3. *A Method for Studying the Relations between the Changes in Geomagnetism and Earth Current.*

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It was often reported that anomalous changes in earth current occured before earthquakes sometimes. As is well known, however, changes in earth current are in the main caused by changes in the earth's magnetic field originated outside the earth. Consequently, it is difficult to abstract change only related to the earthquake itself. Hence, it is desirable to eliminate changes due to those of geomagnetic field of external origin. T. Yosimatu[1] proposed the "Difference method". He measured earth potential with two systems having respectively long and short electrode-distances. Subtracting, then, the values multiplied by a suitable constant from the other values, he assumed that the changes of large scale were approximately eliminated leaving only local or regional changes. According to him, the changes thus observed have some relations to earthquake occurrences.

Meanwhile, the writer[2] studied a method for discussing the relations between geomagnetic field and earth current in the case of periodic variation. That method will be extended to the case of non-periodic variation in this paper intending to apply to the elimination of the geomagnetic effects of external origin.

When we neglect displacement current, electric and magnetic quantities in earth are connected by the relation

$$\mathrm{rot}\,\vec{H} = 4\pi\vec{i}, \tag{1}$$

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

where $\vec{H}$, $\vec{B}$, $\vec{E}$, and $\vec{i}$ denote respectively magnetic field, magnetic induc-

1) T. Yosimatu, *Journ. Meteorol. Soc. Japan* **15** (1937), 145.
2) T. Rikitake, *Bull Earthq. Res. Inst.* **24** (1946), 1.

tion, electric field and electric current-density. Between these quantities we have the next relations

$$\vec{i} = (\sigma)\vec{E}^{3)}, \tag{3}$$

$$\vec{B} = (\mu)\vec{H}, \tag{4}$$

and

$$\operatorname{div} \vec{B} = 0, \tag{5}$$

where σ and μ denote respectively electrical conductivity and magnetic permeability which are in general tensor quantities. From (1) and (3) we have

$$E = \frac{(\rho)}{4\pi} \operatorname{rot} \vec{H} \tag{6}$$

where ρ denotes electrical resistivity being defined by $(\sigma)(\rho) = 1$. Substituting (6) and (4) in (2), we obtain

$$\operatorname{rot} \{(\rho) \operatorname{rot} \vec{H}\} = -4\pi(\mu)p\vec{H} \tag{7}$$

where we write p in place of $\partial/\partial t$. When one of the principal axes of ρ and μ coincides with z-axis that is perpendicular to the earth's surface, the components of the direction of the principal axes satisfy[4)]

$$\left.\begin{aligned} \frac{d^2H_x}{dz^2} &= \frac{4\pi\mu_{xx}p}{\rho_{yy}} H_x, \\ \frac{d^2H_y}{dz^2} &= \frac{4\pi\mu_{yy}p}{\rho_{xx}} H_y, \\ pH_z &= 0, \end{aligned}\right\} \tag{8}$$

where it is assumed that the magnetic field depends only on z. Thus solving (8), we have

$$\left.\begin{aligned} H_x(t) &= H_{ox}(t)e^{-\sqrt{\frac{4\pi\mu_{xx}p}{\rho_{yy}}}z}, \\ H_y(t) &= H_{oy}(t)e^{-\sqrt{\frac{4\pi\mu_{yy}p}{\rho_{xx}}}z}, \end{aligned}\right\} \tag{9}$$

where $\vec{H}_0$ is the magnetic field at the surface $z = 0$.

As to the electric field, we have from (6) and (9)

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3) ( ) denotes tensor quantity.

4) It is assumed that the principal axes of ρ and μ are the same

$$\left.\begin{aligned} E_x(t) &= \sqrt{\frac{\rho_{xx}\mu_{yy}p}{4\pi}}e^{-\sqrt{\frac{4\pi\mu_{yy}p}{\rho_{xx}}}z} H_{oy}(t)\,, \\ E_y(t) &= -\sqrt{\frac{\rho_{yy}u_{xx}p}{4\pi}}e^{-\sqrt{\frac{4\pi\mu_{xx}p}{\rho_{yy}}}z} H_{ox}(t)\,. \end{aligned}\right\} \qquad (10)$$

Solving these operational equations, we obtain

$$\begin{aligned} E_x(t) &= \frac{d}{dt}\int_0^t H_{oy}(t-u)h_x(u)du\,, \\ E_y(t) &= -\frac{d}{dt}\int_0^t H_{oy}(t-u)h_y(u)du\,, \end{aligned} \qquad (11)$$

where $$h(t) = \frac{\sqrt{\rho\mu}}{2\pi}t^{-\frac{1}{2}}\,e^{-\frac{\pi\mu z^2}{\rho t}}. \qquad (12)$$

Thus we can obtain the changes in earth potential due to those of the eaath's magnetic field at the surface expressed by $\vec{H}=0$ $(t<0)$ and $\vec{H}=\vec{H}o\,(t)$ $(t>0)$.

In order to get the expression for $z=0$, we divide the integration into two parts, say $\int_0^\delta$ and $\int_\delta^t$, where δ is so small that the rate of the change in the magnetic field can be regarded as constant during the interval $0\sim\delta$. Hence it becomes

$$\begin{aligned} &\frac{d}{dt}\int_0^\delta H_{oy}(t-u)h_x(u)du \\ &= H'_{oy}(t)\int_0^\delta h_x(u)du \\ &= H'_{oy}(t)\sqrt{\frac{\rho_{xx}\mu_{yy}}{\pi}}a_x\int_{a_x/\sqrt{\delta}}^\infty (e^{-v^2}/v^2)dv \\ &= H'_{oy}(t)\frac{\sqrt{\rho_{xx}\mu_{yy}}}{\pi}\left\{\sqrt{\delta}\,e^{-a_x^2/\delta}-2a_x\int_{a_x/\sqrt{\delta}}^\infty e^{-v^2}dv\right\} \end{aligned}$$

where

$$a_x^2 = \frac{\pi\mu_{yy}z^2}{\rho_{xx}}.$$

Thus making $z \to 0$, we have

$$\lim_{z\to 0}\frac{d}{dt}\int_0^\delta H_{oy}(t-u)h_x(u)du = \frac{\sqrt{\rho_{xx}\mu_{yy}}}{\pi}\sqrt{\delta}\,H'_{oy}(t),$$

$$\lim_{z\to 0}\frac{d}{dt}\int_\delta^t H_{oy}(t-u)h_x(u) = \lim_{z\to 0}\int_\delta^t H'_{oy}(t-u)h_x(u)du$$

$$= \frac{\sqrt{\rho_{xx}\mu_{yy}}}{2\pi}\int_\delta^t H'_{oy}(t-u)u^{-\frac{1}{2}}du\,.$$

Hence it becomes

$$E_{ox} = \frac{\sqrt{\rho_{xx}\mu_{yy}}}{2\pi}\left\{2\sqrt{\delta}\,H'_{oy}(t)+\int_\delta^t H'_{oy}(t-u)u^{-\frac{1}{2}}du\right\} = \frac{\sqrt{\rho_{xx}\mu_{yy}}}{2\pi}F_{ox}(t)\,.$$

In a similar way, we have (13)

$$E_{oy} = -\frac{\sqrt{\rho_{yy}\mu_{xx}}}{2\pi}\left\{2\sqrt{\delta}\,H'_{ox}(t)+\int_\delta^t H'_{ox}(t-u)u^{-\frac{1}{2}}du\right\} = -\frac{\sqrt{\rho_{yy}\mu_{xx}}}{2\pi}F_{oy}(t)\,.$$

Since the components which is parallel to the principal axes are obtained, we can calculate the components of any direction. For instance, the ξ- and η- components shown in Fig. 1 are calculated abbreviating the process of calculation. We have

$$E_{o\xi} = (P\cos 2\alpha + Q)F_{o\xi} - P\sin 2\alpha F_{o\eta}\,,$$

$$E_{o\eta} = -P\sin 2\alpha F_{o\xi} + (P\cos 2\alpha - Q)F_{o\eta}\,, \qquad (14)$$

where $P = \dfrac{\sqrt{\rho_{xx}}+\sqrt{\rho_{yy}}}{4\pi}\,, \quad Q = \dfrac{\sqrt{\rho_{xx}}-\sqrt{\rho_{yy}}}{4\pi}\,.$

and μ is assumed to be unity in electromagnetic unit.

As the magnetic field at the surface produced by the electric current flowing in the earth crust are in general very small[5)], $E_{o\xi}$ and $E_{o\eta}$

5) If we consider a current sheet in which the electric currents flow uniformly in one direction, the magnetic field produced by these currents are given by $H = 2\pi i$. Corresponding to to $H = 1\gamma$, i amounts to $1{\cdot}6\times 10^{-5}$ amp. If the thikness of the sheet is assumed to be 10 km, the current-density becomes $1{\cdot}6\times 10^{-9}$ amp. When we take 10 kilo-ohms as the value of the electrical resistivity, the electric field amounts to 1·6 v/km. Such a large variation is scarecely observed in earth potential. Thus we may say that the changes in the electric currents flowing in the earth crust are not parctically detected by the magnetic observation though it can be measured by the earth potential observation.

deduced from $H_{o\xi}$ and $H_{o\eta}$ can be regarded as the induced field by the changes in the external magnetic field. From this point of view, the differences between the observed earth potential and calculated

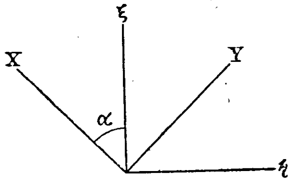

Fig. 1.

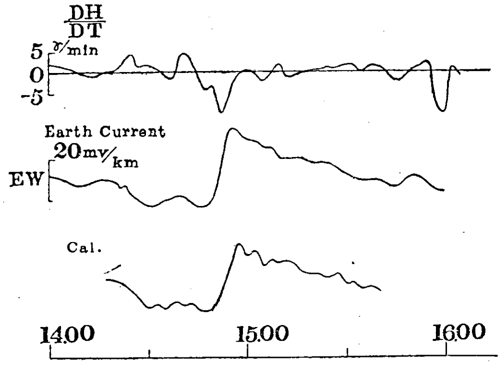

Fig. 2. An example of calculation. The calculated earth potential is shown in arbitrary scale.

one will give, if exist, anomalous changes due to electro-motive forces originated in the earth crust. Thus it is desirable to apply this method to changes which seem to relate to earthquakes

In order to show how well the observed and calculated earth potential coincide each other, for example, the change in earth current are calculated from the rate of changes in the earth's magnetic field measured at the time of solar eclipse Jun. 19, 1936 on the basis of the data obtained at Toyohara Magnetic Observatory where, according to H. Hatakeyama[6], anisotropy in the conductivity is comparatively small. Using the readings of every 2.5 min., the integrals are numerically calculated by means of Simpson's method. As shown in Fig. 2, the observed earth potential agrees well with the calculated one in their shapes.

The writer hopes that observations of earth potential and variability in the earth's magnetic field are both carried out in connection with earthquake occurrences. Applying, then, the method mentioned above, it may be expected to get some conclusions about occurrences of anomalous changes in earth current related to earthquakes.

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6) H. Hatakeyama, *Jonrn. Meteorol. Soc. Japan*, **17** (1939), 209.