hardened at the time of installation of the testing apparatus. Hence, the value of E determined by this test may be expected to be rather larger than that at the natural ground surface.

(b) The second load test was made in a new site of the Tokyo University at Yayoicho, Hongo Ward. In this test, a loading plate of 754 cm². in area was used and the load intensity was limited to be 0.37 kg./cm². in the maximum, this enabled the deformation of the ground to be approximately within the elastic limit. To obtain the value of E just at the ground surface, the testing apparatus was set directly on the ground without removing the loose surface. In Fig. 2, we see that during the first course of loading, the

Table I. Load test on loam at Toyamagahara.

Loading (Loop)	Ultimat load intensity (kg./cm².)	Speed of loading (kg./cm.² per minute)	E Calcuated (kg./cm².)
I	0.525	0.001175	196.6
II	0.725	0.00125	189.0
III	1.075	0.00122	211.7

plate sunk rapidly with increasing load, but the rate of settlement suddenly decreased and only a slight difference of the rate could be seen in the successive courses. It will be unreasonable to consider the first rapid settlement to be an elastic deformation. Surely, it is due to the compaction of the loose soil. The conditions of loading and the calculated values of E are shown in Table II.

Table II. Load test on loam at Yayoicho.

Loading (loop)	Ultimate load intensity (kg./cm².)	Speed of loading (kg./cm². per minuts)	E calculated (kg./cm².)
I	0.370	0.0097	153.8
II	0.370	0.0097	160.0
III	0.370	0.0096	163.1

Now, the result of the determination of the velocities of elastic waves is as follows :

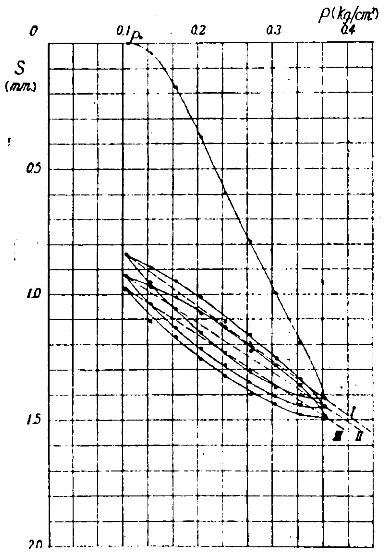

Fig. 2. Load test at Yayoicho, Hongo (loam). Area of loading plate = 754 cm², s settlement, p= load per unit area of the plate, p_0 that due to the dead weight of the testing apparatus = 0.106 kg./cm².

The time-distance curves of elastic waves which were obtained in the place where the second load test was made may be expressed as

$$\Delta = 125.30t + 7215.05t^2, \quad \text{(for longitudinal wave),} \ldots\ldots\ldots\ldots\ldots (7)$$

$$\Delta = 55.02t + 850.00t^2, \quad \text{(for transverse wave),} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where Δ denotes the distance (i. e., the distance of seismograph from falling body) and t the travel-time of the wave (Fig. 3, upper). These expressions may hold for a distance less than 10 metres. The velocities of two types of waves at the ground surface are given by the coefficients of the first terms lying on the right-hand side of the above expressions. From these velocities, we have

$$E = 129.4 \text{ kg./cm.}^2,$$
$$n = 46.9 \text{ kg./cm.}^2,$$
$$\sigma = 0.380.$$

The density, ρ, of the loam in this case is taken as 1.52; the mean obtained from five specimens. The order of E here obtained is in good accordance with that obtained in the second load test, though a difference of about 18 % exists. This will be inevitable within the accuracy of the present test. The wave of extremely low velocity which should be propagated through the uppermost loose soil could not be observed, perhaps such a loose formation has no direct influence upon the propagation of the elastic wave.

The analysis of the time-distance curve will lead us to an interpretation of the wave paths (depths of penetration and others) and the distribution of velocity below the ground surface[3] (Fig. 3, lower). From the velocities thus given, we can determine the variation of elastic property of soil with depth (Table III).

According to F. Kishinouye and K. Iwama[4], the mean velocity of longitudinal wave was 300 m./sec. and that of transverse wave was 115 m./sec. in the loam (deposited in the site of the Tokyo University), when they were measured within a

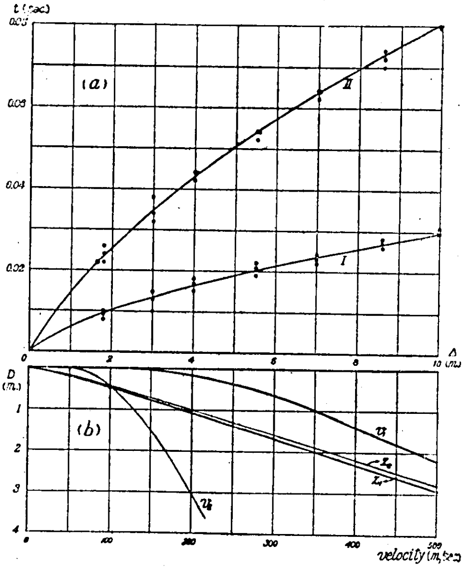

Fig. 3. (a) Time-distance curves. I, longitudinal wave, II transverse wave.) (b) Depth of penetration for given value of Δ (z_1 for longitudinal wave, z_2 for transverse wave) and the variation of velocity with depth *D*.

Table III. Velocities of waves and elastic constants.
(loam in the up-town of Tokyo.)

Depth (m.)	V_1 (m./sec.)	V_2 (m./sec.)	*E* (kg./cm.^{2})	*n* (kg./cm.^{2})	σ
0	125	55	129.4	46.9	0.380
0.1	176	71	219.0	78.2	0.403
0.3	233	91	362.2	128.4	0.410
0.5	274	104	478.7	167.8	0.415
1.0	353	129	750.3	258.1	0.421
1.5	414	151	1012.9	353.6	0.423
2.0	470	170	1287.5	448.2	0.424
2.5	526	187	1560.4	542.4	0.427
3.0	559	201	1762.3	626.6	0.427

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[3] See 1)

[4] F. Kishinouye and K. Iwama, *Bull. Earthq. Res. Inst.*, **22**, (1944), 170.

distance of less than 10 metres. If so, the elastic constants which are deduced from these velocities become as $E=576$ kg./cm.^{2}, $n=206$ kg./cm.^{2} and $\sigma=0.406$. As these values are far larger than those at the ground surface, they may be recognized as the mean values which prevail in the loam lying within a depth of 0.5~1.0 m. measured from the surface. Further, the result of the first load test will account for the elastic property of the same soil at a depth of about 10 cm. below the natural surface of the ground.

It should be added that the loam on which the experiments mentioned above were made was clayey loam and the bearing power of such a loam has been already known by many experiements to be about 5 tons per square metre. The bearing power which was deducible from the experiment made at Toyamagahara also showed a good agreement with this value.

The data for the physical properties of various soil layers are necessary when the effect of the soil layers upon the vibration of the structures is taken into consideration. Further, the data are also helpful for the study of the subsidence of the lowland which has been continuously observed for recent years near Tokyo and Osaka. The study is being continued with the Fund for Science Research of the Ministry of Education for the year 1949.