6. *Local Phenomena of Tsunami (Part 2).*

By Nobuji NASU,

Earthquake Research Institute.

(Read Feb. 15, 1949.—Received March 31, 1949.)

The estimation of the speed with which the seismic sea waves sweep over the land is necessary for the determination of the extent of their dynamical effects upon structures of various kinds. It will furnish excellent data for the mitigation plan of the damage caused by the tsunami. There are, however, no data which are available for this purpose, because no systematic investigation relating to the intensity of the tsunami was tried in the past destructive tsunamis. During our inspection tour to the districts which suffered a considerable damage in the recent Nankaido tsunami of 1946, good examples were found which enabled us to estimate the speed of the tsunami. Generally speaking, in these estimations only simple cases have to be taken, like in the determination of thc seismic intensity from the damage of simple structures. An assumption must be introduced herein, such that the motion of the water is steady; this corresponds to the assumption of the simple harmonic motion which is usually made for the earthquake motion. The principles of the estimation of the velocity treated in this paper are based on the following facts.

First, in a certain area, the change in the height of the tsunami is much influenced by the speed of the flowing water and the geographical feature of this area. This change and the land-feature will be made clear by means of the precise leveling survey, and from the result of it we can estimate the flowing velocity of the tsunami.

Secondly, the force acts on the body submerged in the water varies with the square of the velocity with which the water impinges upon the body. Hence, the estimation of the velocity of the water is possible from the damage caused on the simple structures, if the strengths of these structures are known.

There are, however, various metdods of estimations basing on the facts above mentioned. In the following, the methods which are of practical use will be described.

We shall take up four examples basing on the fact first cited.

(1) *Curved river.* The flowing velocity of the tsunami can be determined from the amount of elevation of the water-surface measured on both sides of the curved river. When the sea water runs up the curved course, the surface of the water becomes higher on the outer river bank than on the inner one by the action of the centrifugal force (Fig. 1,). The difference of height, H, in this case is given by

$$H = \frac{v^2}{g} \log_e \frac{b}{a}, \qquad (1)$$

where v denotes the velocity, g the gravity-acceleration, a and b are the radii of curvature of the inner and outer banks respectively. An actual example was the brook which flowed into the sea at the head of Yura Bay, Wakayama Prefecture. Here, the velocity was estimated to be 2.26 m./sec.; the value being calculated for the actually obtained values of $H = 0.19$ m., $a = 50$ m. and $b = 60$ m. Another similar example was at Inami about 500 m. up the river mouth where water-course curved with the radii of $a = 140$ m. and $b = 160$ m. From the elevation of the water-surface which mersured 0.21 m., the velocity worked out as 3.16 m./sec. This value fairly agreed with that determined by another method of estimation which will be cited later in this paper, that is, 3.00 m./sec., the value estimated from the overturning of a concrete wall which stood at a distance of 50 m. from this point.

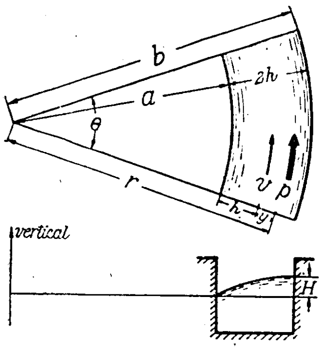

Fig. 1.

In the above example, the velocity is assumed to be uniform throughout the cross-section of the river. It varies, however, considerably across the river. Of course, the velocity is the maximum in the middle of a river and the minimum near the banks, but the point of the maximum velocity is not exactly at the centre of the width of the river, it leans to the inner bank. Putting $b = a + 2h$, $\frac{h}{a} = \alpha$, the velocity at any point where $r = a + h + y$ is given by

$$v = -\frac{1}{2\mu}\frac{\partial p}{\partial \theta}\frac{h}{\alpha}\left\{\left[\frac{(1+2\alpha)^2 \log(1+2\alpha)}{4\alpha+4\alpha^2}\right]\left[1+\alpha+\frac{\alpha y}{h}-\frac{1}{1+\alpha+\frac{\alpha y}{h}}\right] - \left(1+\alpha+\frac{\alpha y}{h}\right)\log\left(1+\alpha+\frac{\alpha y}{h}\right)\right\}, \qquad (2)$$

and $\bar{v}$, the mean value of v is

$$\bar{v} = -\frac{1}{4\mu}\frac{\partial p}{\partial \theta}\frac{h}{\alpha}\left\{\alpha+\alpha^2-\frac{(1+2\alpha)^2}{4(\alpha+\alpha^2)}[\log(1+2\alpha)]^2\right\}, \qquad (3)$$

or in other forms,

$$v = -\frac{1}{2\mu}\frac{\partial p}{\partial \theta}\left[\frac{(b^2\log b-a^2\log a)}{b^2-a^2}r-\frac{a^2b^2}{b^2-a^2}\left(\log\frac{b}{a}\right)\frac{1}{r}-r\log r\right], \qquad (4)$$

and

$$\bar{v} = -\frac{1}{2\mu}\cdot\frac{1}{b-a}\frac{\partial p}{\partial \theta}\left[\frac{(b^2-a^2)}{4}-\frac{a^2b^2}{b^2-a^2}\log\left(\frac{b}{a}\right)^2\right], \qquad (5)$$

where μ denotes the viscosity, $\frac{\partial p}{\partial \theta}$ the pressure-gradient along the river course and is constant in this case. In Fig. 2, the distribution of the ratio $v:\bar{v}$ in a cross-section is shown for given values of α[1]

It will be seen that if the curvature of the river is small, that is, α is large, the effect of the curvature is noticeable, but otherwise it is not. Further, the maximum velocity which is expected becomes one and half times as large the estimated mean value.

(2) *Straight water-course with corrugated bottom.* In a street where the flow of the water is limited only in one direction by the houses and walls standing on both sides of the street, a noticeable change in the height of the water can be seen along the street, if the surface of the ground is corrugated. Assuming that such a street is a straight canal having a rectangular cross-section and a sinuous bottom in the from $y=a\sin mx$, Where x is the lengthwise direction of the canal, m a constant and a is also a constant which is small compared with the mean depth of the water in the canal, h.

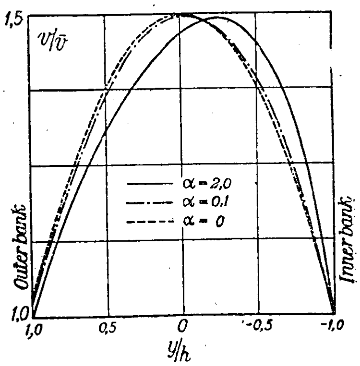

Fig. 2.

The elevation of the free surface of water has the form $y'=b\sin mx$. Then, for a given value of x, the ratio $y':y$, that is, $b:a$ is

$$\frac{b}{a} = \frac{1}{\cosh mh-\frac{g}{mv^2}\sinh mh}, \qquad (6)$$

1) Modern Developments in Fluid Dynamics, Oxford Eng. Sc. Ser. 1, 1938.

where v denotes the general velocity of the water in the canal, g the acceleration due to gravity.[2] The velocity of the water can be determined if $b:a$ is known. The crests and troughs of the free surface and the bottom correspond or are opposite as

$$v^2 \gtrless \frac{g}{m}\tanh mh \tag{7}$$

or

$$v^2 \gtrless c^2 \tag{8}$$

where c is the velocity of propagation of the wave of the length $2\pi/m$ in water of depth h. As an extreme case, if $v=c$, the ratio b/a becomes infinite, this means that the free surface cannot be represented by a simple sine-curve, the assumption from which this calculation started breaks down in this case.

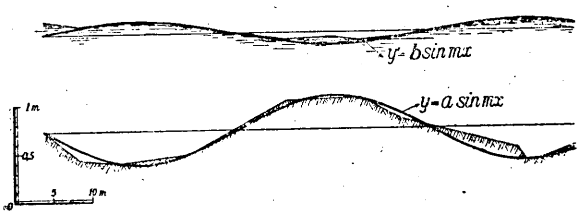

Fig. 3.

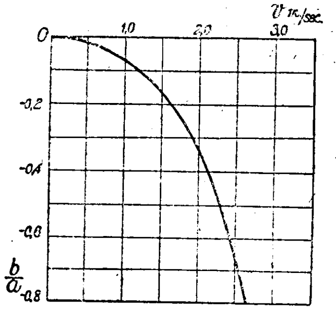

Fig. 4.

The actual example which was found at Kainan is shown in Fig. 3. In this example, the numerical values actually determined are,

$$a = 0.35\,\text{m.},\quad b = 0.12\,\text{m.},\quad b/a = 0.343.$$

By the aid of Fig. 4, we can determine the velocity as 2.0 m./sec. It must be remembered that the estimation is impossible, if $v = 3.95$ m./sec., because in this case the ratio b/a becomes infinite. However, the value estimated here is only one-half of this extreme value, thus there is no objection in this estimation.

(3) *Tsunami dissipation zone.* For convenience, a brief explanation of the tsunami dissipation zone[3] will be given here. The idea is that

2) The expression is derivable from a complex potential,

$$w = -Uz + \frac{aU}{\sinh mh}\cos m(z-ih) - \frac{bU}{\sinh mh}\cos mz, \qquad z = x+iy.$$

3) A. IMAMURA: *Theoretical and Applied Seismology*, p. 134, 1937.

the energy of the tsunami will be dissipated if the water spreads over a wide area which is devoted to this zone after passing through a narrow water course. As the result, the important part of the city or town can escape the full force of tsunami. Hence, in cities which have in their immediate vicinity the water-courses, such as valleys or inhabited lowland, flat ground, these areas should be utilized for this purpose. The lowland which acts as such a safety measure is called the "tsunami dissipation zone."

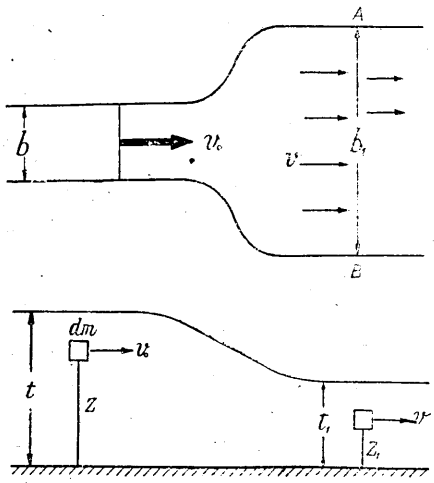

Fig. 5.

Taking up an ideal case as is schematically shown in Fig. 5, the decrease of the water-height in the dissipation zone will be calculated in the following.

Let the cross-section of the narrow course of water be b, the velocity of the elementary volume of water, dm, be v_0 and the height of dm above the ground be z. After spreading over a wider area, of which the width is b_1, dm has a velocity of v and a height of z_1. Then, the difference of height is $z-z_1$. Neglecting the change in pressure, we have

$$\frac{1}{2}dm(v^2-v_0^2) = g\,dm(z-z_1)\,. \tag{9}$$

Put, $$dm = \frac{\rho}{g}b\,dz\,v_0\,dt = \frac{\rho}{g}b_1\,dz_1\,v\,dt\,, \tag{10}$$

where ρ denotes the density of the water.

It follows, $$b\,v_0\,dz = b_1\,v\,dz_1\,,$$

and $$v = \frac{1}{a}\frac{dz}{dz_1}\,,\quad \left(a = \frac{b_1}{v_0\,b}\right). \tag{11}$$

Therefore, expression (9) may be written as

$$\frac{dz}{dz_1} = a\sqrt{v_0^2+2g(z-z_0)} = aZ\,. \tag{12}$$

After integration we have

$$a(agz_1+v_0-z) = \log\frac{aZ-1}{av_0-1}. \tag{13}$$

Let t be the depth of water in cross-section b, t_1 be that in cross-section b_1, ($z=t$, $z_1=t_1$), and the decrease of height $h=t-t_1$. For h we have the following expression.

$$\frac{b_1}{b}+\frac{b_1}{bv_0}\left[\frac{b_1 g}{bv_0}(t-h)-\sqrt{v^2+2gh}\right] = \log\frac{b_1\sqrt{v_0^2+2gh}-bv_0}{v_0(b_1-b)}. \tag{14}$$

For a given value of b_1/b, the above equation may be solved graphically. An example of the solution is shown in Fig. 6. In this case, the initial height of the water is assumed to be 5 m., and $\frac{v_0^2}{2g}=2$, that is, $v_0 = 6.26$ m./sec., a rather large velocity of the tsunami.

Decrease of height for $\frac{v_0^2}{2g}=1, 2, 3$ and

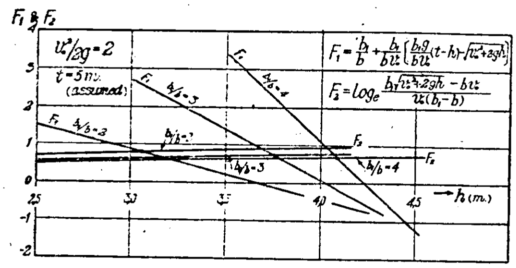

Fig. 6.

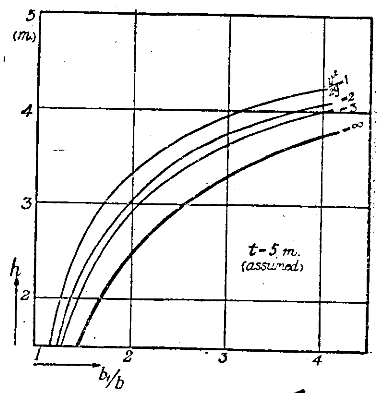

Fig. 7.

∞ is shown in Fig. 7, in which h is plotted against b_1/b, As is clear in this figure, when the area of the dissipation zone is wider, the decrease of the height of tsunami becomes greater, but there are limits in the amount of this decrease as shown by a curve in thick line which corresponds to the extreme case $v=\infty$. In this case, h is given by

$$h = \frac{b_1-b}{b_1}t. \tag{15}$$

Further, it can be seen that the dissipation zone is more effective for a smaller velocity of the tsunami for a given value of b_0/b.

Unfortunately, there could be found no good example in the recent tsunami of 1946, except the lowland which was situated near the village of Hiro, Wakayama Prefecture, where it acted as a dissipation zone to some extent. The height of the water inrushing there was so low that no serious damage was caused. The decrease of the water-height could be seen, but its exact value was uncertain.

(4) *Inclination of the ground and the decrease of the height of tsunami.* As already mentioned in Part 1 of this paper,[4] the velocity of tsunami can be determined from the decrease of the water-height and the inclination of the ground by Chézy's formula. As the actual example has been already given, the description of this method will be abbreviated here.

Now in the following, we shall explain the methods of estimation of the speed which are based on the second fact cited at the top of this paper. As already mentioned, the force exerted by the water upon the submerged body is given by the following expression.

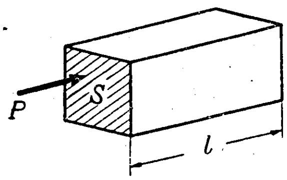

Fig. 8.

$$P = \rho k S \frac{v^2}{2g},$$
$$= Kv^2, \qquad \left(K = \rho k \frac{S}{2g}\right), \tag{16}$$

where P is the pressure, ρ the specific gravity of the water, S the cross-section of the body and g the acceleration due to gravity. k is a constant which is determined by experiments, such that, for prismatic bodies of length l and of cross-section S, it takes values as

$$k = 1.86, \quad 1.47, \quad 1.35, \quad 1.33,$$
$$\text{for} \quad \frac{l}{\sqrt{S}} = 0.03, \quad 1, \quad 2, \quad 3. \tag{17}$$

When the force of tsunami acts on the simple structures, such as stone pillar, monument, wall and others, they will be removed or sometime will be broken if they cannot resist the attack of the waves. Actually in the field which was invaded by the tsunami there can be seen the overturning, sliding and fracturing of these bodies in many places. There are other structures which are heavier than those just mentioned but may be treated as simple structures, that is, breakwater, sea wall and others of this kind. From the relation above given, the force by which these phenomena are displayed can be determined with a fair approximation, also the velocity of the water can be estimated easily.

As treated usually in the harbor engineering, the conditions of the sliding and the overturning of the wall set on the rubble-mound (Fig. 9) are,

4) N. Nasu; *Bull. Earthq. Res. Inst.*, **25** (1947), 81–84.

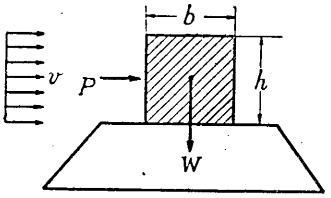

Fig. 9.

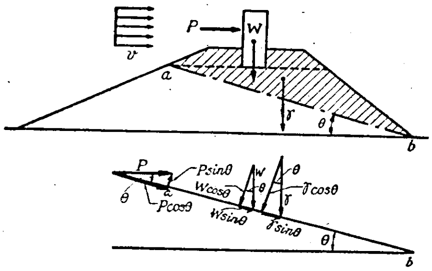

Fig. 10.

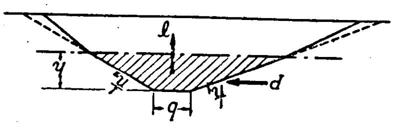

Fig. 11.

$$P \geqq fW, \quad \text{for sliding,} \qquad (18)$$

$$Ph \geqq Wb, \quad \text{for overturning,} \qquad (19)$$

where W denotes the weight of the wall measured in the water and f the coefficient of friction between the base of the concrete wall and the rubbles; f is usually taken as 0.6—0.7.

If the following condition is satisfied in the part of the rubbles, the breakwater of this kind is safe for the sliding and the collapse. (Fig. 10)

$$f\{(W+\gamma)\cos\theta - P\sin\theta\} \geqq (W+\gamma)\sin\theta + P\cos\theta. \qquad (20)$$

As usually, $f=1.0$ in this case, the condition may be written as

$$(W+\gamma)\cos\theta - P\sin\theta \geqq (W+\gamma)\sin\theta + P\cos\theta. \qquad (21)$$

The similar idea of sliding is applicable to the collapse of the rubble mound which has no vertical wall as shown in Fig. 11. In this case, the condition of the collapse is

$$P > \gamma, \qquad (f=1.0) \qquad (22)$$

The calculations above mentioned are all related to the determination of the minimum dimensions of the structures which are necessary for resisting the expectable water pressure. In our case, however, the purpose lies in the determination of the pressure or the velocity of the water by the action of which the damage of structures will be caused. For the help of this determination, the relations between the dangerous velocity and the dimensions of the structures as shown in Figs. 9, 10 and are formulated in the following.

Let the dangerous velocity of the water be v, then the conditions of the sliding and the overturning of the wall (Fig. 9) are

$$v^2 \geqq \frac{2f\rho' g}{\rho_0 k}(b), \qquad \text{for sliding,} \qquad (23)$$

$$v^2 \geqq \frac{2\rho' g}{\rho_0 k}\left(\frac{b^2}{h}\right), \quad \text{for overturning,} \tag{24}$$

where ρ_0 is the specific gravity of the water and ρ' is that of the wall measured in the water.

In the case of a concrete wall submerged in the sea water, $\rho_0 = 1.03$, $\rho' = 2.35-1.03 = 1.32$. Let $f = 0.65$ and $k = 1.5$ in their means, then, the relations given above become, (v in m./sec., b and h in m.),

$$v^2 = 10.9\,b\,, \quad \text{for sliding,} \tag{25}$$

$$v^2 = 16.7\left(\frac{b^2}{h}\right), \quad \text{for overturning.} \tag{26}$$

The similar relation can be obtained in the case of the rubble mound as shown in Fig. 11.

$$v^2 = \frac{h+0.89\,b}{0.0358}\,. \tag{27}$$

In this case, the inclinations of the mound are $n_1 = 2.5$, $n_2 = 2.0$ and the specific gravity of the rubble in the water is assumed to be 0.98.

Formulae given above were proved to be convenient in the estimation of the flowing velocity of the tsunami in the recent Nankaido tsunami. At Kainan and Yura, the velocities that were estimated from the damage of the breakwaters showed a good accordance with those obtained by other methods.