# 2. On Seismic Sea Waves caused by Deformations of the Sea Bottom. The 3rd Report. The One-dimensional Source.

By Ryutaro TAKAHASI,

Earthquake Research Institute.

(Read Mar. 18, 1947.—Received June 30, 1948.)

Travel-times of the seismic sea waves, observed at various tide guage stations in the case of the Nankai Earthquake on Dec. 21, 1946, suggest a belt-like source elongated in the direction EW. The present paper deals with the seismic sea waves that will be generated from such an elongated source as can be regarded one-dimensional. As in the previous papers[1], we take the origin of the coordinates at the undisturbed sea surface, $x$-axis horizontally and $z$-axis vertically downwards. The depression of the sea surface $\zeta$ and the velocity of depression of the sea bottom $\eta$ are assumed to be functions of $x$ and $t$ only. Denoting by $\phi$ the velocity potential, we have

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0, \tag{1}$$

and

$$\left[\frac{\partial\phi}{\partial t}+2\mu\phi\right]_{z=\zeta}=g\zeta, \tag{2}$$

where $\mu$ is the coefficient of fictitious viscosity. As the boundary condition, we have at the sea bottom, where $z=H$,

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=H}=\eta(x,t)\equiv f(x)\cdot\chi(t), \tag{3}$$

where $\eta(x,t)$ is assumed to be separable into space and time factors. From these expressions we can easily obtain the formal solution as follows:

$$\zeta=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}d\alpha\int_{-\infty}^{\infty}\chi(\tau)e^{i\alpha(t-\tau)}d\tau\int_{-\infty}^{\infty}Gdk\int_{-\infty}^{\infty}f(\lambda)e^{ik(x-\lambda)}d\lambda, \tag{4}$$

1) *Bull. Earthq. Res. Inst.*, **20** (1943), 375; **23** (1946), 23.

where

$$G = -(\alpha i + 2\mu)/[(\alpha^2 - 2\mu\alpha i)\cosh kH - gk \sinh kH]. \tag{5}$$

The change of order of the integration being allowed, we integrate first with respect to $\alpha$. Since $G$ has poles at $\alpha = i\mu \pm \gamma$, we get, just in the same way as in the lst paper,

$$\left.\begin{aligned}\int_{-\infty}^{\infty} G e^{i\alpha(t-\tau)} d\alpha &= 2\pi[\mu \sin \gamma(t-\tau) + \gamma \cos \gamma(t-\tau)] e^{-\mu(t-\tau)}/\gamma \cosh kH \\ &\qquad\qquad \text{if} \quad (t-\tau) > 0, \\ &= 0 \qquad \text{if} \quad (t-\tau) < 0,\end{aligned}\right\} \tag{6}$$

where
$$\gamma = \sqrt{gk \tanh kH - \mu^2}. \tag{7}$$

Therefore in the limit of $\mu \to 0$, we have

$$\zeta = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{dk}{\gamma \cosh kH} \int_{-\infty}^{t} \chi(\tau)\gamma \cos \gamma(t-\tau) d\tau \int_{-\infty}^{\infty} f(\lambda) e^{ik(x-\lambda)} d\lambda \tag{8}$$

Next we will execute the integration with respect to $\tau$. We assume that

$$\left.\begin{aligned}\chi(t) &= 0 \qquad t < 0, \quad t > T, \\ &= 1/T \qquad 0 < t < T,\end{aligned}\right\} \tag{9}$$

that is, the deformation of the sea bottom is assumed to grow uniformly and simultaneously at each point and attain to the final amount $f(x)$ at $t = T$. We have then

$$\left.\begin{aligned}\int_{-\infty}^{t} \chi(\tau)\gamma \cos \gamma(t-\tau) d\tau &= -\frac{1}{T} \sin \gamma(t-\tau)\Big|_0^T \quad \text{if} \quad t > T, \\ &= -\frac{1}{T} \sin \gamma(t-\tau)\Big|_0^T \quad \text{if} \quad t < T.\end{aligned}\right\} \tag{10}$$

Therefore, if we write

$$\xi(t) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin \gamma t}{\gamma \cosh kH} dk \int_{-\infty}^{\infty} f(\lambda) e^{ik(x-\lambda)} d\lambda \tag{11}$$

we have simply

$$\left.\begin{aligned}\zeta &= [\xi(t) - \xi(t-T)]/T \qquad t > T \\ &= [\xi(t) - \xi(0)]/T \qquad t < T\end{aligned}\right\} \tag{12}$$

Therefore we can put with $\xi(t)$ in place of $\zeta$. In the case of $T\rightarrow 0$, that is, when the deformation occur instantaneously, we have obviously $\zeta = \partial\xi/\partial t$.

Now changing variables from $k, \lambda, x, H$ to $m, s, \rho, h$, by the relations

$$m = ka, \quad \lambda = sa, \quad x = \rho a, \quad H = ha \tag{13}$$

where $a$ is a parameter expressing the width of the deformed area, we have

$$\xi(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\sin \gamma t}{\gamma \cosh mh} dm \int_{-\infty}^{\infty} f(s) e^{im(\rho - s)} ds \tag{14}$$

$$\gamma = \sqrt{(gm \tanh mh)/a} \tag{15}$$

In most cases of seismic sea waves, deformed area measures several hundred km in width, whereas the sea is only a few km deep, so we have $h \ll 1$. In such cases, $\gamma \doteqdot mc/a$, $c = \sqrt{gH}$; then we obtain

$$\xi(t) = \frac{a}{2\pi c}\int_{-\infty}^{\infty} \sin m\frac{ct}{a} e^{im(\rho-s)} \frac{dm}{m}\int_{-\infty}^{\infty} f(s) ds \tag{16}$$

$$= \frac{a}{2c}\int_{\rho - ct/a}^{\rho + ct/a} f(s) ds, \tag{17}$$

in virtue of the integral formula:

$$\left.\begin{aligned} \frac{1}{\pi}\int_{-\infty}^{\infty} \sin \rho x \cdot e^{i\sigma x} \frac{dx}{x} &= 0 \qquad |\sigma/\rho| > 1 \\ &= 1 \qquad |\sigma/\rho| < 1. \end{aligned}\right\} \tag{18}$$

By means of (17), $\xi(t)$ can be calculated easily for any $f(x)$. In the case of $T\rightarrow 0$, since $\zeta = \partial\xi/\partial t$, we have

$$\zeta(t) = \frac{1}{2}f(\rho + ct/a) + \frac{1}{2}f(\rho - ct/a). \tag{19}$$

In the case when $T$ is finite, we can obtain $\zeta(t)$ from (12).

Now if

$$f(s) = 0 \quad \text{when} \quad s > 1 \quad \text{or} \quad s < -1, \tag{20}$$

we have from (17)

$$\text{when} \quad \rho > 1, \quad \xi(t) = \frac{a}{2c}\int_{-1}^{1} f(s)ds \quad \text{if} \quad ct/a > (\rho+1),$$

$$= \frac{a}{2c}\int_{\rho-ct/a}^{1} f(s)ds \quad \text{if} \quad (\rho+1) > ct/a > (\rho-1),$$

$$= 0 \qquad \text{if} \quad ct/a < (\rho-1),$$

and when

$$1 > \rho > 0, \qquad \xi(t) = \frac{a}{2c}\int_{-1}^{1} f(s)ds \quad \text{if} \quad ct/a > (\rho+1),$$

$$= \frac{a}{2c}\int_{\rho-ct/a}^{1} f(s)ds \quad \text{if} \quad (\rho+1) > ct/a > (1-\rho),$$

$$= \frac{a}{2c}\int_{\rho-ct/a}^{\rho+ct/a} f(s)ds \quad \text{if} \quad ct/a < (1-\rho).$$

$\rho$ can be considered positive without loss of generality. As examples, wave forms for the two cases:

*a)* $f(s) = L \qquad |s| < 1, \quad f(s) = 0 \quad |s| > 1, \quad T = a/2c;$

*b)* $f(s) = L \sin \pi s \qquad |s| < 1, \quad f(s) = 0 \quad |s| > 1, \quad T = a/2c;$

are shown in Figs. 1. and 2.

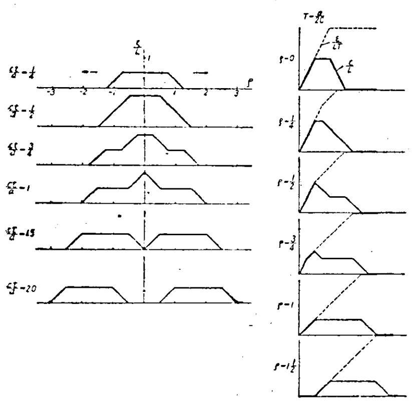

Fig. 1. Shape of seismic sea waves when the deformation is $f(\rho) = L, \ |\rho| < 1.$

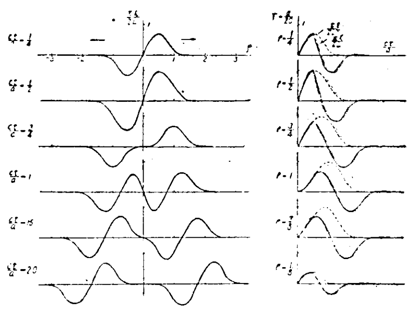

Fig. 2. Shape of seismic sea waves when the deformation is $f(\rho) = L \sin \pi\rho, \ |\rho| < 1.$